CSE 373: Data Structures and Algorithms

Lecture 11: Finish AVL Trees; Priority Queues; Binary Heaps

Instructor: Lilian de Greef
Quarter: Summer 2017

Today

- Announcements
- Finish AVL Trees
- Priority Queues
- Binary Heaps

Announcements

- Changes to Office Hours: from now on…
  - **No** more Wednesday morning & **shorter** Wednesday afternoon office hours
  - **New** Thursday office hours!
  - Kyle's Wednesday hour is now 1:30-**2:30**pm
  - Ben's office hours is now Thursday 1:**00**-3:**00**pm

- AVL Tree Height
  - In section, computed that minimum # nodes in AVL tree of a certain height is $S(h) = 1 + S(h-1) + S(h-2)$ where h = height of tree
  - Posted a proof next to these lecture slides online for your perusal

Announcements

- Midterm
  - Next Friday! (at usual class time & location)
  - Everything we cover in class until exam date is fair game (minus clearly-marked "bonus material"). That includes next week's material!
  - Today's hw3 due date designed to give you time to study.

- Course Feedback
  - Heads up: official UW-mediated course feedback session for part of Wednesday
  - Also want to better understand an anonymous concern on course pacing → Poll

The AVL Tree Data Structure

An **AVL tree** is a *self-balancing* binary search tree.

Structural properties

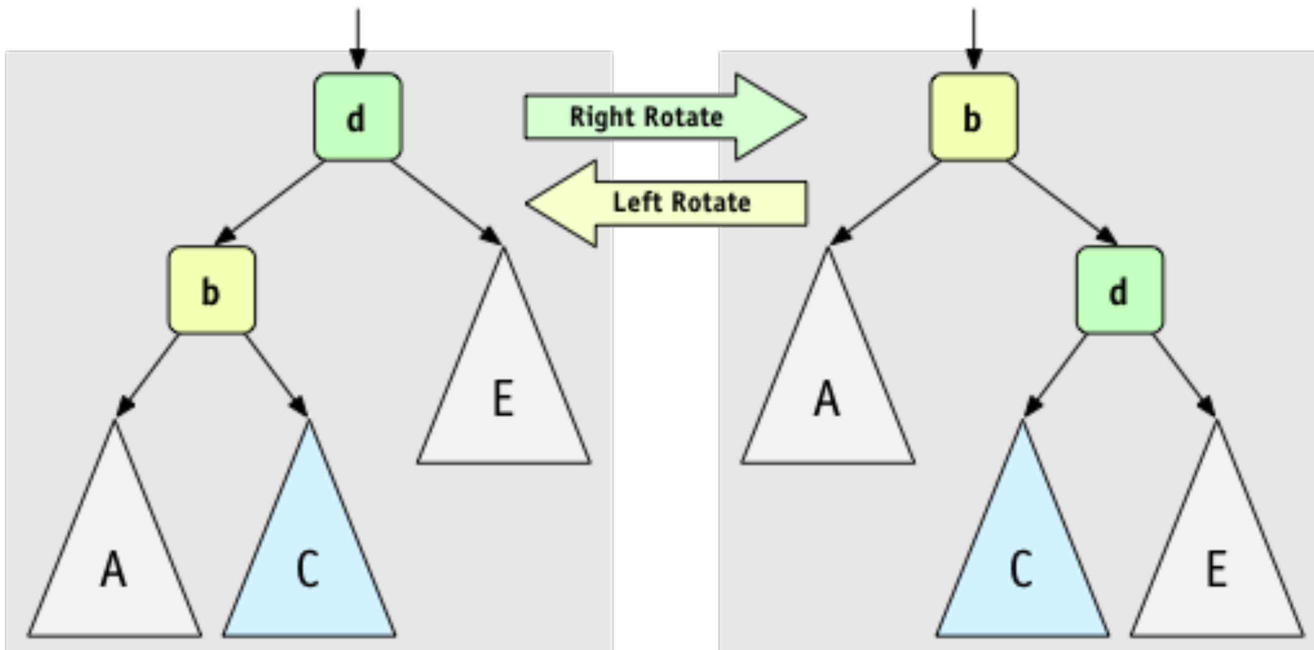
1. Binary tree property (same as BST)
2. Order property (same as for BST)

3. Balance condition:
   balance of every node is between -1 and 1

   where **balance**(*node*) = height(*node*.left) – height(*node*.right)

Result: **Worst-case** depth is O(log n)

Single Rotations



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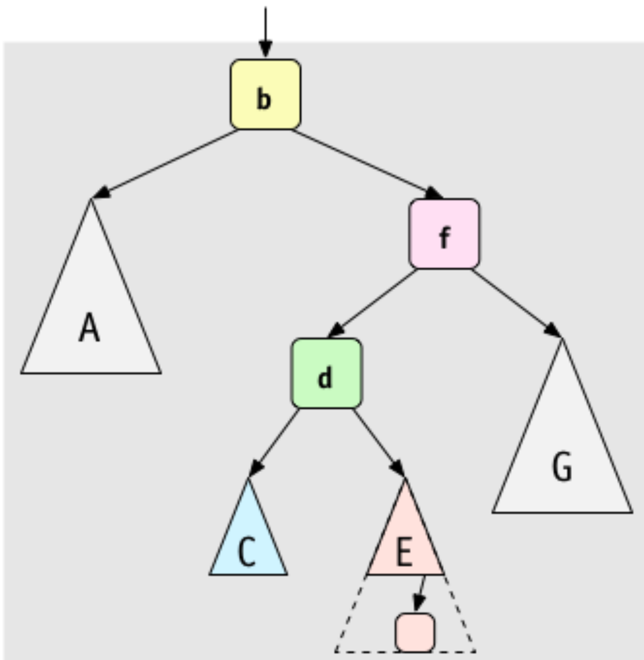
Case #3: Right-Left Case



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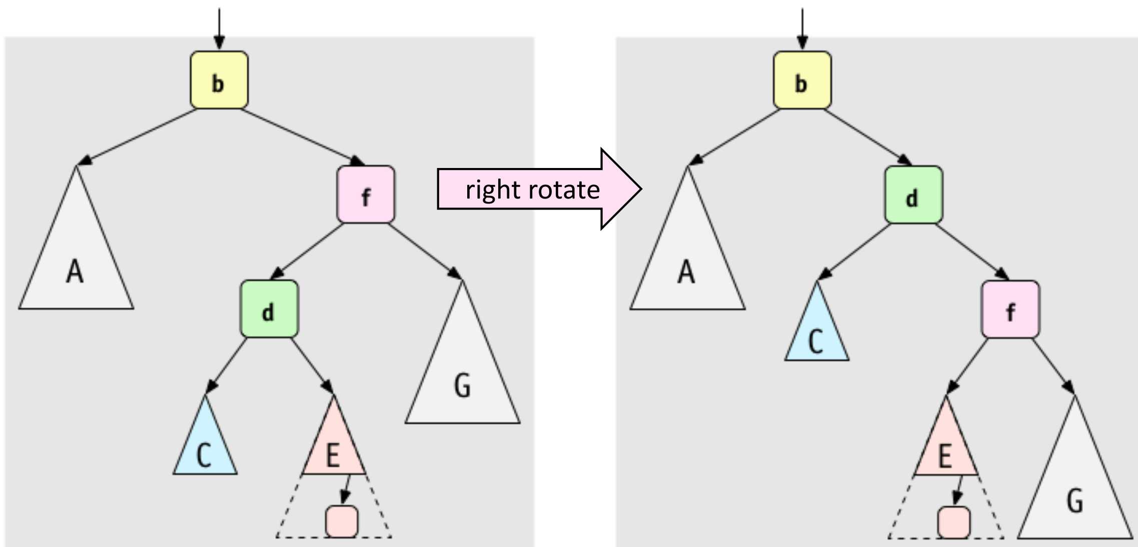
Case #3: Right-Left Case (after one rotation)



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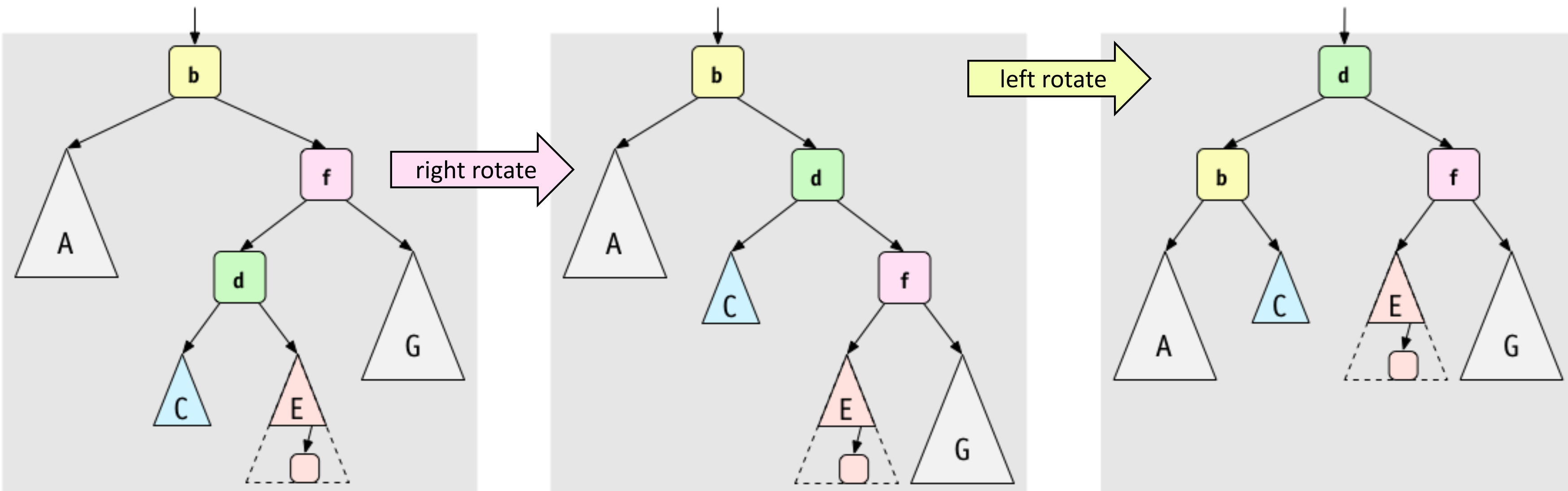
Case #3: Right-Left Case (after two rotations)



A way to remember it:
Move d to grandparent's position. Put everything else in their only legal positions for a BST.

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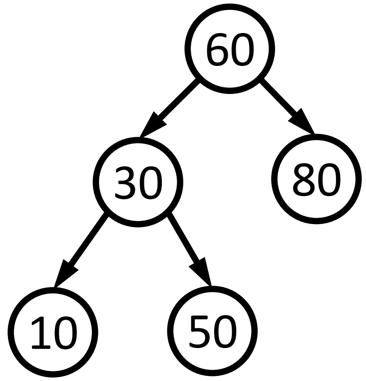
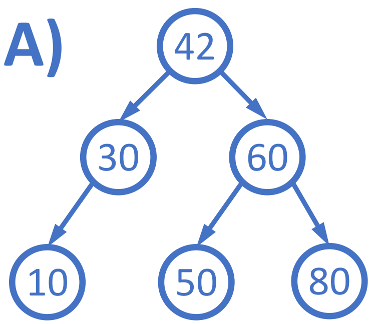
Practice time! Example of Case #4

Starting with this AVL tree:

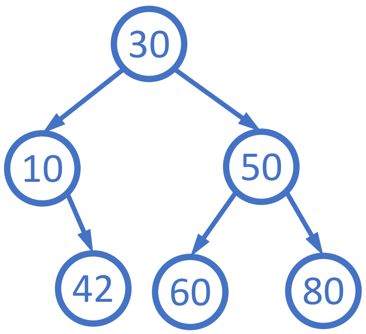


Which of the following is the updated AVL tree after inserting 42?

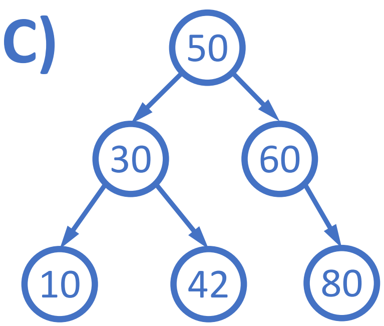
A)



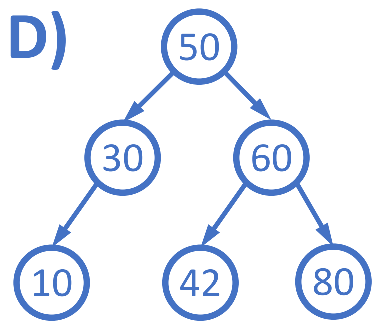
B)



C)



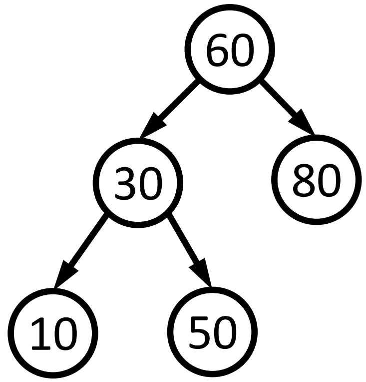
D)



(extra space for your scratch work and notes)

Practice time! Example of Case #4

Starting with this AVL tree:



Which of the following is the updated AVL tree after inserting 42?

What's the name of this case?

What rotations did we do?

Insert, summarized

- Insert as in our generic BST
- Check back up path for imbalance, which will be 1 of 4 cases:
  - Node's left-left grandchild is too tall
  - Node's left-right grandchild is too tall
  - Node's right-left grandchild is too tall
  - Node's right-right grandchild is too tall
- Only occurs because
- After the appropriate single or double rotation, the smallest-unbalanced subtree has the same height as before the insertion
  - So all ancestors are now balanced

AVL Tree Efficiency

- Worst-case complexity of `find`:

- Worst-case complexity of `insert`:

- Worst-case complexity of `buildTree`:

Takes some more rotation action to handle `delete`…

Pros and Cons of AVL Trees

Arguments for AVL trees:

1. All operations logarithmic worst-case because trees are *always* balanced
2. Height balancing adds no more than a constant factor to the speed of `insert` and `delete`

Arguments against AVL trees:

1. Difficult to program & debug [but done once in a library!]
2. More space for height field
3. Asymptotically faster but rebalancing takes a little time
4. If *amortized* logarithmic time is enough, use splay trees (also in the text, not covered in this class)

Lots of cool Self-Balancing BSTs out there!

Popular self-balancing BSTs include:

- AVL tree
- Splay tree
- 2-3 tree
- AA tree
- Red-black tree
- Scapegoat tree
- Treap

(Not covered in this class, but several are in the textbook and all of them are online!)

(From https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree#Implementations)

Wrapping Up: Hash Table vs BST Dictionary

Hash Table advantages:

BST advantages:

- Can get keys in sorted order without much extra effort
- Same for ordering statistics, finding closest elements, range queries
- Easier to implement if don't have hash function (which are hard!).
- Can guarantee O(log n) *worst-case* time, whereas hash tables are O(1) *average* time.

Priority Queue ADT & Binary Heap data structure

Like a Queue, but with priorities for each element.

An Introductory Example…

Gill Bates, the CEO of the software company Millisoft,
built a robot secretary to manage her hundreds of emails.
During the day, Bates only wants to look at a few emails every now and then so she can stay focused.
The robot secretary sends her the most important emails each time.
To do so, he assigns each email a *priority* when he gets them
and only sends Bates the *highest priority* emails upon request.

All of your computer servers are on fire!

Priority:

Here’s a reminder for our meeting in 2 months.

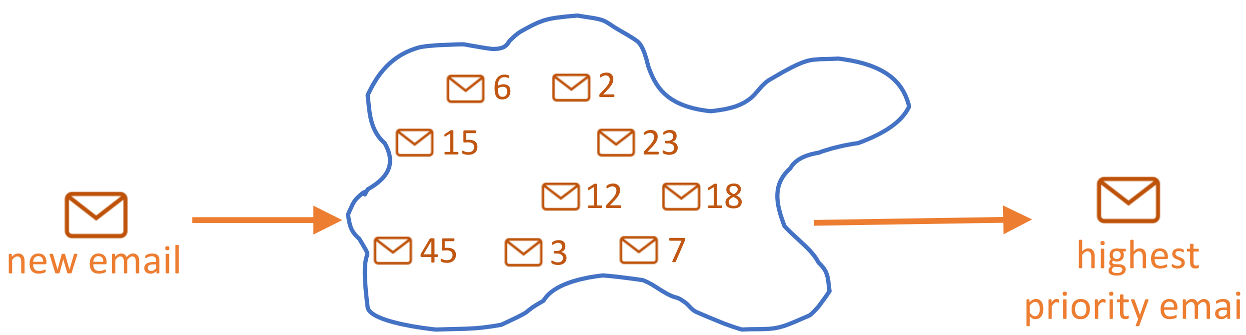
Priority:

Priority Queue ADT

A **priority queue** holds *compare-able data*

In our introductory example:

- Like dictionaries, we need to *compare items*
  - Given x and y, is x less than, equal to, or greater than y
  - Meaning of the ordering can depend on your data



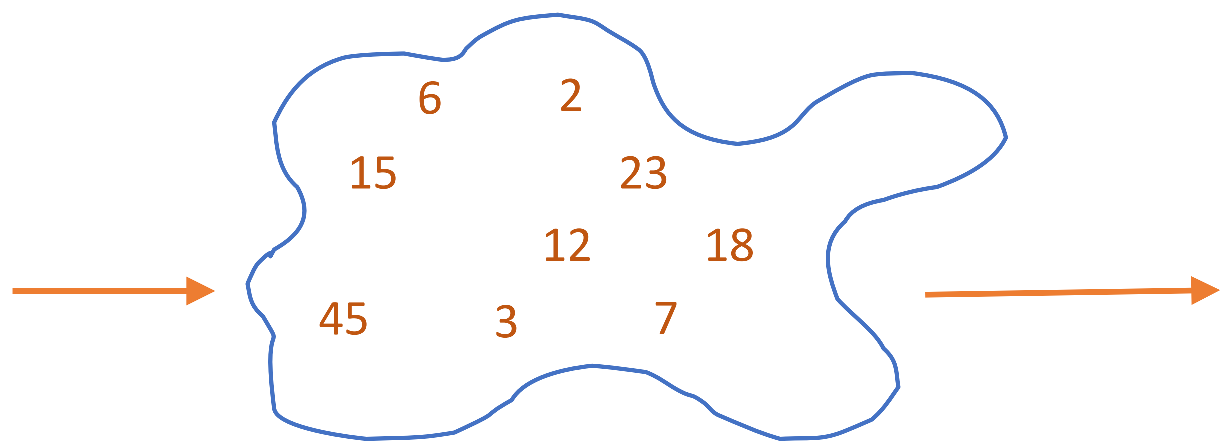
- The data typically has two fields:
  - We'll now use integers for examples, but can use other types /objects for priorities too!

Priority Queue ADT

Meaning:

- A **priority queue** holds *compare-able data*
- Key property:

Operations:



Priority Queue: Example

`insert` x_1 with priority *5*

`insert` x_2 with priority *3*

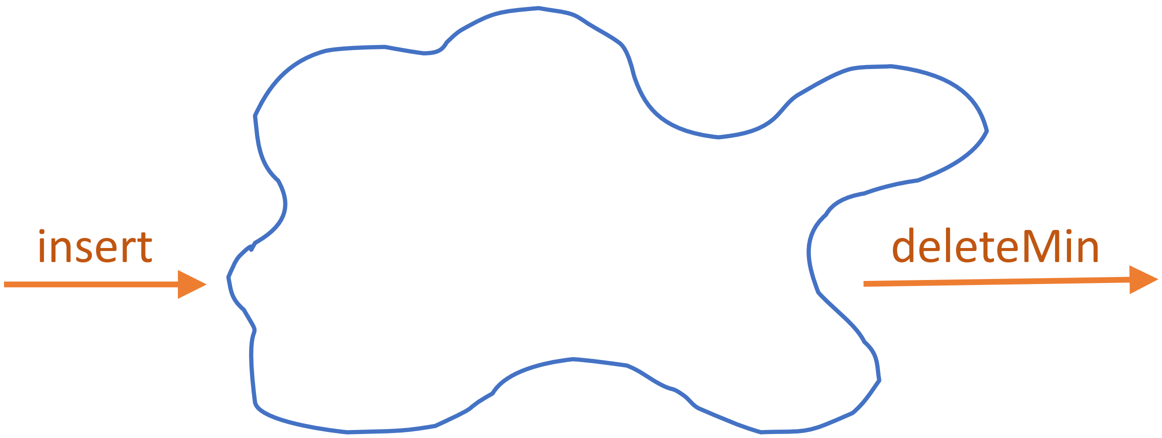
a = `deleteMin`

`insert` x_3 with priority *2*

`insert` x_4 with priority *6*

c = `deleteMin`

d = `deleteMin`



Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`

But the whole point is to use priorities instead of FIFO

Applications

Like all good ADTs, the priority queue arises often

- Run multiple programs in the operating system
  - “critical” before “interactive” before “compute-intensive”
- Treat hospital patients in order of severity (or triage)
- Forward network packets in order of urgency
- Select most frequent symbols for data compression
- Sort (first `insert` all, then repeatedly `deleteMin`)
  - Much like Homework 1 uses a stack to implement reverse

Finding a good data structure

Will show an efficient, non-obvious data structure for this ADT

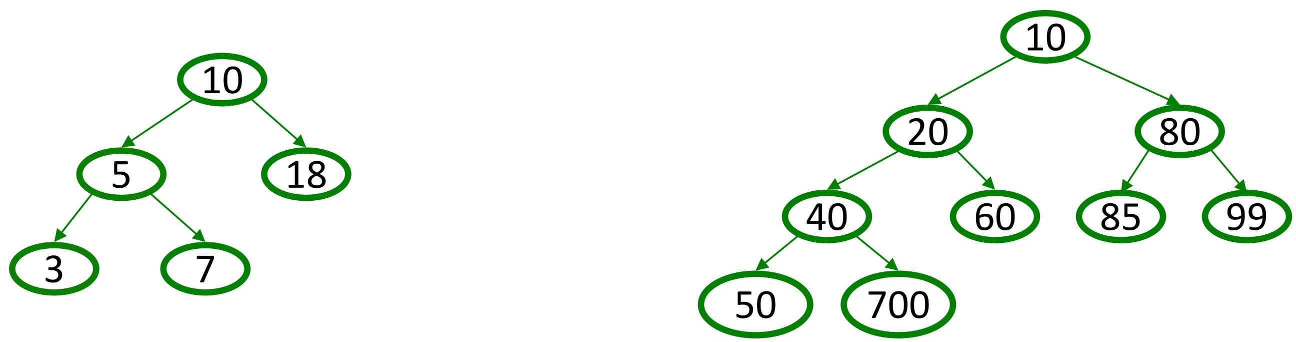
But first let's analyze some "obvious" ideas for *n* data items

data	*insert algorithm / time*	*deleteMin algorithm / time*
unsorted array	add at end	search
unsorted linked list	add at front	search
sorted circular array	search / shift	move front
sorted linked list	put in right place	remove at front
binary search tree	put in right place	leftmost
AVL tree	put in right place	leftmost

Our data structure

A *binary min-heap* (or just *binary heap* or just *heap*) has:

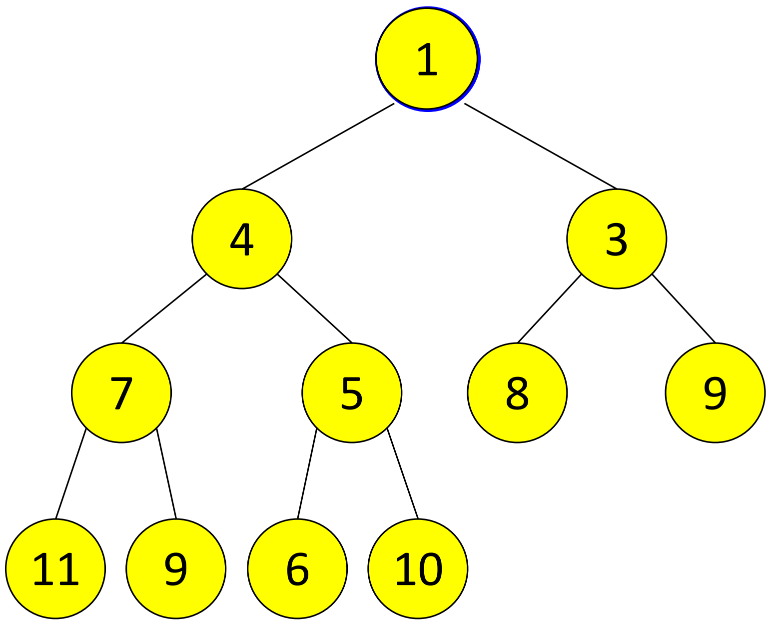
- Structure property:
- Heap property: The priority of every (non-root) node is less important than the priority of its parent



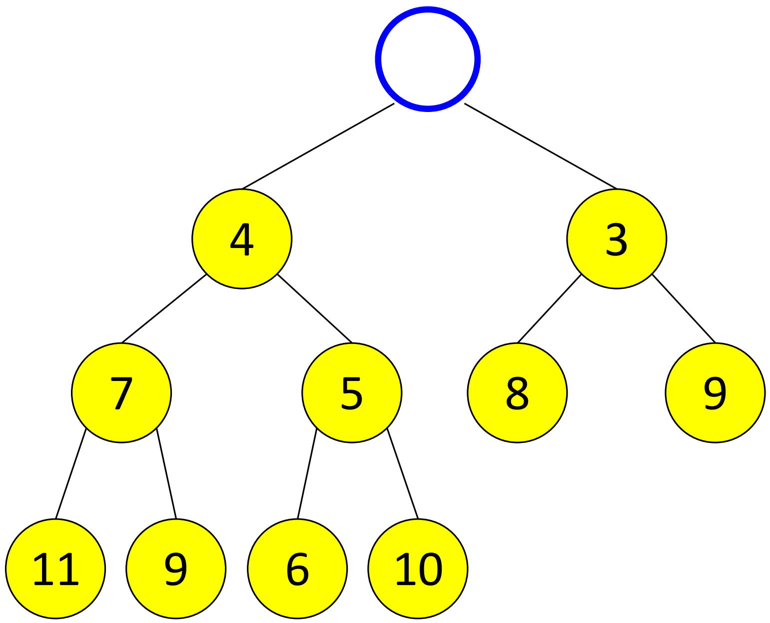
So:

- Where is the highest-priority item?
- Where is the lowest priority?
- What is the height of a heap with *n* items?

`deleteMin`: Step #1

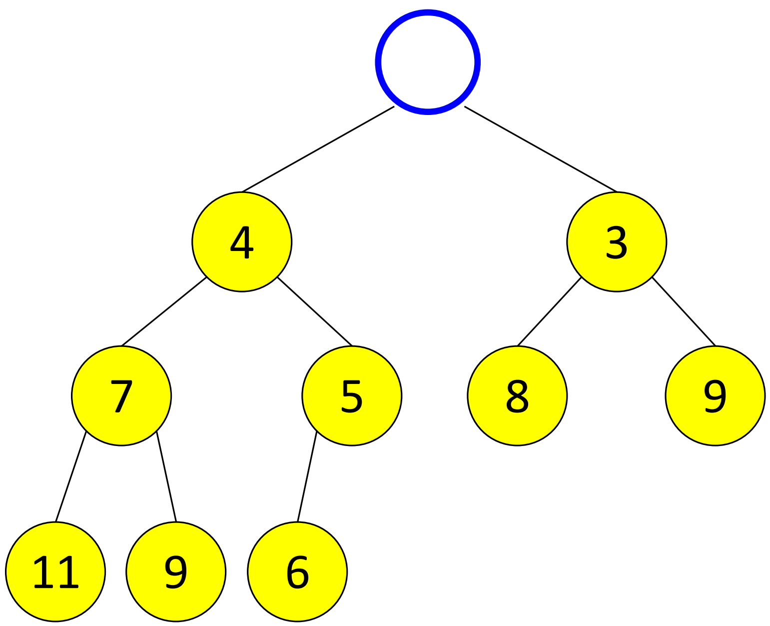


`deleteMin`: Step #2 (Keep Structure Property)



Want to keep structure property

`deleteMin`: Step #3

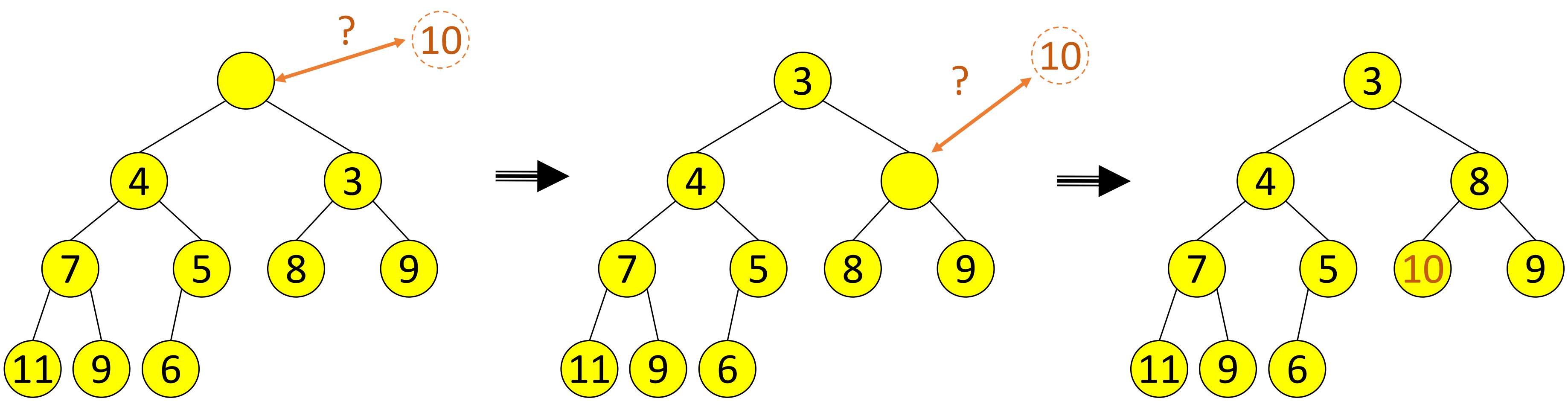


Want to restore heap property

`deleteMin`: Step #3 (Restore Heap Property)

Percolate down:

- Compare priority of item with its children
- If item has lower priority, swap with the most important child
- Repeat until both children have lower priority or we've reached a leaf node

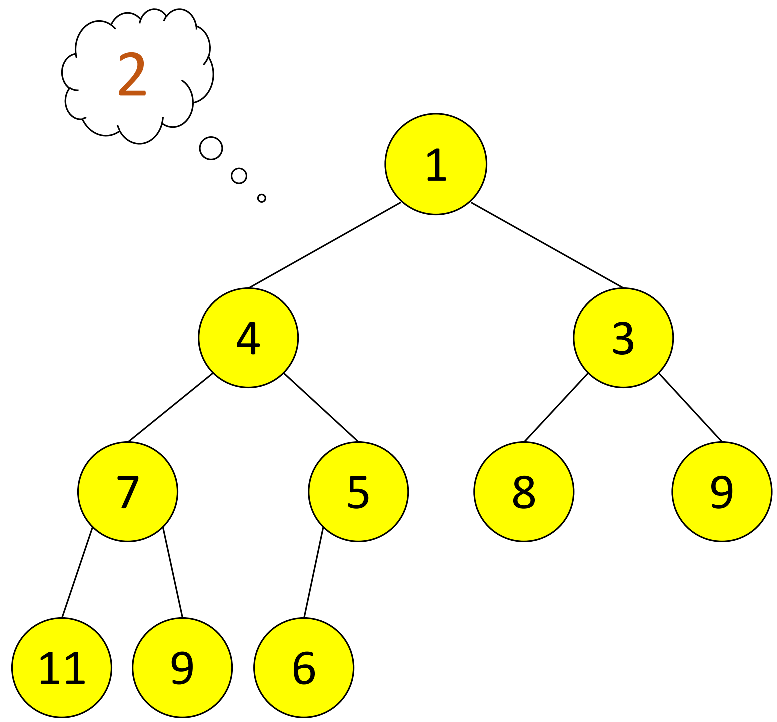


What is the run time?

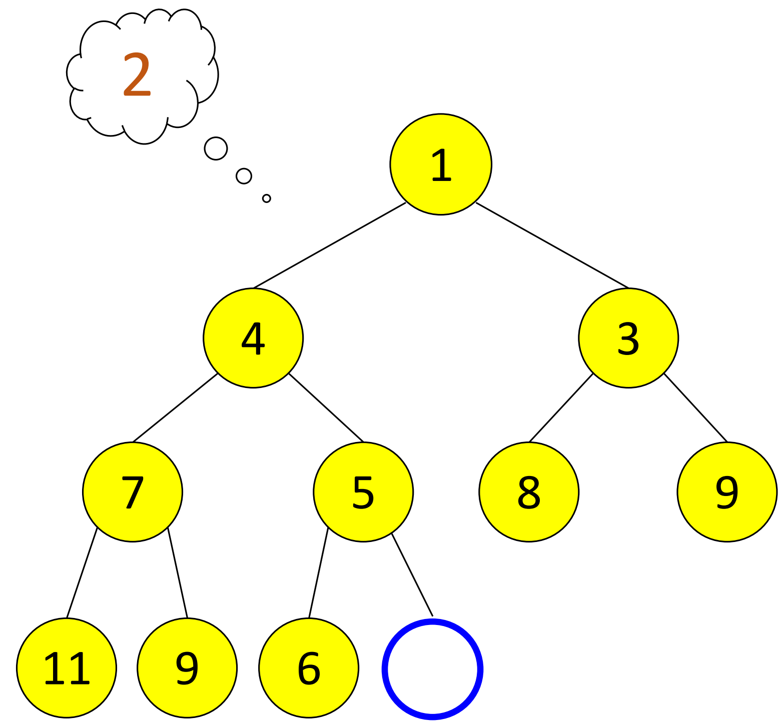
`deleteMin`: Run Time Analysis

- Run time is
- A heap is a
- So its height with *n* nodes is
- So run time of `deleteMin` is

`insert`: Step #1



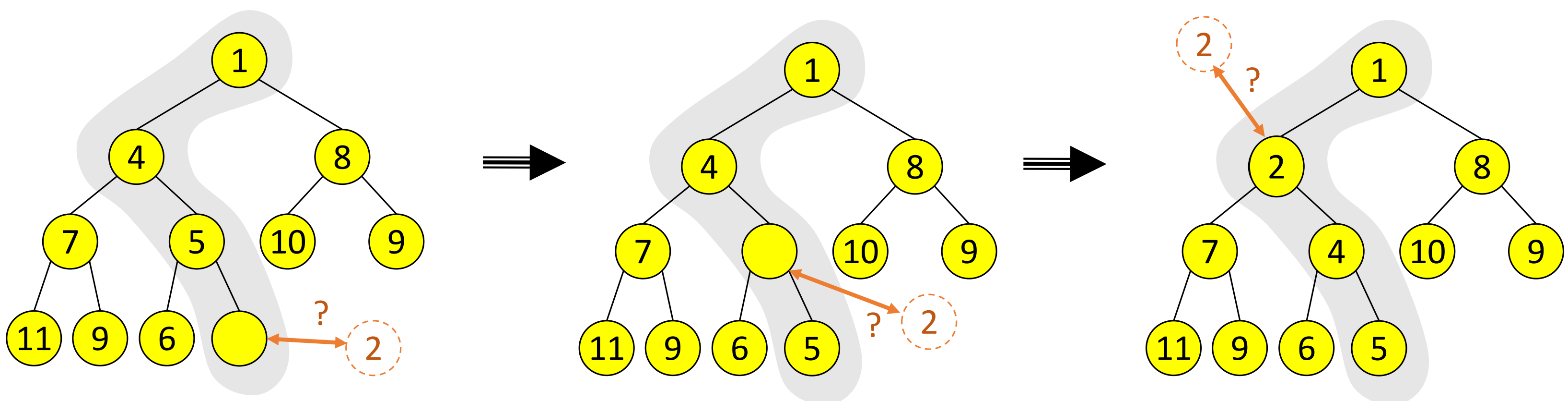
`insert`: Step #2



`insert`: Step #2 (Restore Heap Property)

Percolate up:

- Put new data in new location
- If higher priority than parent, swap with parent
- Repeat until parent is more important or reached root



What is the running time?

Summary: basic idea for operations

`findMin`: **return** `root.data`

`deleteMin`:

1. `answer = root.data`
2. Move right-most node in last row to root to restore structure property
3. "Percolate down" to restore heap property

`insert`:

1. Put new node in next position on bottom row to restore structure property
2. "Percolate up" to restore heap property

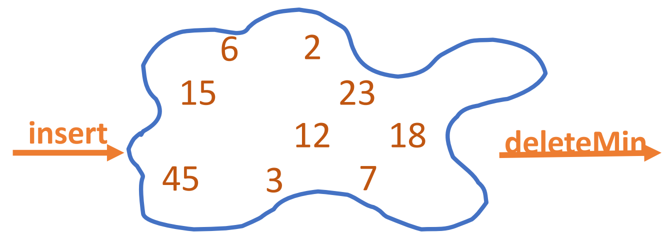
Overall strategy:

1. *Preserve structure property*
2. *Restore heap property*

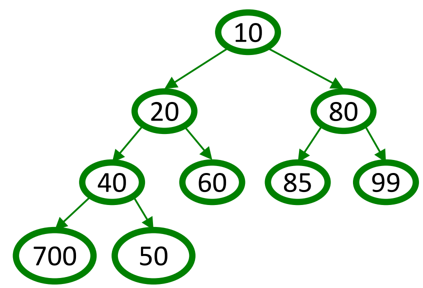
Binary Heap

- Operations
  - $O(\log n)$ `insert`
  - $O(\log n)$ `deleteMin` *worst-case*
  - *Very* good constant factors
  - *If* items arrive in random order, then `insert` is $O(1)$ on *average*

Summary: Priority Queue ADT



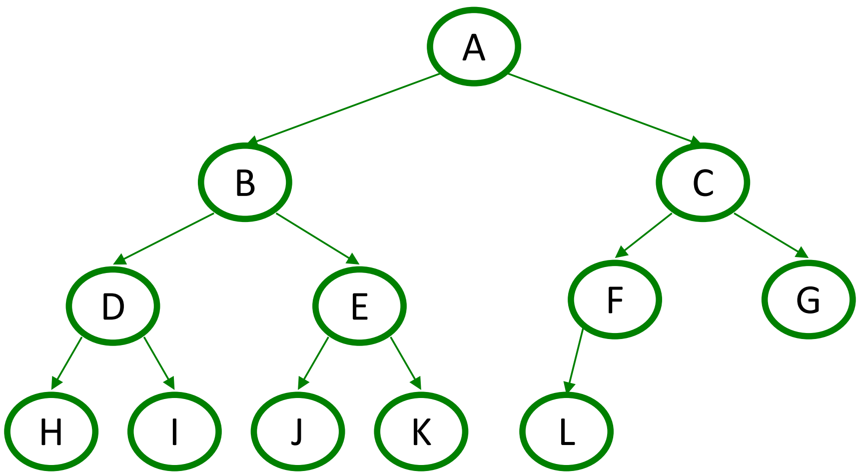
- Priority Queue ADT:
  - `insert` comparable object,
  - `deleteMin`



- Binary heap data structure:
  - Complete binary tree
  - Each node has less important priority value than its parent

- `insert` and `deleteMin` operations = O(height-of-tree)=O(`log` n)
  - `insert`: put at new last position in tree and percolate-up
  - `deleteMin`: remove root, put last element at root and percolate-down

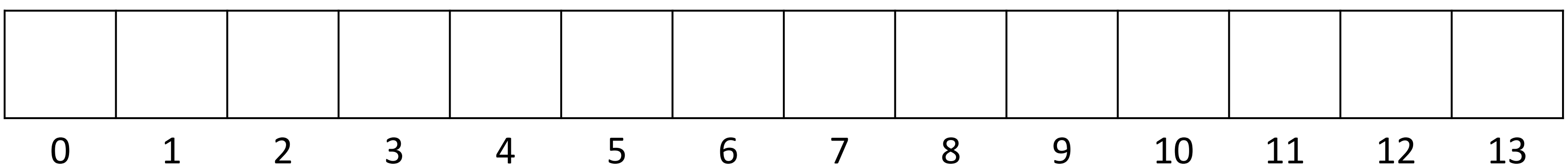
Binary Trees Implemented with an **Array**



From node `i`:

left child: `i*2`
right child: `i*2+1`
parent: `i/2`

(wasting index 0 is convenient for the index arithmetic)



Judging the array implementation

Pros:

- Non-data space: just index 0 and unused space on right
  - In conventional tree representation, one edge per node (except for root), so n-1 wasted space (like linked lists)
  - Array would waste more space if tree were not complete
- Multiplying and dividing by 2 is very fast (shift operations in hardware)
- Last used position is just index

Cons:

- Same might-be-empty or might-get-full problems we saw with array-based stacks and queues (resize by doubling as necessary)

Pros outweigh cons: min-heaps almost always use array implementation

Practice time!

Starting with an empty array-based binary heap, which is the result after

1. `insert` (in this order): 16, 32, 4, 67, 105, 43, 2
2. `deleteMin` once

0	1	2	3	4	5	6	7

A)

	4	15	32	43	67	105	
0	1	2	3	4	5	6	7

B)

	16	32	4	67	105	43	
0	1	2	3	4	5	6	7

C)

	4	32	16	43	105	67	
0	1	2	3	4	5	6	7

D)

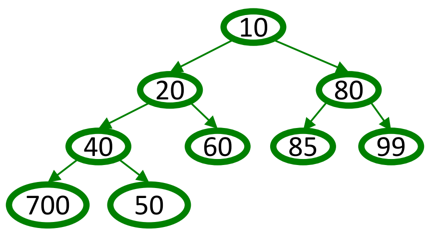
	4	32	16	67	105	43	
0	1	2	3	4	5	6	7

(extra space for your scratch work and notes)

Semi-Pseudocode: `insert` into binary heap

```
void insert(int val) {
  if(size==arr.length-1)
    resize();
  size++;
  i=percolateUp(size,val);
  arr[i] = val;
}
```

```
int percolateUp(int hole,
                int val) {
  while(hole > 1 &&
          val < arr[hole/2])
    arr[hole] = arr[hole/2];
    hole = hole / 2;
  }
  return hole;
}
```

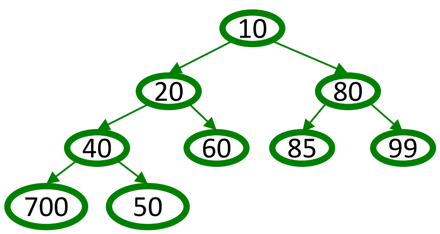


This pseudocode uses ints. In real use, you will have data nodes with priorities.

	10	**20**	**80**	**40**	**60**	**85**	**99**	**700**	**50**				
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Semi-Pseudocode: `deleteMin` from binary heap

```
int deleteMin() {
  if(isEmpty()) throw…
  ans = arr[1];
  hole = percolateDown
           (1,arr[size]);
  arr[hole] = arr[size];
  size--;
  return ans;
}
```




```
int percolateDown(int hole,
                  int val) {
 while(2*hole <= size) {
  left  = 2*hole;
  right = left + 1;
  if(right > size ||
     arr[left] < arr[right])
    target = left;
  else
    target = right;
  if(arr[target] < val) {
    arr[hole] = arr[target];
    hole = target;
  } else
      break;
 }
 return hole;
}
```

	10	**20**	**80**	**40**	**60**	**85**	**99**	**700**	**50**				
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Example

1. `insert` (in this order): 16, 32, 4, 67, 105, 43, 2
2. `deleteMin` once

0	1	2	3	4	5	6	7

Example

1. `insert` (in this order): 16, 32, 4, 67, 105, 43, 2
2. `deleteMin` once

0	1	2	3	4	5	6	7

Other operations

- `decreaseKey`: given pointer to object in priority queue (e.g., its array index), lower its priority value by *p*
    - Change priority and percolate up
- `increaseKey`: given pointer to object in priority queue (e.g., its array index), raise its priority value by *p*
    - Change priority and percolate down
- `remove`: given pointer to object in priority queue (e.g., its array index), remove it from the queue
    - `decreaseKey` with $p = \infty$, then `deleteMin`

Running time for all these operations?