CSE 373: Data Structures and Algorithms

Asymptotic Analysis

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Code Analysis

How do we compare two piece of code? Lots of metrics we could pick!

- Time needed to run
- Memory used
- Number of network calls made
- Amount of data we save to the disk
- Specialized vs. generic
- Code reusability
- Security

(Some metrics are intangible and hard to measure those, e.g., security, code reusability)

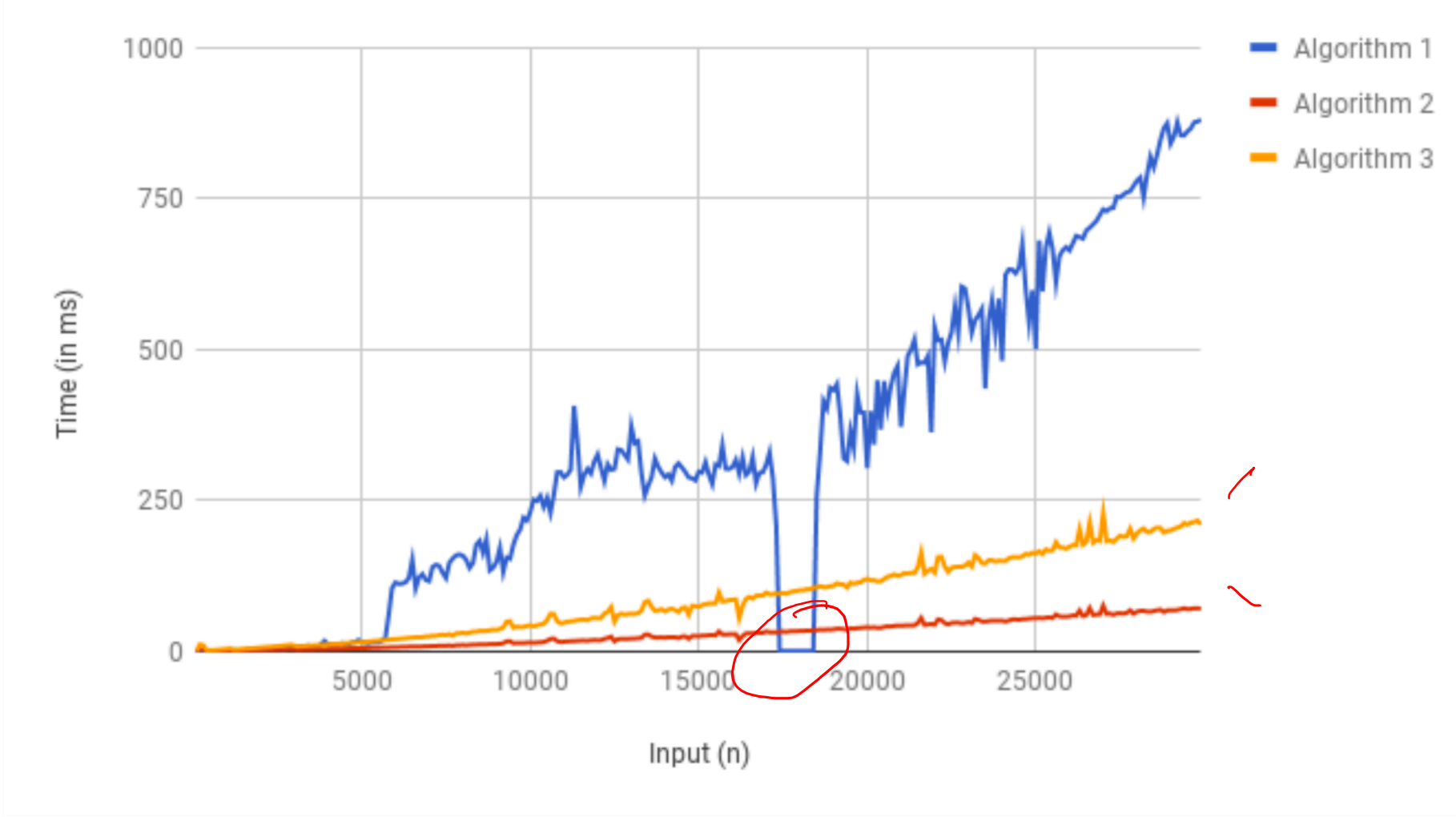
Today: Focus on comparing algorithms based on *how long it takes them to run in the worst case.*

Which of these algorithms is better?

Algorithm	Time (in ms)
Algorithm 1	1
Algorithm 2	30
Algorithm 3	100

This is a trick question. Why isn't this table enough to let us decide which algorithm is better?

Which of these algorithms is better?



Comparing Algorithms

We want:

- To see **overall** trends as input increases
  - Considering a single data point isn't helpful
  - We really care about large inputs
- Final result is independent of incidental factors
  - (CPU speed, programming language, other programs running, etc.)
- Rigorously discover overall trends without resorting to testing
  - What if we miss worst-case input?
- A way to analyze before coding!

What Are We Counting?

Worst case analysis

- For a given input size, what's the running time for the worst state our data structure we can be in or the worst input we can give?

Best case analysis

- What is the number of steps for the best state of our structure and the best question?

Average case analysis

- How are we doing on average over all possible inputs/states of our data structure?
- Have to ask this question very carefully to get a meaningful answer

We usually do worst case analysis.

Asymptotic Analysis: Two step process

1. **Model** what we care about as a mathematical function
2. **Analyze** that function using asymptotic analysis

Modeling: What Are We Counting?

Consecutive statements
- Sum of time of each statement

Function calls
- Time of function's body

Conditionals
- Time of condition + max(if branch, else branch)

Loops
- Number of iterations x time of loop body

Modeling: Assumptions

Assume basic operations take the same constant amount of time.

What's a basic operation?

- Adding ints or doubles
- Assignment
- Incrementing a variable
- A return statement
- Accessing an array index or an object field

What's not a basic operation?

- Making a method call.

This is a LIE but it's a very useful lie.

Modeling Case Study

Goal: return 'true' if a sorted array of ints contains duplicates

Solution 1: compare each pair of elements

```
public boolean hasDuplicate1(int[] array) {
   for (int i = 0; i < array.length; i++) {
      for (int j = 0; j < array.length; j++) {
         if (i != j && array[i] == array[j]) {
            return true;
         }
      }
   }
   return false;
}
```

Solution 2: compare each consecutive pair of elements

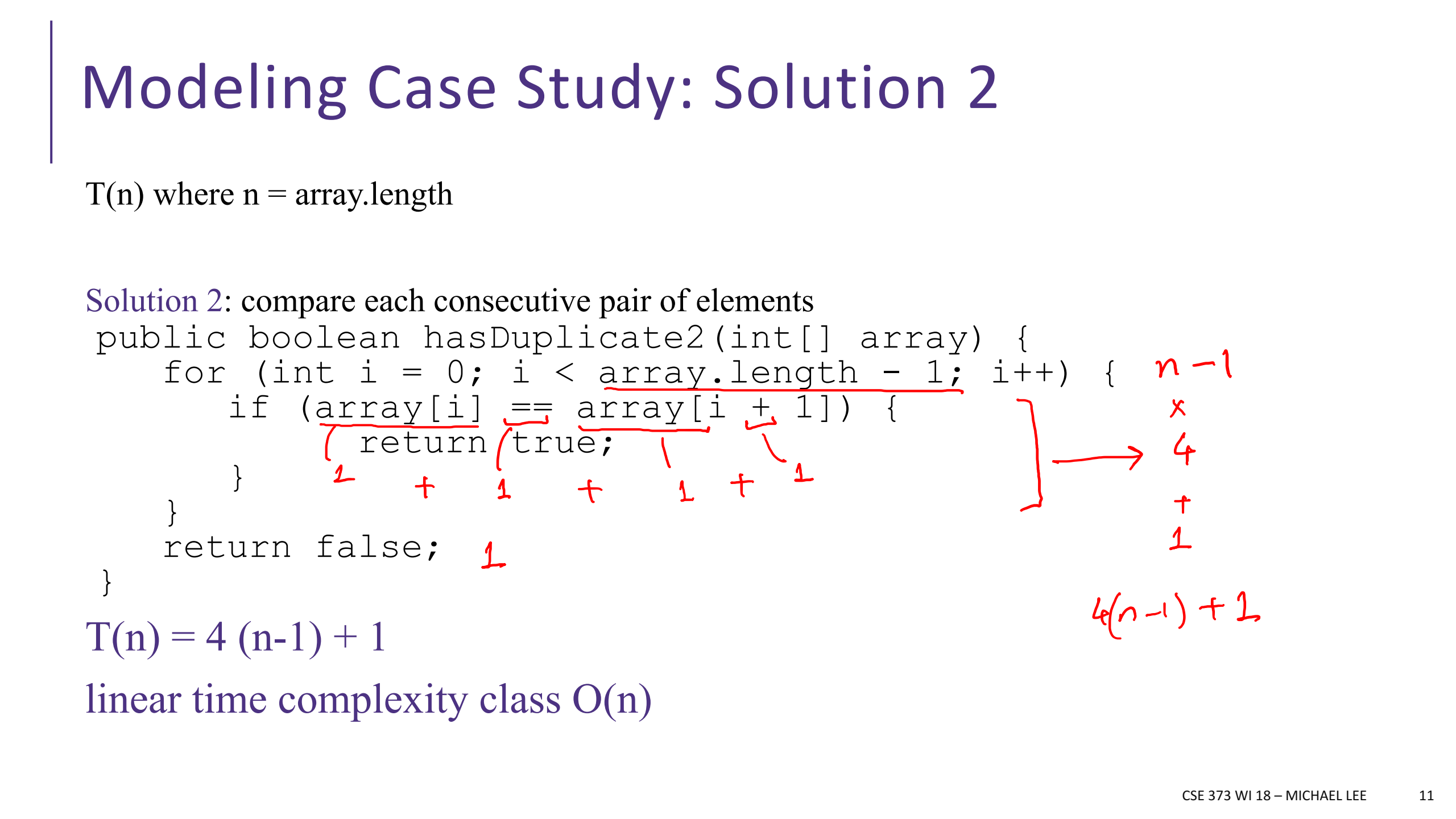
```
public boolean hasDuplicate2(int[] array) {
   for (int i = 0; i < array.length - 1; i++) {
      if (array[i] == array[i + 1]) {
            return true;
      }
   }
   return false;
}
```

Modeling Case Study: Solution 2

T(n) where n = array.length

Solution 2: compare each consecutive pair of elements

```
public boolean hasDuplicate2(int[] array) {
    for (int i = 0; i < array.length - 1; i++) {
        if (array[i] == array[i + 1]) {
            return true;
        }
    }
    return false;
}
```



$T(n) = 4 (n-1) + 1$

linear time complexity class O(n)

Modeling Case Study: Solution 1

Solution 1: compare each consecutive pair of elements

```
public boolean hasDuplicate1(int[] array) {
    for (int i = 0; i < array.length; i++) {
        for (int j = 0; j < array.length; j++) {
            if (i != j && array[i] == array[j]) {
                return true;
            }
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    }
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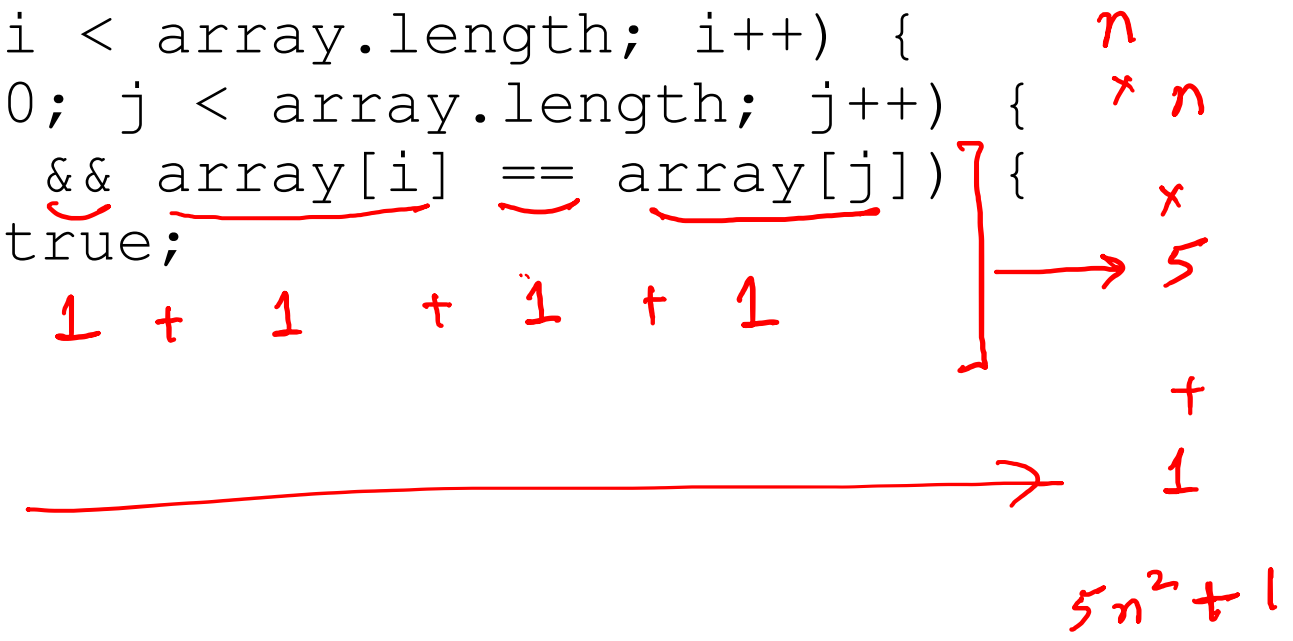

$T(n) = 5\ n^2 + 1$

quadratic time complexity class $O(n^2)$

Asymptotic Analysis: Two step process

1. **Model** what we care about as a mathematical function

2. **Analyze** that function using asymptotic analysis
   - Specifically: have a way to compare two functions
   - Even more specifically: define a "less then or equal to" operator for functions