Quickcheck 02: Solutions

Name:

Definition: Dominated by

A function $f(n)$ is dominated by $g(n)$ when...

- There exists two constants $c > 0$ and $n_0 > 0$...
- Such that for all values of $n \geq n_0$...
- $f(n) \leq c \cdot g(n)$ is true.

Demonstrate that $2n^3 - 3 + 9n^2 + \sqrt{n}$ is dominated by n^3 by finding a c and n_0. Show your work.

Solution:

We'll go term by term in the first function.

$2n^3 \leq 2 \cdot n^3$ for all n.

$-3 \leq 0 \cdot n^3$ for all n.

$9n^2 \leq 1 \cdot n^3$ for all $n \geq 9$

$\sqrt{n} \leq 1 \cdot n^3$ for all $n \geq 1$ (since $n \geq 1$ means $n^{2.5} \geq 1$).

All inequalities are true as long as $n \geq \max\{0, 0, 9, 1\} = 9$. If all the inequalities are true we can sum them to get

$$2n^3 - 3 + 9n^2 + \sqrt{n} \leq (2 + 0 + 1 + 1)n^3 = 4n^3$$

for all $n \geq 9$.

Thus we take $n_0 = 9$ and $c = 4$. This is not the only solution; many others are possible.