CSE 373: Data Structures and Algorithms

Lecture 9: Binary Search Trees

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Quarter: Summer 2017

Today

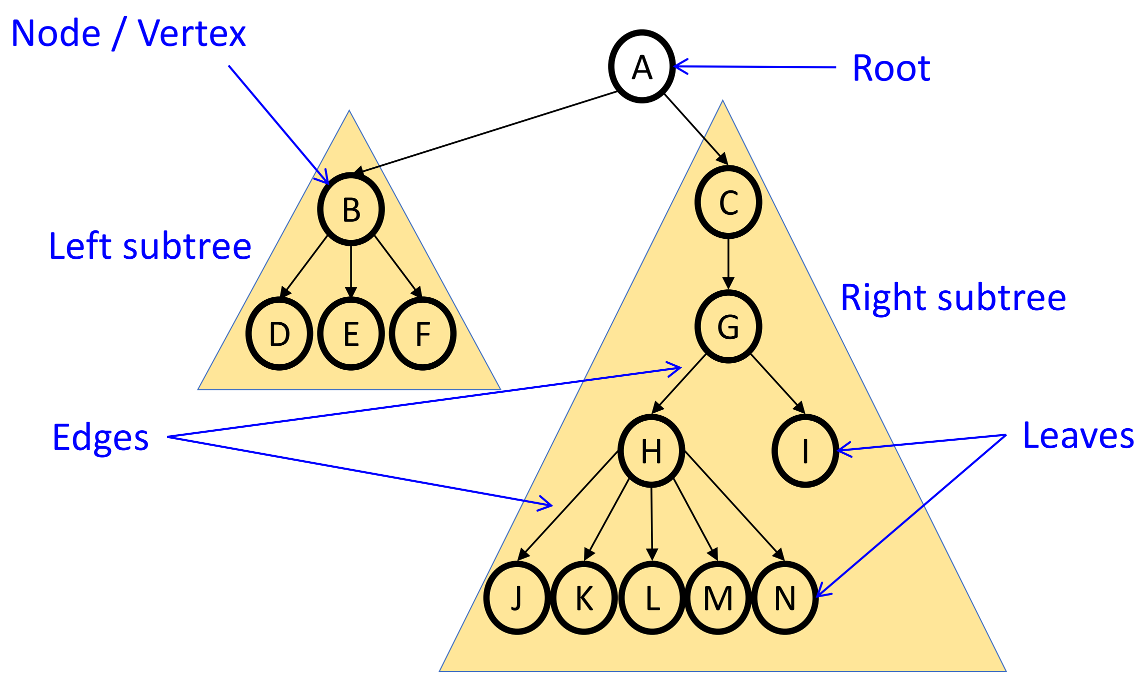
- Announcements
- Binary Trees
  - Height
  - Traversals
- Binary Search Trees
  - Definition
  - `find`
  - `insert`
  - `delete`
  - `buildTree`

Announcements

- Change to office hours for just this week
  - Tuesday's "office" office hours / private office hours
    - 12:00pm – 12:30pm
    - (not at 1:30pm!)
  - Dorothy and I trading 2:00pm - 3:00pm office hours this week
    - Same time and location

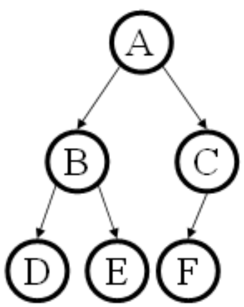
- Homework 1 Statistics
  - Mean: 39.7/50 (+1 extra credit)
  - Median: 42.5/50 (+0 extra credit)
  - Max: 49/50 (+1) or 47/50 (+4)
  - Standard Deviation: 10.18

Reminder: Tree terminology

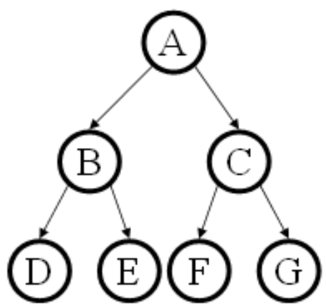


Binary Trees

- Binary tree: Each node has at most 2 children (branching factor 2)
- Binary tree is
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*
- Special Cases:



Complete Tree



Perfect Tree

(Last week's practice) What does the following method do?

```
int height(Node root){
  if (root == null),
      return -1;
  return 1 + max(height(root.left),
                 height(root.right);
}
```

A. It calculates the number of nodes in the tree.

B. It calculates the depth of the nodes.

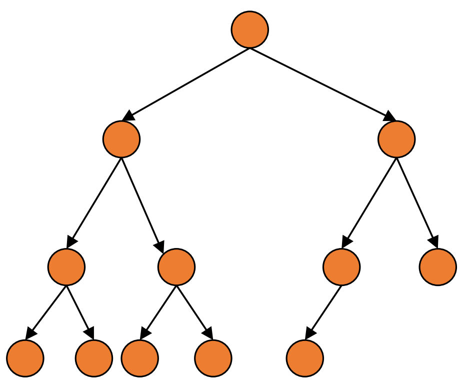
C. It calculates the height of the tree.

D. It calculates the number of leaves in the tree.

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:



- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

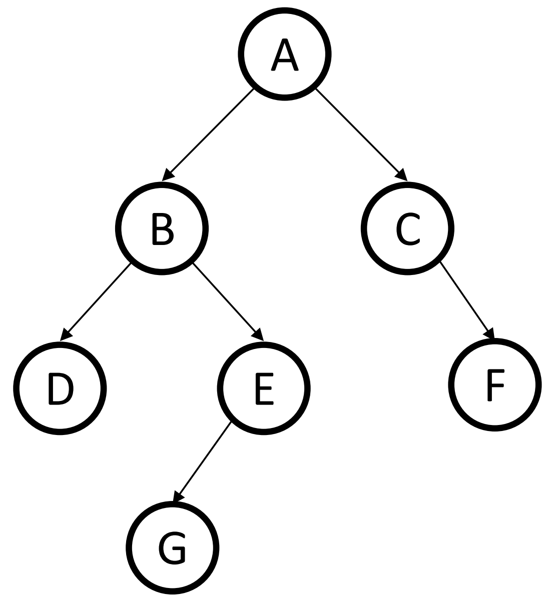
For *n* nodes, the min height (best-case) is

the max height (worst-case) is

Tree Traversals

A **traversal** is an order for visiting all the nodes of a tree

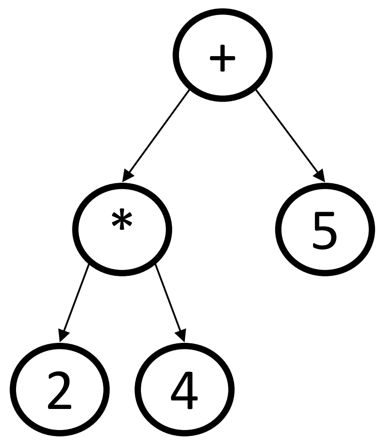
- ***Pre-order***: root, left subtree, right subtree
- ***In-order***: left subtree, root, right subtree
- ***Post-order***: left subtree, right subtree, root



Tree Traversals: Practice

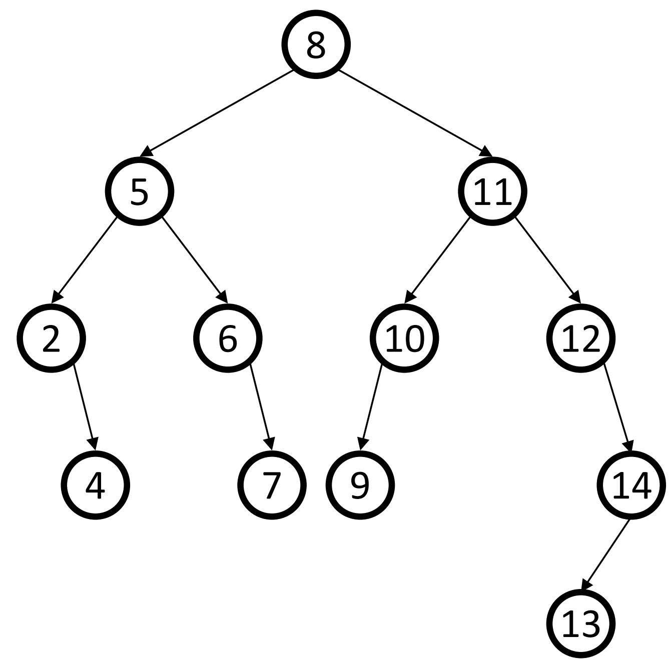
Which one makes sense for evaluating this *expression tree?*

- ***Pre-order***: root, left subtree, right subtree
- ***In-order***: left subtree, root, right subtree
- ***Post-order***: left subtree, right subtree, root



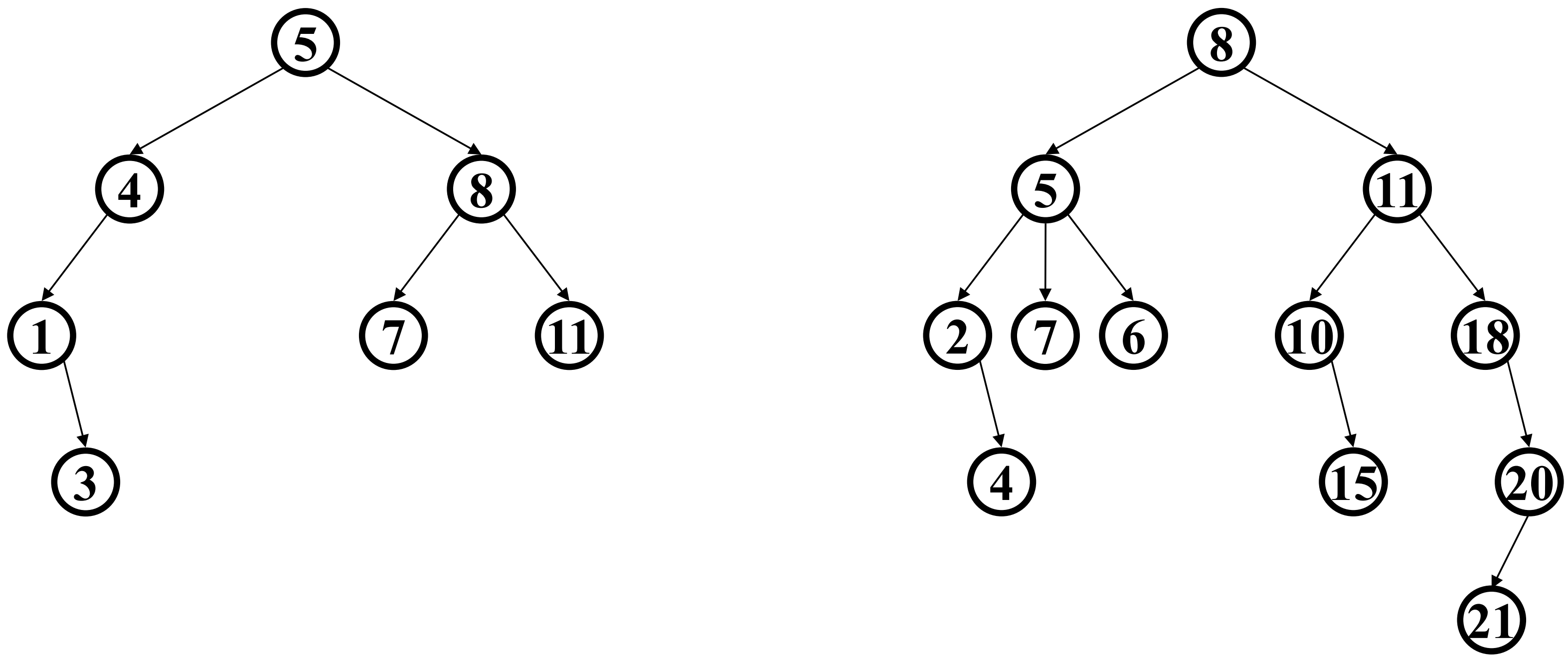
Binary Search Tree (BST) Data Structure

- Structure property (binary tree)
  - Each node has ≤ 2 children
  - Result: keeps operations simple
- Order property
  - Result: straight-forward to find any given value

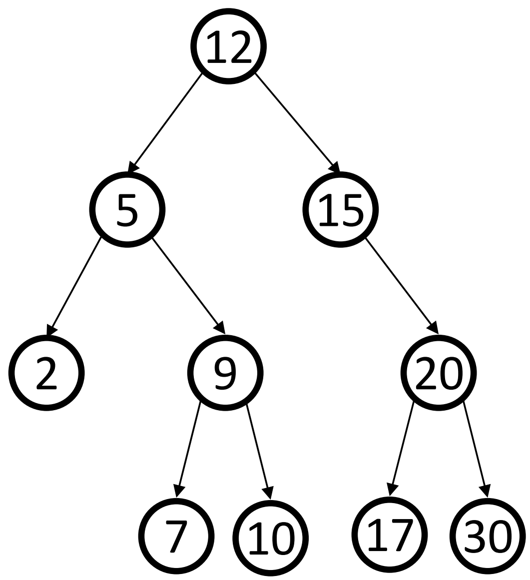


A binary *search* tree is a type of binary tree
(but not all binary trees are binary search trees!)

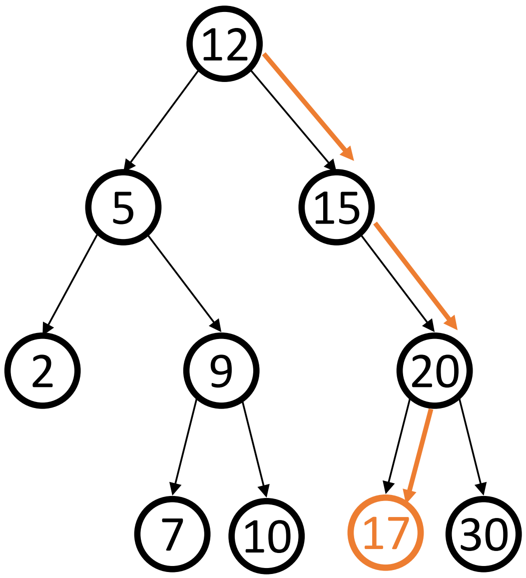
Practice: are these BSTs?



How do we `find(value)` in BST's?



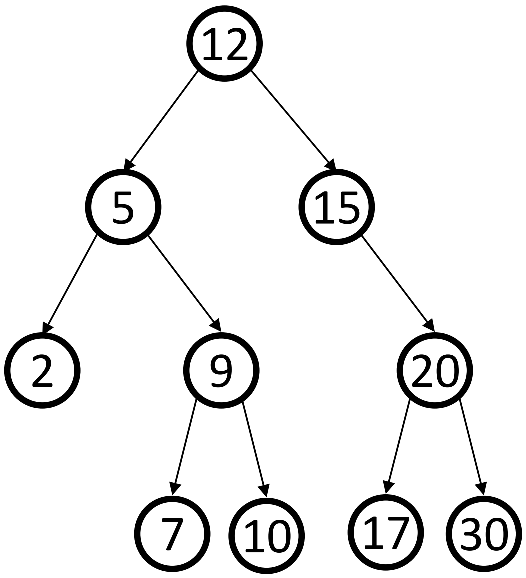
`find` in BST: Recursive Version




```
Data find(Data value, Node root){
 if(root == null)
   return null;
 if(key < root.value)
   return find(value, root.left);
 if(key > root.value)
   return find(value, root.right);
 return root.value;
}
```

What is the running time?

`find` in BST: Iterative Version

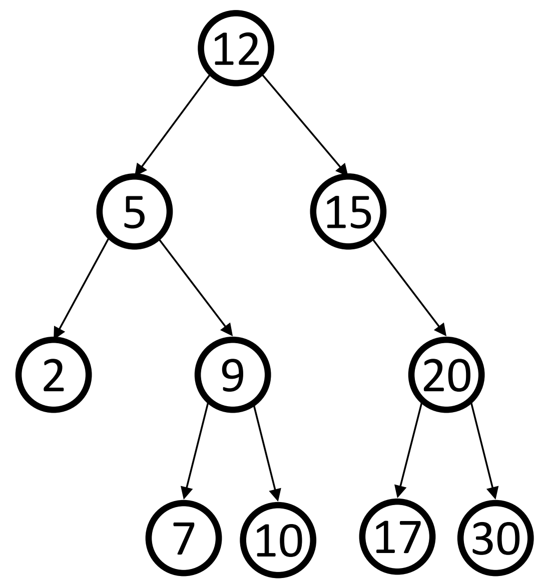



```
Data find(Object value, Node root){
 while(root != null
        && root.value != value) {
  if (value < root.value)
    root = root.left;
  else (value > root.value)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.value;
}
```

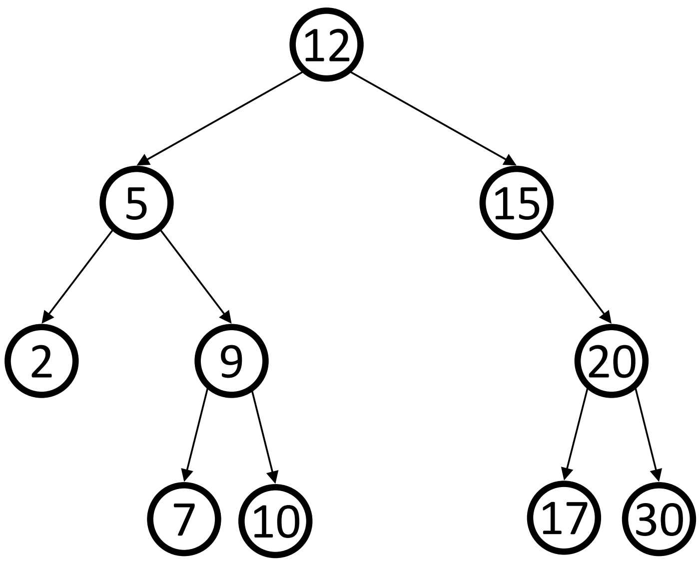
Other BST "Finding" Operations

`findMin`: Find *minimum* node

`findMax`: Find *maximum* node



`insert` in BST



`insert(13)`
`insert(8)`
`insert(31)`

Worst-case running time:

Practice with `insert`, primer for `delete`

Start with an empty tree. Insert the following values, in the given order:
`14, 2, 5, 20, 42, 1, 4, 16`

Then, changing as few nodes as possible, delete the following in order:
`42, 14`

What would the root of the resulting tree be?

A. `2`
B. `4`
C. `5`
D. `16`

(Extra space for scratch work / notes)

`delete` in BST

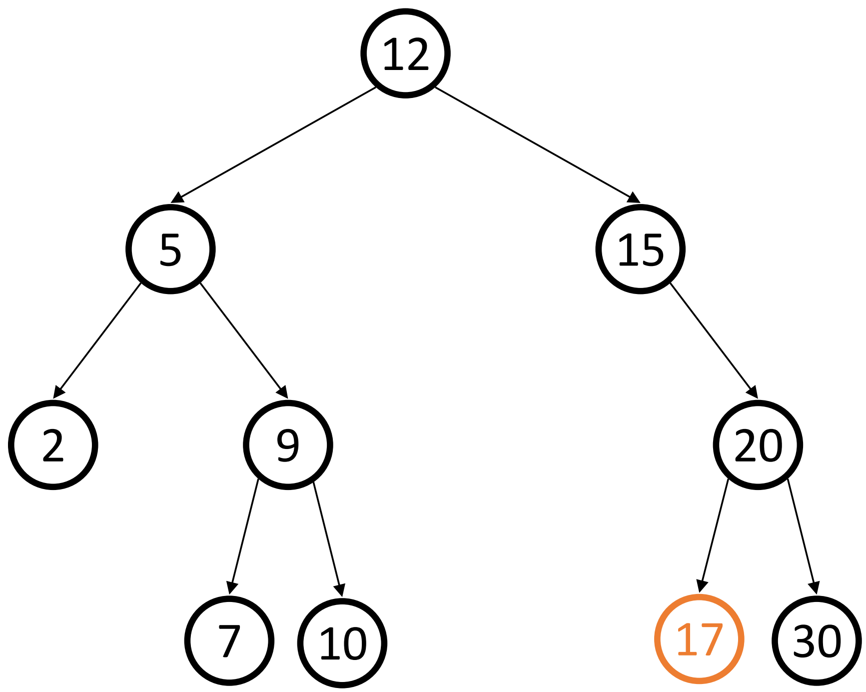
- Why might `delete` be harder than `insert`?

- Basic idea:

- Three potential cases to fix:

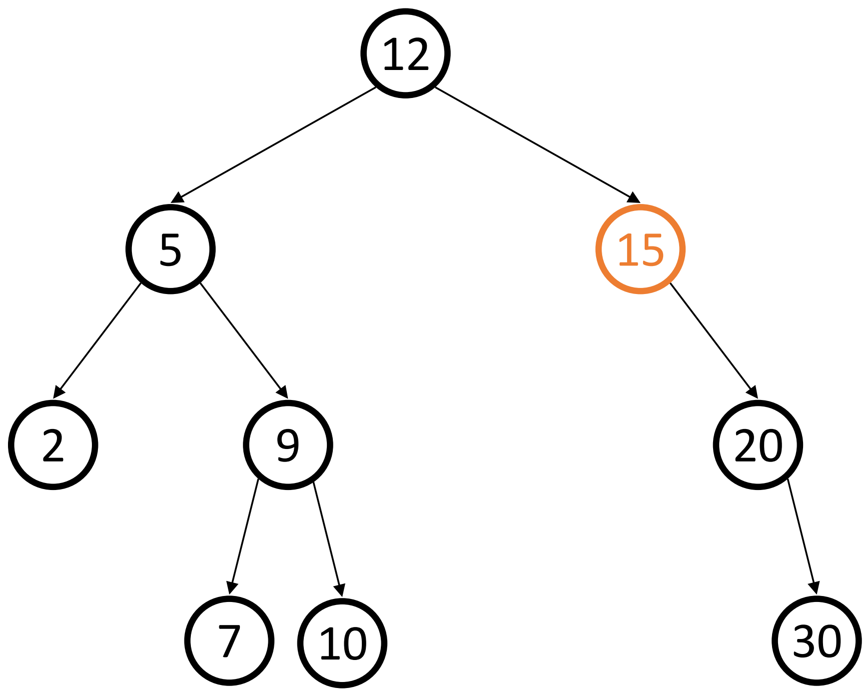
`delete` case: Leaf

`delete(17)`



`delete` case: One Child

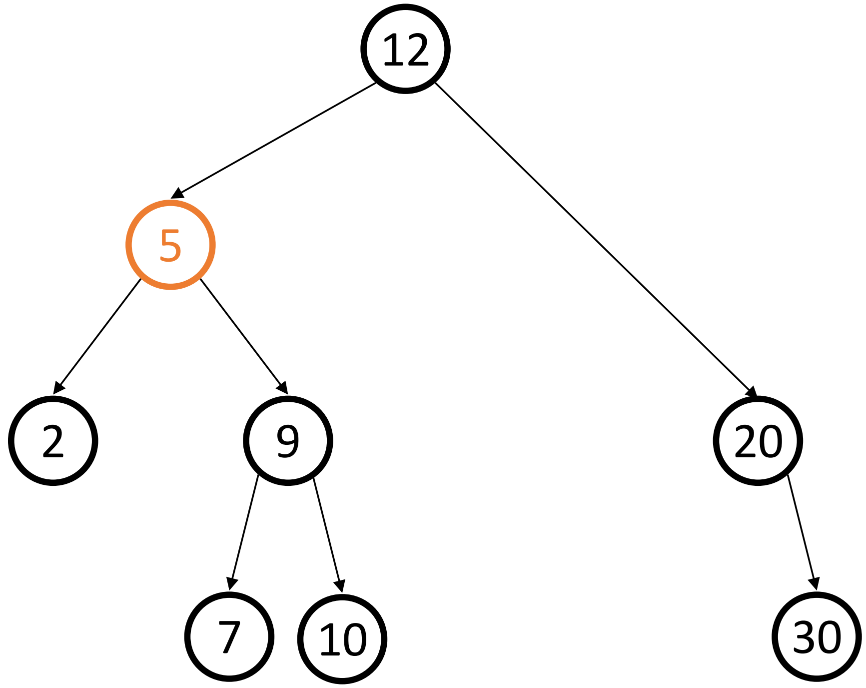
`delete(15)`



`delete` case: Two Children

`delete(5)`

What can we replace 5 with?



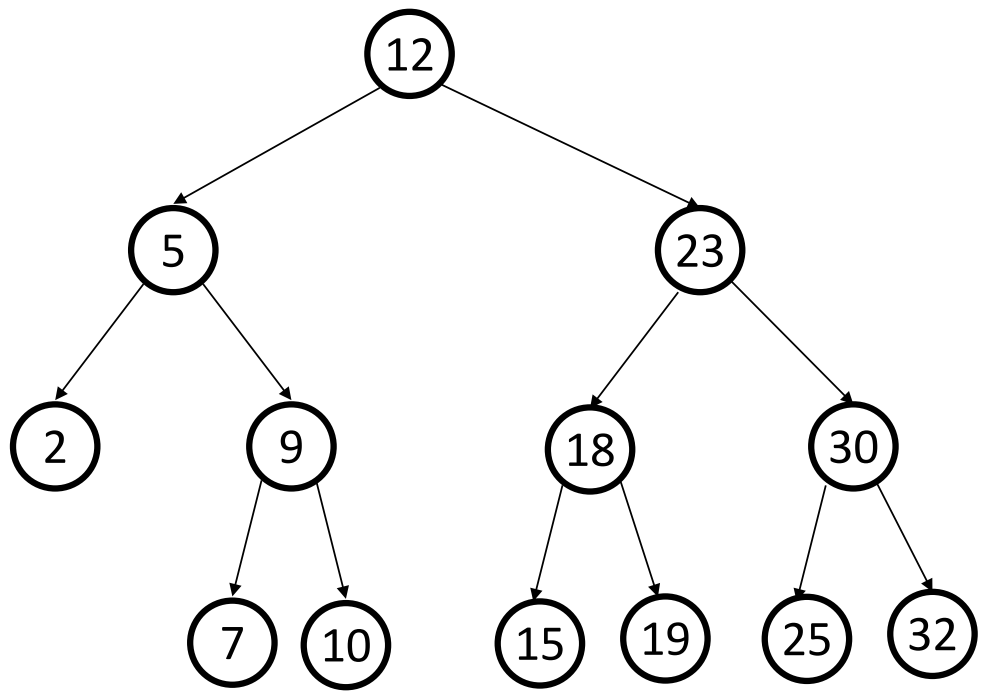
`delete` case: Two Children

What can we replace the node with?

Options:

`delete` case: Two Children (example #2)

`delete(23)`

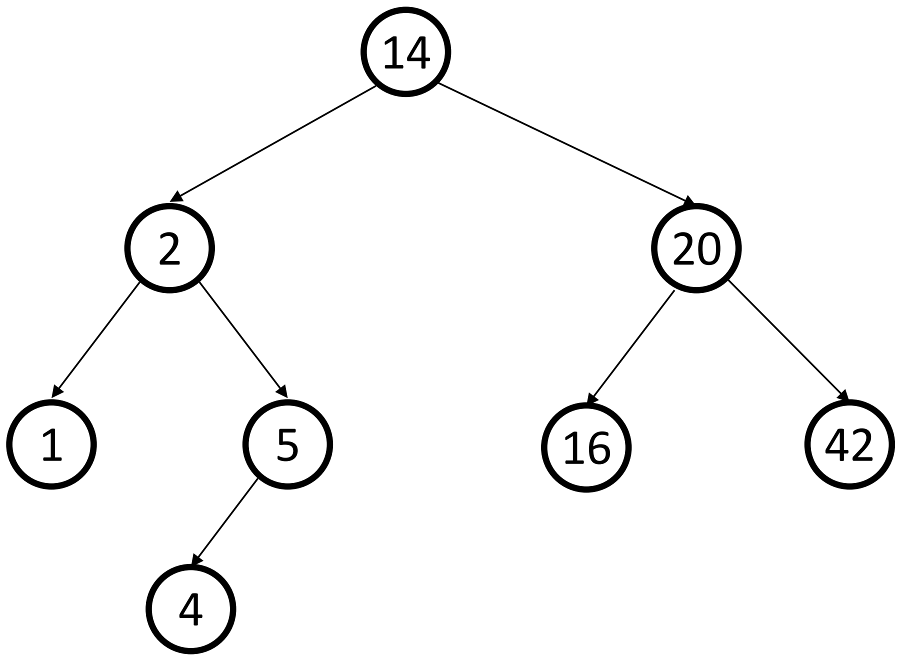


Practice with `insert`, primer for `delete`

REVISITED

Changing as few nodes as possible, delete the following in order:

`42, 14`



`delete` through Lazy Deletion

- Lazy deletion can work well for a BST
  - Simpler
  - Can do “real deletions” later as a batch
  - Some inserts can just “undelete” a tree node

- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - e.g., **`findMin`** and **`findMax`**?

`buildTree` for BST

Let's consider `buildTree` (insert values starting from an empty tree)

Insert values 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

- If inserted in given order, what is the tree?
- What big-O runtime for buildTree on this sorted input?
- Is inserting in the reverse order any better?

`buildTree` for BST

Insert values 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

What we if could somehow re-arrange them

- median first, then left median, right median, etc.
  5, 3, 7, 2, 1, 4, 8, 6, 9

- What tree does that give us?

- What big-O runtime?