CSE 373: Data Structures and Algorithms

Lecture 19: Comparison Sorting Algorithms

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Today

- Intro to sorting
- Comparison sorting
  - Insertion Sort
  - Selection Sort
  - Heap Sort
  - Merge Sort

Mini-Announcements

- Homework 4 due today

- Homework 5 coming out today, due Friday 5:00pm
  - Can get started using material covered today
  - Can complete using material covered by Monday

Sorting

Now looking at algorithms instead of data structures!

Introduction to Sorting

- Stacks, queues, priority queues, and dictionaries all focused on providing one element at a time
- But often we know we want "all the things" in some order
  - Humans can sort, but computers can sort fast
  - Very common to need data sorted somehow
    - Alphabetical list of people
    - List of countries ordered by population
    - Search engine results by relevance
    - List store catalogue by price
    - …
- Algorithms have different asymptotic and constant-factor trade-offs
  - No single "best" sort for all scenarios
  - Knowing one way to sort just isn't enough

- sorting midterms by name
- sorting dates into chronological order

More Reasons to Sort

General technique in computing:

Preprocess data to make subsequent operations faster

Example: Sort the data so that you can

- Find the k^{th} largest in constant time for any k
- Perform binary search to find elements in logarithmic time

Whether the performance of the preprocessing matters depends on

- How often the data will change (and how much it will change)
- How much data there is

The main problem, stated carefully

For now, assume we have *n* comparable elements in an array and we want to rearrange them to be in increasing order

Input:

- An array `A` of data records
- A key value in each data record
- A comparison function

Effect:

- Reorganize the elements of `A` such that for any `i` and `j`, if `i < j` then $A[i] < A[j]$
- (Also, `A` must have exactly the same data it started with)
- Could also sort in reverse order, of course

An algorithm doing this is a **comparison sort**

Variations on the Basic Problem

1. Maybe elements are in a linked list (could convert to array and back in linear time, but some algorithms needn't do so)
2. Maybe ties need to be resolved by "original array position"
   - Sorts that do this naturally are called stable sort
3. Maybe we must not use more than $O(1)$ "auxiliary space"
   - Sorts meeting this requirement are called in-place sort
4. Maybe we can do more with elements than just compare
   - Sometimes leads to faster algorithms
5. Maybe we have too much data to fit in memory
   - Use an " external " algorithm

Sorting: The Big Picture

Surprising amount of neat stuff to say about sorting:

Simple algorithms: $O(n^2)$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort	Heap sort		Bucket sort	External sorting
Selection sort	Merge sort		Radix sort	
Shell sort	Quick sort			
…	…			

Real-world example demo time!

Help me sort some cards!

Insertion Sort

- Idea: At step `k`, put the `k`th element in the correct position among the first `k` elements

- Alternate way of saying this:
  - Sort first two elements
  - Now insert 3rd element in order
  - Now insert 4th element in order
  - …

- "Loop invariant": when loop index is `i`, first `i` elements are sorted

- Time?

  Best-case $O(n)$ almost sorted   Worst-case $O(n^2)$ sorted in reverse   "Average" case $O(n^2)$ (see text)

Selection sort

Always

$T(1) = 1$

$T(n) = n + T(n-1)$

- Idea: At step `k`, find the smallest element among the not-yet-sorted elements and put it at position k
- Alternate way of saying this:
  - Find smallest element, put it 1st
  - Find next smallest element, put it 2nd
  - Find next smallest element, put it 3rd …
- "Loop invariant": when loop index is `i`, first `i` elements are the `i` smallest elements in sorted order
- Time?

  Best-case $O(n^2)$ Worst-case $O(n^2)$ "Average" case $O(n^2)$

Insertion Sort vs. Selection Sort

- Different algorithms
- Solve the same problem
- Have the same worst-case and average-case asymptotic complexity
  - Insertion-sort has better best-case complexity; preferable when input is "mostly sorted"
- Other algorithms are more efficient *for large arrays that are not already almost sorted*
  - Insertion sort may do well on small arrays

The Big Picture

Surprising amount of juicy computer science: 2-3 lectures…

Simple algorithms: $O(n^2)$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort Selection sort Shell sort …	Heap sort Merge sort Quick sort (avg) …		Bucket sort Radix sort	External sorting

Heap sort

- Sorting with a heap:
  - `insert` each `arr[i]`, or better yet use `buildHeap`
  - `for(i=0; i < arr.length; i++)`
    `arr[i] =`
- Worst-case running time: $O(n \log n)$
- We have the array-to-sort and the heap
  - So this is not an in-place sort
  - There's a trick to make it in-place...

But this reverse sorts – how would you fix that?

In-place heap sort

maxHeap

- Treat the initial array as a heap (via `buildHeap`)
- When you delete the `i`th element, put it at `arr[n-i]`
  - That array location isn't needed for the heap anymore!

4	7	5	9	8	6	10	3	2	1

heap part | sorted part

(min)

`arr[n-i]= deleteMin()`

5	7	6	9	8	10	4	3	2	1

heap part | sorted part

“AVL sort”

- We can also use a balanced tree to:
  - `insert` each element: total time $O(n \log n)$
  - Repeatedly `deleteMin`: total time $O(n \log n)$
    - Better: in-order traversal $O(n)$, but still $O(n \log n)$ overall

- Compared to heap sort
  - both are $O(n \log n)$ in worst, best, and average case
  - neither parallelizes well
  - heap sort is can be done in-place, has better constant factors

Better Space efficiency

Design decision: which would you choose between Heap Sort and AVL Sort? Why?

“Hash sort”???

Nope!

Finding min item in a hashtable is *O*(n), so this would be a slower, more complicated selection sort

already terrible

Divide and conquer

Very important technique in algorithm design

1. Divide problem into smaller parts

2. Independently solve the simpler parts
   - Think recursion
   - Or parallelism

3. Combine solution of parts to produce overall solution

Two great sorting methods are fundamentally divide-and-conquer
(Merge Sort & Quicksort)

Merge Sort

Merge Sort: recursively…

- Sort the left half of the elements
- Sort the right half of the elements
- Merge the two sorted halves into a sorted whole

Real-world example demo time!

Help me sort some cards!

Merge sort

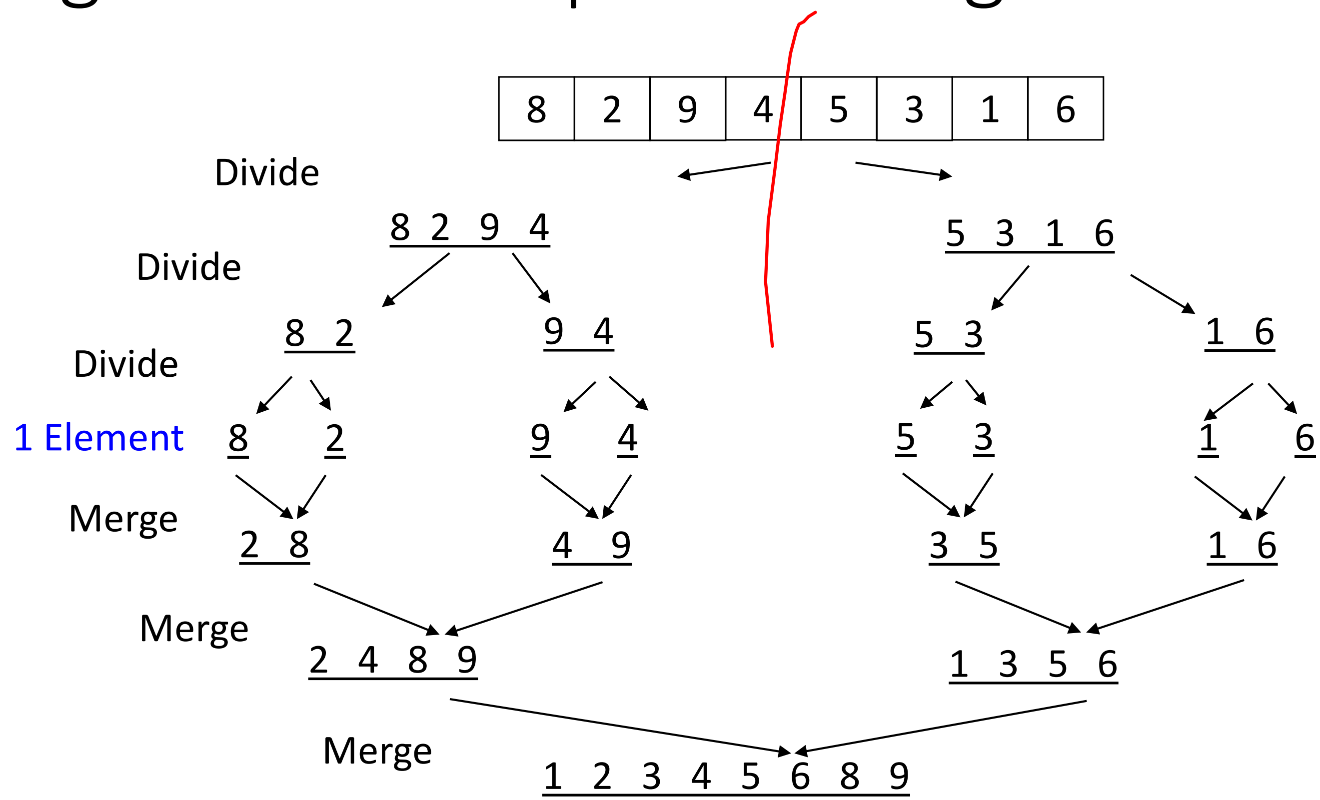
8	2	9	4	5	3	1	6

- To sort array from position `lo` to position `hi`:
  - If range is 1 element long, it is already sorted! ← Base Case
  - Else:
    - Sort from `lo` to `(hi+lo)/2`
    - Sort from `(hi+lo)/2` to `hi`
    - Merge the two halves together
- Merging takes two sorted parts and sorts everything
  - $O(n)$ but requires auxiliary space…

Merge Sort: Example focused on merging

Start with:	8	2	9	4	5	3	1	6	Main array
After recursion: (not magic ☺)	2	4	8	9	1	3	5	6	Main array
Merge: Use 3 “fingers” and 1 more array	1	2	3	4	5	6	8	9	Auxiliary array
(After merge, copy back to original array)	1	2	3	4	5	6	8	9	Main array

Merge Sort: Example showing recursion

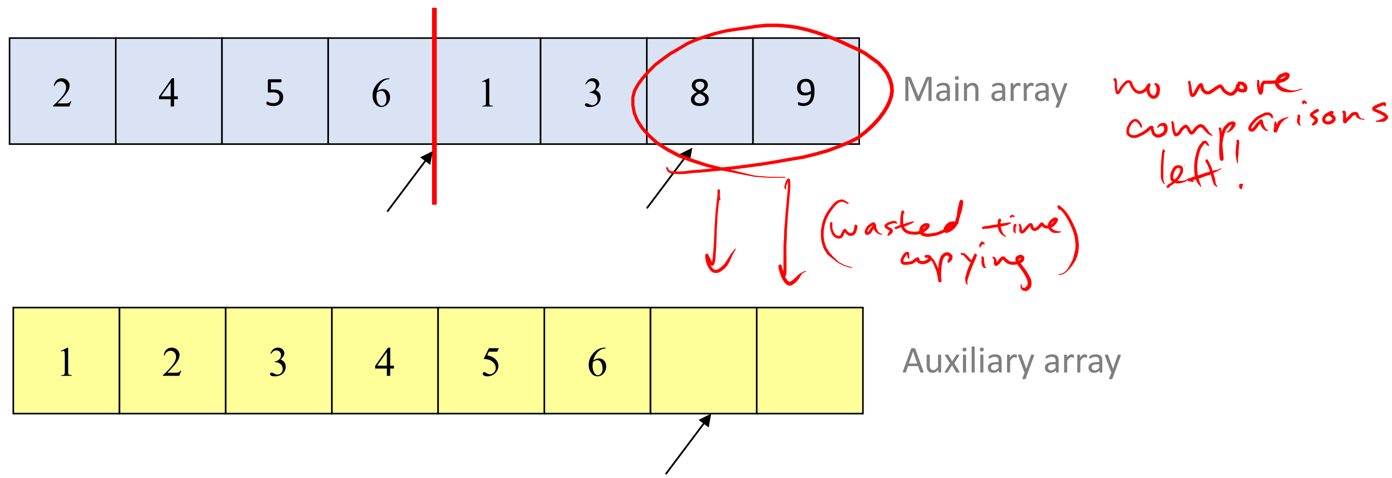


One way to practice on your own time:

- Make yourself an unsorted array
- Try using one of the sorting algorithms on it
- You know you got the right end result if it comes out sorted
- Can use the same example for merge sort as the previous slide to double check in-between steps

Some details: saving a little time

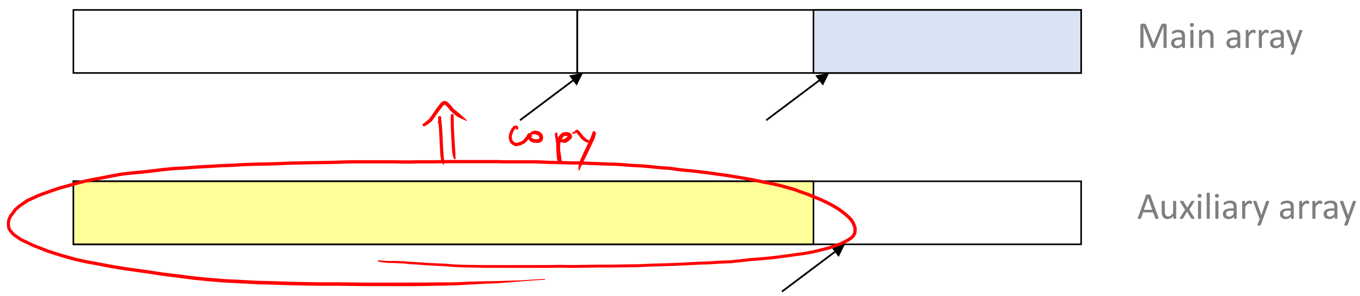
- What if the final steps of our merge looked like this:



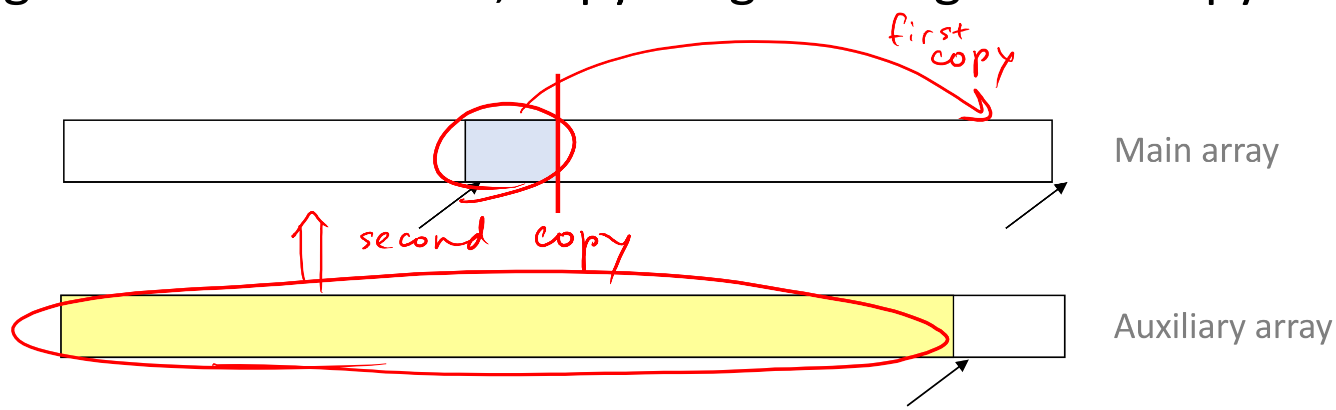
- Wasteful to copy to the auxiliary array just to copy back…

Some details: saving a little time

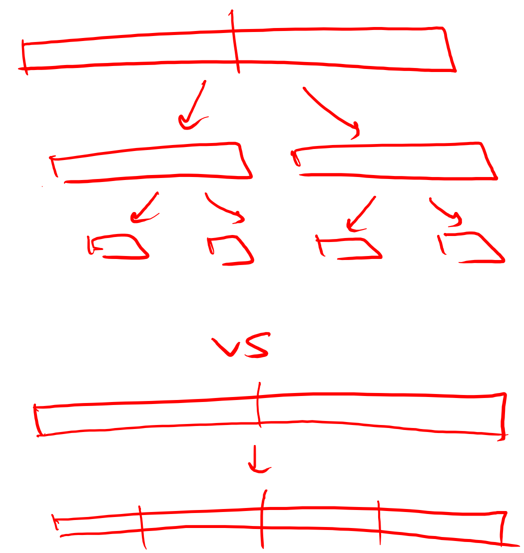
- If left-side finishes first, just stop the merge and copy back:



- If right-side finishes first, copy dregs into right then copy back



Some details: saving space and copying



Simplest / Worst:

Use a new auxiliary array of size `(hi-lo)` for every merge

Better:

Use a new auxiliary array of size `n` for every merging stage

Better:

Reuse same auxiliary array of size `n` for every merging stage

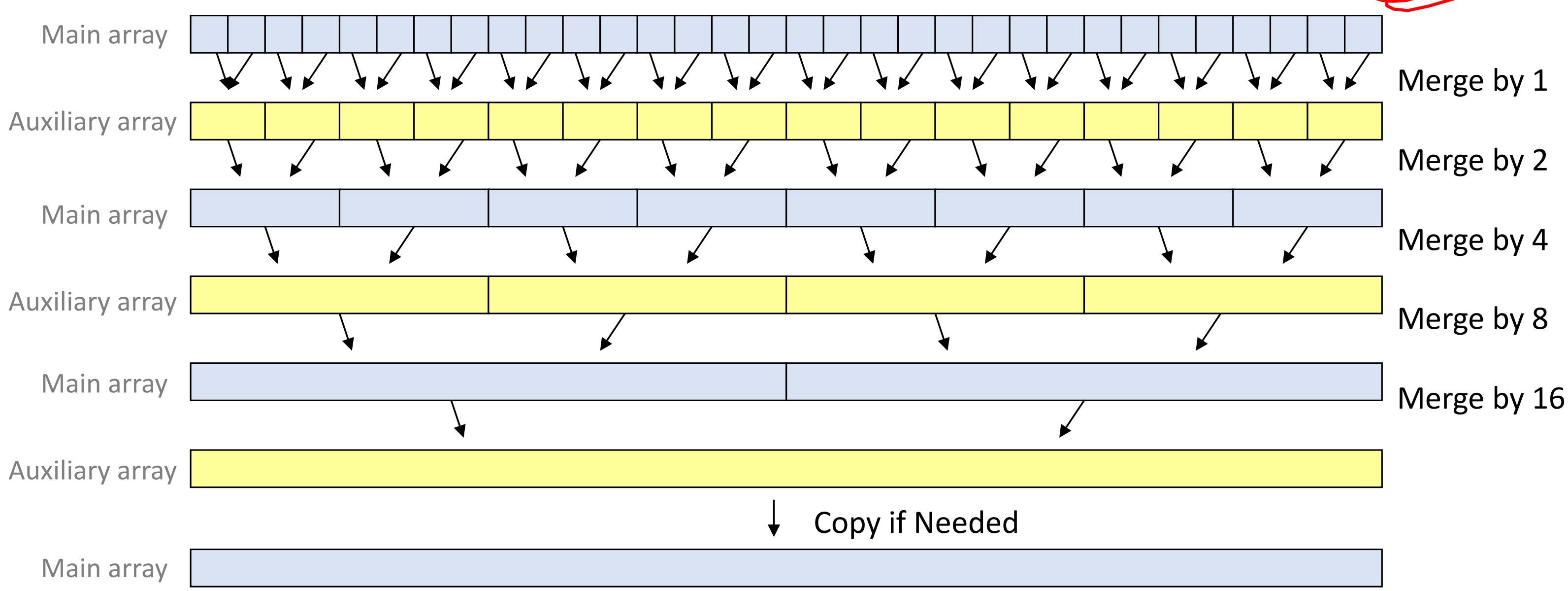
Best (but a little tricky):

Don't copy back – at 2^{nd}, 4^{th}, 6^{th}, … merging stages, use the original array as the auxiliary array and vice-versa

- Need one copy at end if number of stages is odd

Swapping Original / Auxiliary Array ("best")

- First recurse down to lists of size 1
- As we return from the recursion, swap between arrays



(Arguably easier to code up without recursion at all)

Cool Resources

- http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
- http://www.sorting-algorithms.com/
- https://www.youtube.com/watch?v=t8g-iYGHpEA