CSE 373: Data Structures & Algorithms

Pseudocode; ADTs; Priority Queues; Heaps

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Course Logistics

- HW 1 released Monday. Due a week from Tuesday.
- Java Review Session early next week, room and time TBA and posted on the course website
- Slides posted and updated from last time with links correctly working in PDF version

Pseudocode

Describe an algorithm in the steps necessary, write the shape of the code but ignore specific syntax.

Algorithm: Count all elements in a list greater than x

Pseudocode:

```
int counter // keeps track of number > x
while list has more elements {
   increment counter if current element is > than x
   move to next element of list
}
```

More Pseudocode

Algorithm: Given a list of names in the format "firstName lastName", make a Map of all first names as keys with sets of last names as their values

Pseudocode:

```
create the empty result map
while list has more names to process {
    firstName is name split up until space
    lastName is name split from space to the end
    if firstName not in the map yet {
       put firstName in map as a key with an empty
       set as the value
    }
    add lastName to the set for the first name
    move to the next name in the list
}
```

Pseudocode Practice

Come up with pseudocode for the following algorithm:

Algorithm: Given a list of integers, find the index of the maximum integer in the list.

Pseudocode Practice Solution

Algorithm: Given a list of integers, find the index of the maximum integer in the list.

```
if list is not empty:
  int maxIndex starts at 0 for first index
  for each index i in the list:
    if the element at index i is greater than
    the element at index maxIndex:
      reset maxIndex to i
  return maxIndex
else:
  error case: return -1? throw exception?
```

Terminology Review

- Abstract Data Type (ADT)
  - Mathematical description of a "thing" with set of operations
- Algorithm
  - A high level, language-independent description of a step-by-step process
- Data structure
  - A specific organization of data and family of algorithms for implementing an ADT
- Implementation of a data structure
  - A specific implementation in a specific language

Another ADT: Priority Queue

A **priority queue** holds *comparable data*

- Given *x* and *y*, is *x* less than, equal to, or greater than *y*
- Meaning of the ordering can depend on your data
- Many data structures require this: dictionaries, sorting
- Typically elements are comparable types, or have two fields: the *priority* and the *data*

Priority Queue vs Queue

Queue: follows First-In-First-Out ordering

Example: serving customers at a pharmacy, based on who got there first.

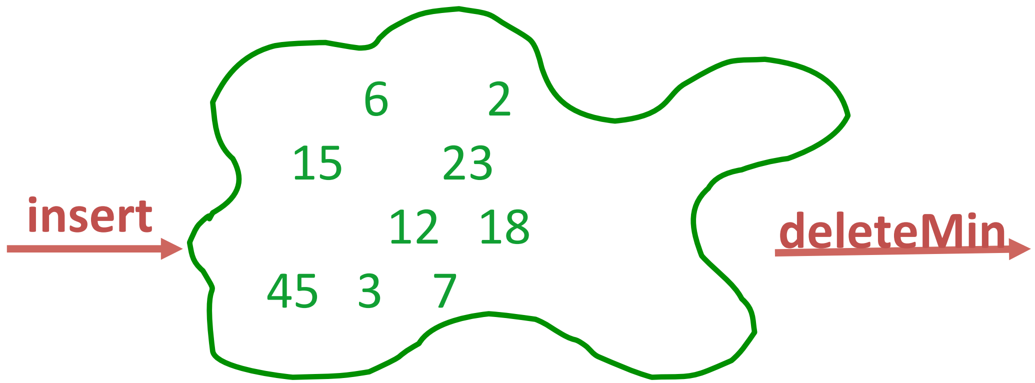
Priority Queue: compares priority of elements to determine ordering

Example: emergency room, serves patients with priority based on severity of wounds

Priorities

- Each item has a "priority"
  - The *lesser* item is the one with the *greater* priority
  - **So "priority 1" is more important than "priority 4"**
  - Can resolve ties arbitrarily

- Operations:
  - **`insert`**
  - **`deleteMin`**
  - **`is_empty`**



- **`deleteMin`** *returns* and *deletes* the item with greatest priority (lowest priority value)
- **`insert`** is like **`enqueue`**, **`deleteMin`** is like **`dequeue`**
  - But the whole point is to use priorities instead of FIFO

Priority Queue Example

Given the following, what values are **a**, **b**, **c** and **d**?

`insert` *element1* with priority *5*

`insert` *element2* with priority *3*

`insert` *element3* with priority *4*

a = **`deleteMin`** `// a = ?`

b = **`deleteMin`** `// b = ?`

`insert` *element4* with priority *2*

`insert` *element5* with priority *6*

c = **`deleteMin`** `// c = ?`

d = **`deleteMin`** `// d = ?`

Priority Queue Example Solutions

`insert` *element1* with priority *5*

`insert` *element2* with priority *3*

`insert` *element3* with priority *4*

a = **`deleteMin`** `// a = element2`

b = **`deleteMin`** `// b = element3`

`insert` *element4* with priority *2*

`insert` *element5* with priority *6*

c = **`deleteMin`** `// c = element4`

d = **`deleteMin`** `// d = element1`

Some Applications

- Run multiple programs in the operating system
  - "critical" before "interactive" before "compute-intensive", or let users set priority level
- Select print jobs in order of decreasing length
- "Greedy" algorithms (we'll revisit this idea)
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (Huffman CSE 143)
- Sorting (first **`insert`** all, then repeatedly **`deleteMin`**)

Possible Implementations

- Unsorted Array
  - **`insert`** by inserting at the end
  - **`deleteMin`** by linear search

- Sorted Circular Array
  - **`insert`** by binary search, shift elements over
  - **`deleteMin`** by moving "front"

More Possible Implementations

- Unsorted Linked List
  - **`insert`** by inserting at the front
  - **`deleteMin`** by linear search

- Sorted Linked List
  - **`insert`** by linear search
  - **`deleteMin`** remove at front

- Binary Search Tree
  - **`insert`** by search traversal
  - **`deleteMin`** by find min traversal

One Implementation: Heap

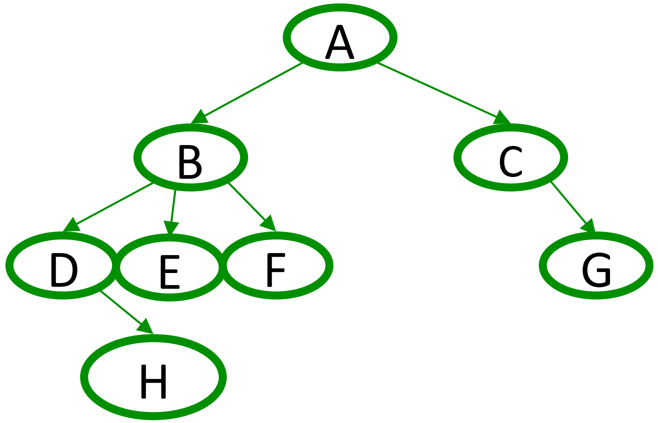
Heaps are implemented with Trees

A *binary min-heap* (or just *binary heap* or *heap*) is a data structure with the properties:

- Structure property: A *complete* binary tree
- Heap property: The priority of every (non-root) node is greater than the priority of its parent
  - ***Not*** a binary search tree

Tree Review

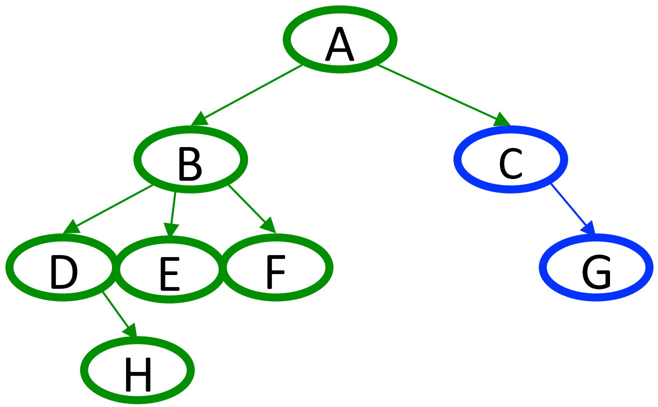
- root of tree:
- leaves of tree:
- children of B:
- parent of C:
- subtree C:
- height of tree:
- height of E:
- depth of E:
- degree of B:



- perfect tree:
- complete tree:

Tree Review

- root of tree: A
- leaves of tree: H,E,F,G
- children of B: D, E, F
- parent of C: A
- subtree C: in blue
- height of tree: 3
- height of E: 0
- depth of E: 2
- degree of B: 3

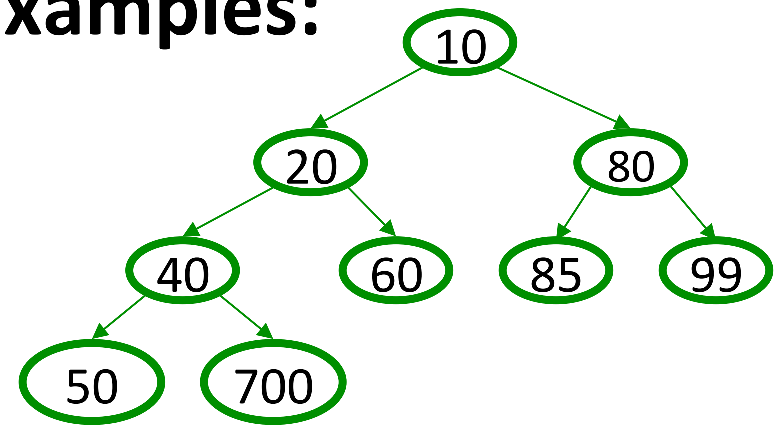


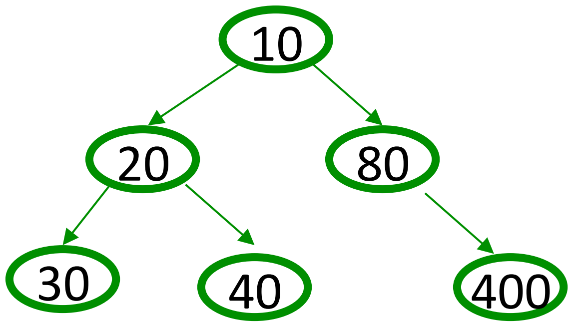
- perfect tree:
  every level is completely full
- complete tree:
  all levels full, with a possible exception being the bottom level, which is filled left to right

Structure Property: Completeness

- A Binary Heap is a **complete** binary tree:
  - A binary tree with all levels full, with a possible exception being the bottom level, which is filled left to right

Examples:





are these trees *complete*?

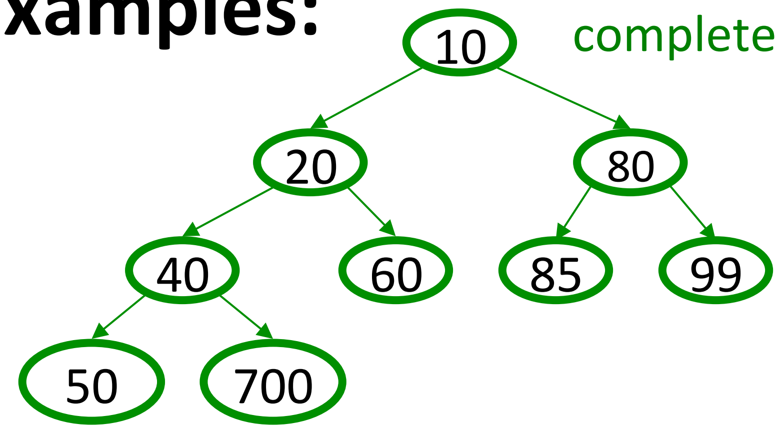
Structure Property: Completeness

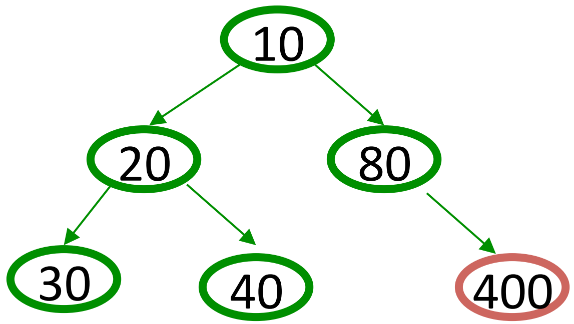
- A Binary Heap is a **complete** binary tree:
  – A binary tree with all levels full, with a possible exception being the bottom level, which is filled left to right

incomplete

Examples:

complete





Heap Order Property

- The priority of every (non-root) node is greater than (or equal to) that of it's parent. AKA the children are always greater than the parents.

which of these follow the *heap order property*?

Heap Order Property

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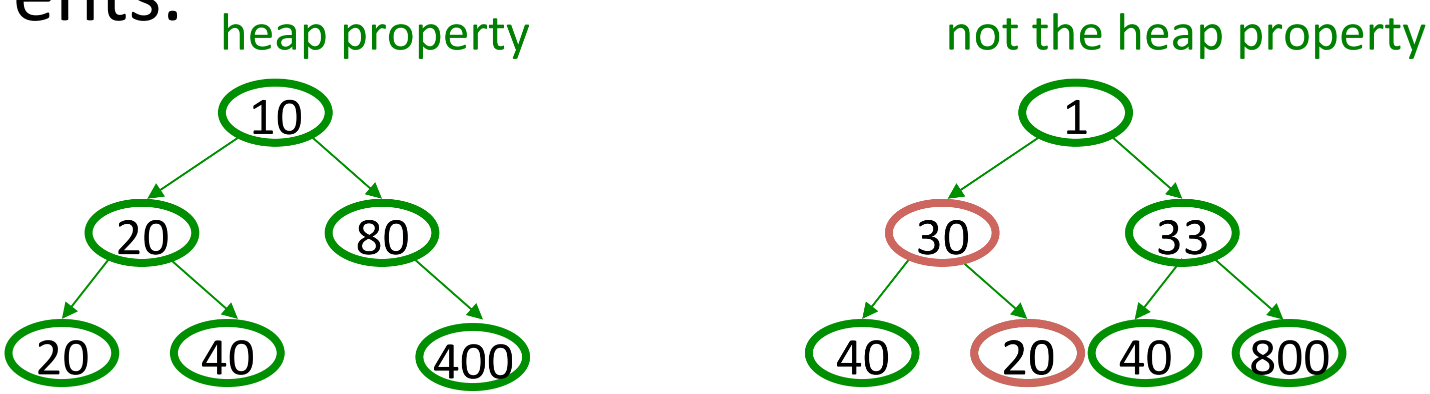
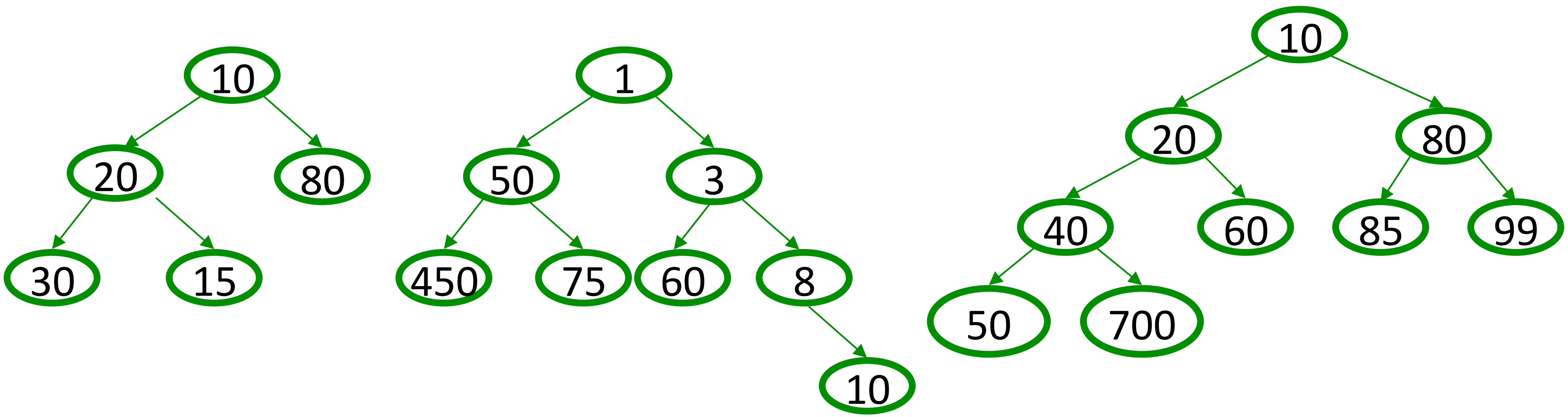


Heaps

A *binary min-heap* (or just *binary heap* or just *heap*) is:

- Structure property: A *complete* binary tree
- Heap property: The priority of every (non-root) node is greater than (or equal to) the priority of its parent. AKA the children are always greater than the parents.
  - ***Not*** a binary search tree



which of these are *heaps*?

Heaps

A *binary min-heap* (or just *binary heap* or just *heap*) is:

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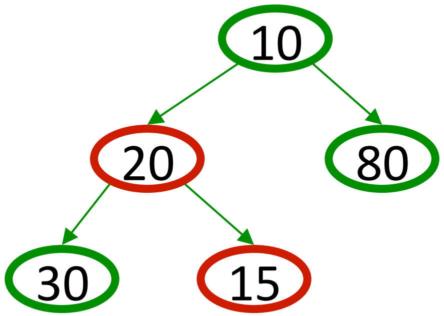
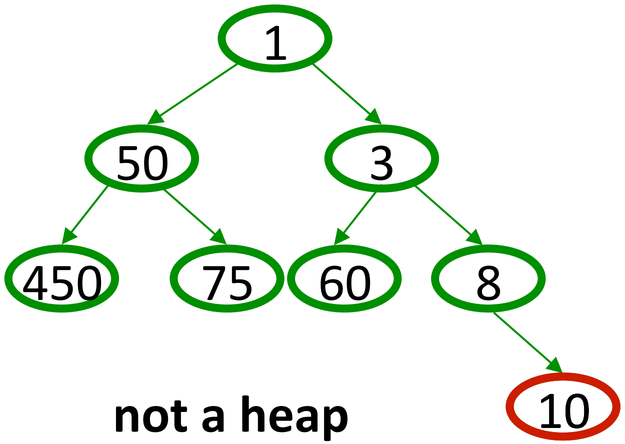
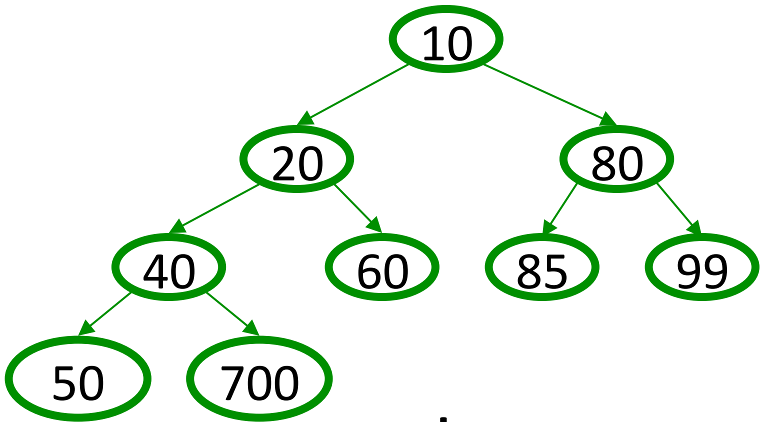


not a heap



not a heap



a heap

Heaps

- Where is the highest-priority item?

- What is the height of a heap with *n* items?

- How do we use heaps to implement the operations in a Priority Queue ADT?

Heaps

- Where is the highest-priority item?

    At the root (at the top)

- What is the height of a heap with *n* items

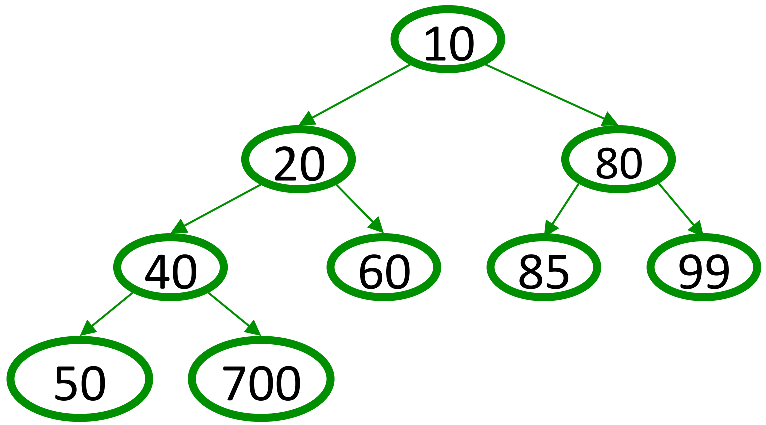
    $\log_2 n$ (We'll look at computing this next week)

- How do we use heaps to implement the operations in a Priority Queue ADT?

    See following slides

Operations: basic idea

- **`deleteMin`**:
  1. **`answer = root.data`**
  2. Move right-most node in last row to root to restore structure property
  3. "Percolate down" to restore heap property
- **`insert:`**
  1. Put new node in next position on bottom row to restore structure property
  2. "Percolate up" to restore heap property

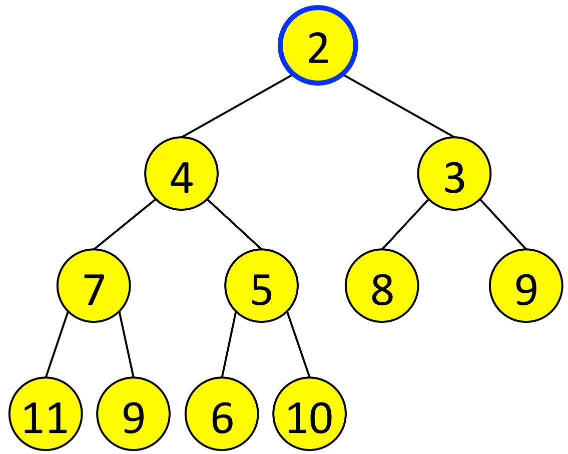


Overall strategy:

- *Preserve structure property*
- *Break and restore heap property*

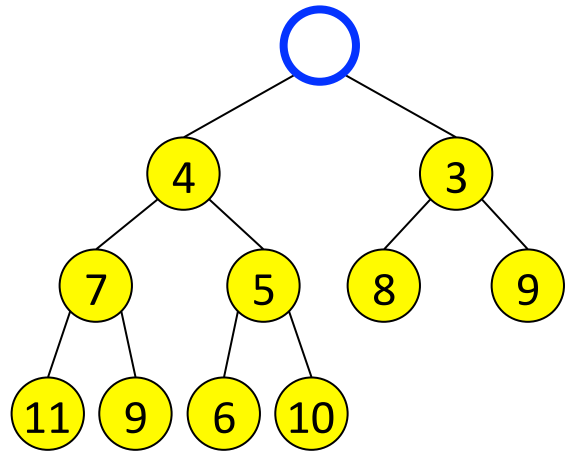
deleteMin

1. Delete (and later return) value at root node

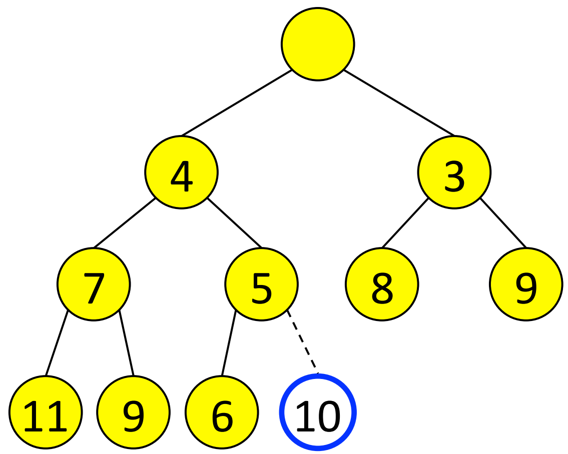


2. Restore the Structure Property

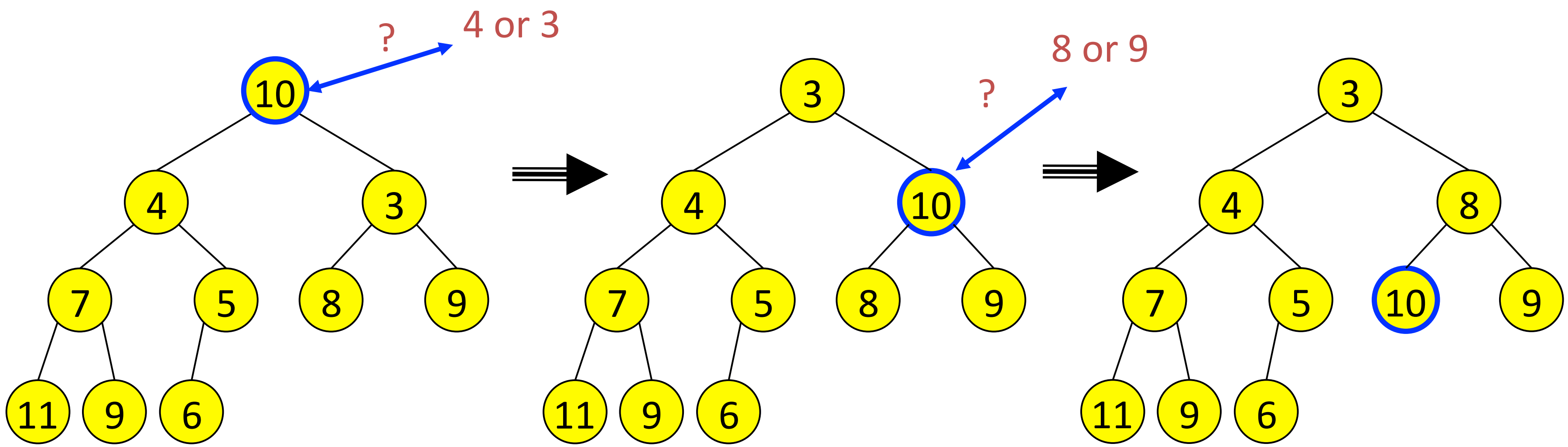
- We now have a "hole" at the root
  - Need to fill the hole with another value



- When we are done, the tree will have one less node and must still be complete



3. Restore the Heap Property

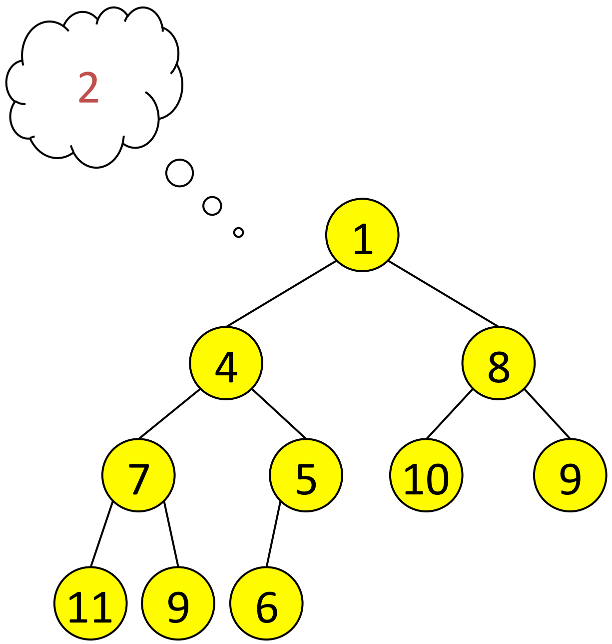


Percolate down:

- Keep comparing with both children
- Swap with lesser child and go down one level
    - What happens if we swap with the larger child?
- Done if both children are ≥ item or reached a leaf node

Insert

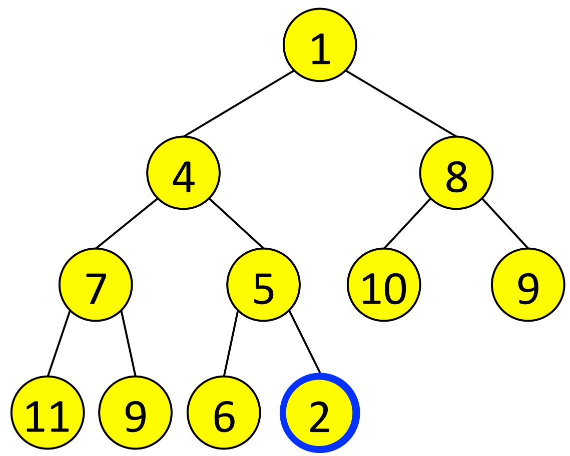
- Add a value to the tree
- Afterwards, structure and heap properties must still be correct
- Where do we insert the new value?



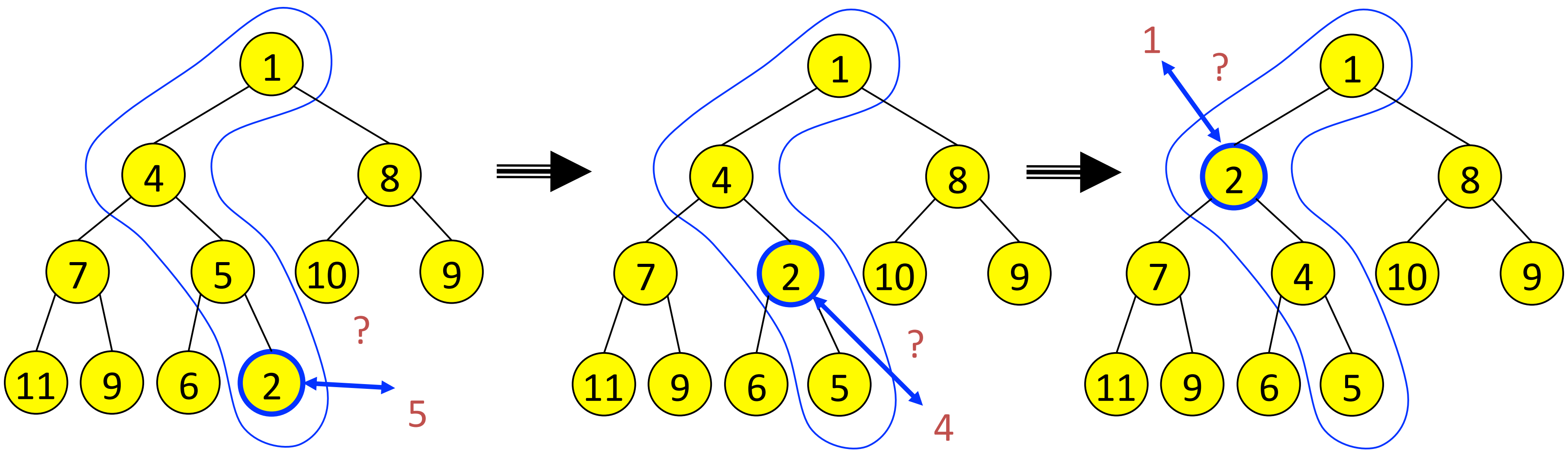
Insert: Maintain the Structure Property

- There is only one valid tree shape after we add one more node

- So put our new data there and then focus on restoring the heap property



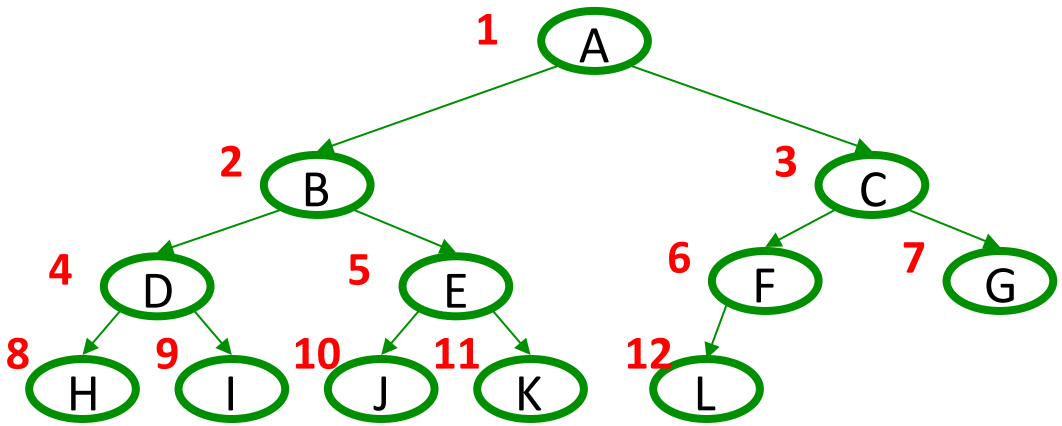
Maintain the heap property



Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent ≤ item or reached root

Array Representation of Binary Trees



From node `i`:

left child: `i*2`
right child: `i*2+1`
parent: `i/2`

(wasting index 0 is convenient for the index arithmetic)

implicit (array) implementation:

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Judging the array implementation

Plusses:

- Less "wasted" space
  - Just index 0 and unused space on right
  - In conventional tree representation, one edge per node (except for root), so *n*-1 wasted space (like linked lists)
  - Array would waste more space if tree were not complete
- Multiplying and dividing by 2 is very fast (shift operations in hardware)
- Last used position is just index **`size`**

Minuses:

- Same might-be-empty or might-get-full problems we saw with stacks and queues (resize by doubling as necessary)

Plusses outweigh minuses: "this is how people do it"