CSE 373 18au: Final Exam Reference Sheet

Splitting a sum

$$\sum_{i=a}^{b}(x+y) = \sum_{i=a}^{b} x + \sum_{i=a}^{b} y$$

Gauss's identity

$$\sum_{i=0}^{n-1} i = 0 + 1 + \ldots + n - 1 = \frac{n(n-1)}{2}$$

Factoring out a constant

$$\sum_{i=a}^{b} cf(i) = c\sum_{i=a}^{b} f(i)$$

Finite geometric series

$$\sum_{i=0}^{n-1} r^i = \frac{r^n - 1}{r - 1}$$

Log of a product

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

Log of a fraction

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

Log of a power

$$\log_b(x^y) = y \cdot \log_b(x)$$

Power of a log

$$x^{\log_b(y)} = y^{\log_b(x)}$$

Change-of-base identity

$$\log_a(n) = \frac{\log_b(n)}{\log_b(a)}$$

Log of the same base

$$\log_b(b) = 1$$

Master theorem

Given a recurrence of the form:

$$T(n) = \begin{cases} d & \text{if } n = 1 \\ aT\left(\frac{n}{b}\right) + n^c & \text{otherwise} \end{cases}$$

We know that:

- If $\log_b(a) < c$ then $T(n) \in \Theta\left(n^c\right)$
- If $\log_b(a) = c$ then $T(n) \in \Theta\left(n^c \log(n)\right)$
- If $\log_b(a) > c$ then $T(n) \in \Theta\left(n^{\log_b(a)}\right)$