# CSE 373: Data Structures and Algorithms

## Lecture 7: Hash Table Collisions

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Quarter: Summer 2017

# Today

- Announcements
- Hash Table Collisions
- Collision Resolution Schemes
  - Separate Chaining
  - Open Addressing / Probing
    - Linear Probing
    - Quadratic Probing
    - Double Hashing
- Rehashing

# Announcements

- Reminder: homework 2 due tomorrow
- Homework 3: Hash Tables
  - Will be out tomorrow night
  - Pair-programming opportunity! (work with a partner)
  - Ideas for finding partner: before/after class, section, Piazza
- Pair-programming: write code together
  - 2 people, 1 keyboard
  - One is the "navigator," the other the "driver"
  - Regularly switch off to spend equal time in both roles
  - Side note: our brains tend to edit out when we make typos
  - Need to be in same physical space for entire assignment, so partner and plan accordingly!

# Review: Hash Tables & Collisions

# Hash Tables: Review

- A data-structure for the dictionary ADT
- *Average case* O(1) `find`, `insert`, and `delete`
  (when under some often-reasonable *assumptions*)
- An array storing (key, value) pairs
- Use *hash value* and table size to calculate array index
- Hash value calculated from key using *hash function*

`find`, `insert`, or `delete`
(key, value)

⇩

apply *hash function*
h(key) = *hash value*

⇩

index = hash value %
table size

⇩

if collision, apply
collision resolution ?

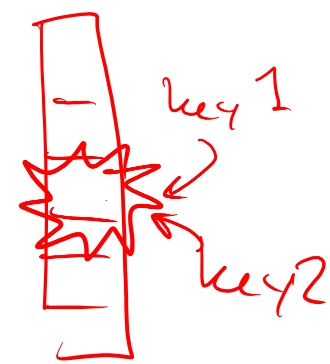
⇩

array[index] = (key, value)

# Hash Table Collisions: Review

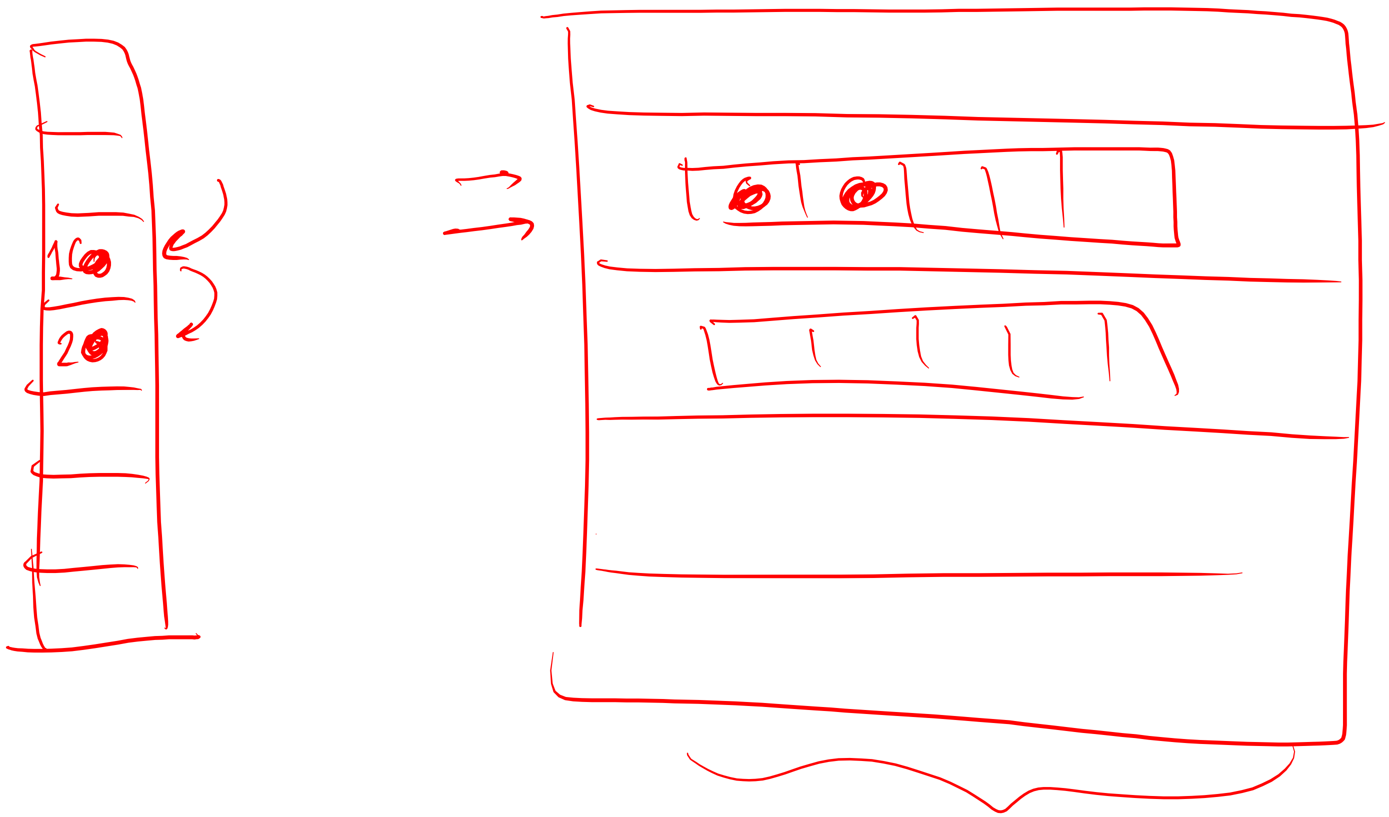
- Collision: when two keys map to the same location in the hash table

- We try to *avoid* them by having a good hash function (unique indexes)

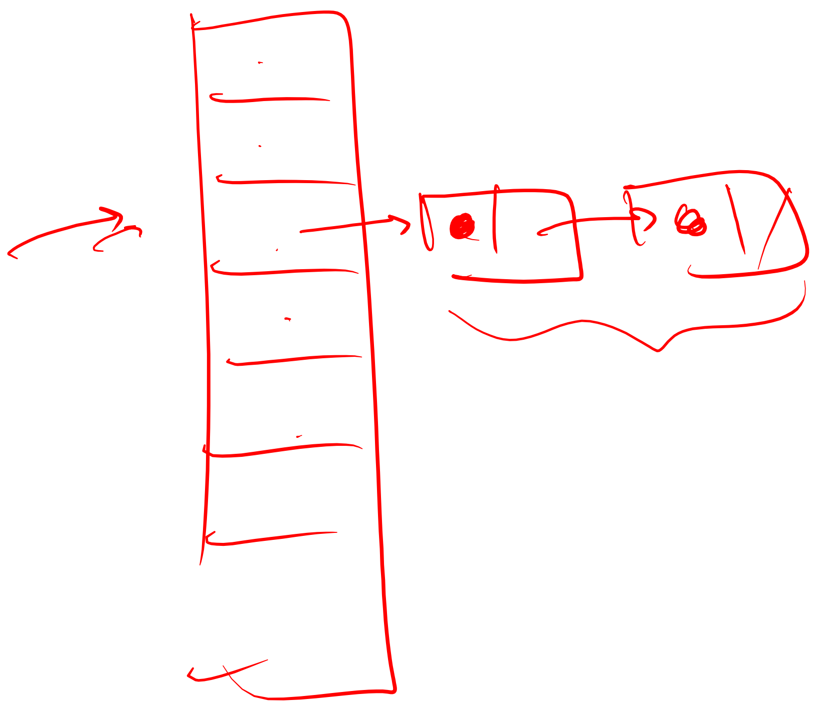



- Unfortunately, collisions are unavoidable in practice
  - Number of possible keys >> table size
  - No perfect hash function & table-index combo

# Collision Resolution Schemes: your ideas



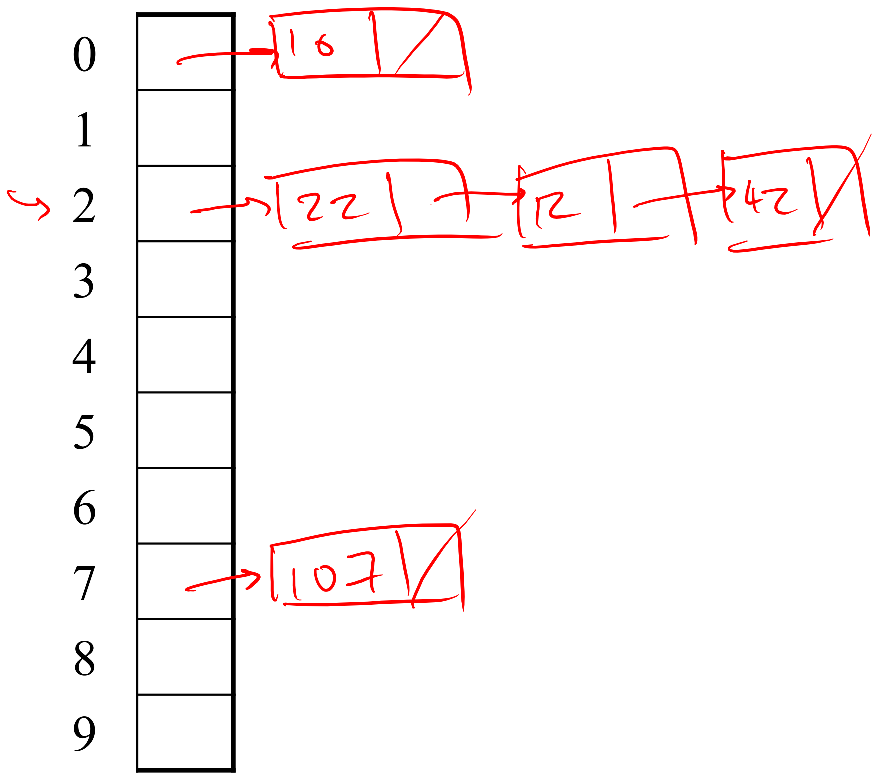
# Collision Resolution Schemes: your ideas



# Separate Chaining

One of several collision resolution schemes

# Separate Chaining




All keys that map to the same table location (aka "bucket") are kept in a list ("chain").

Example:

insert 10, 22, 107, 12, 42

and **`TableSize`** = 10

(for illustrative purposes, we're inserting hash values)

# Separate Chaining: Worst-Case

What's the worst-case scenario for `find`?

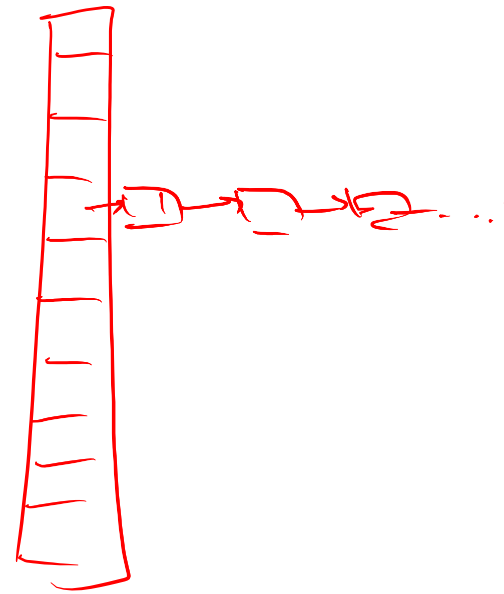
all keys indexed to the same bucket

What's the worst-case running time for `find`?

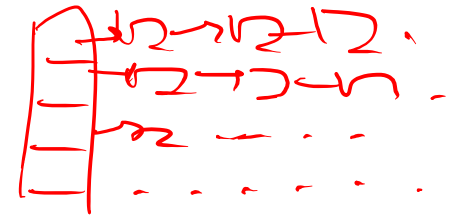
$O(n)$ linear

But only with really bad luck or really bad hash function

↳ not worth avoiding worst-case



# Separate Chaining: Further Analysis



- How can `find` become slow when we have a good hash function?

way bigger

\# elements >> table size

mean long chains

- How can we reduce its likelihood?

Maintain a good ratio of
\# elements to the table size
(resize the table as needed)

# Rigorous Analysis: Load Factor

**Definition:** The **load factor** ($\lambda$) of a hash table with *N* elements is

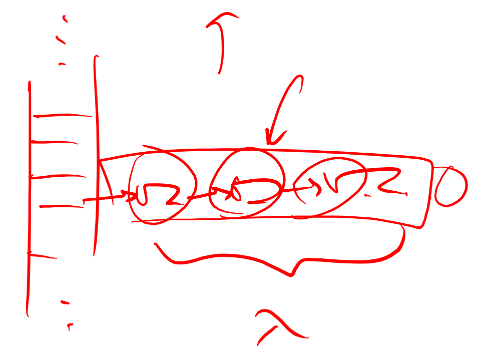
$$\lambda = \frac{N}{table\ size}$$

N = table size
average: 1 element/bucket

Under separate chaining, the average number of elements per bucket is $\lambda$

For a *random* `find`, on average

- an unsuccessful `find` compares against $\lambda$ items
- a successful `find` compares against $\lambda/2$ items



# Rigorous Analysis: Load Factor

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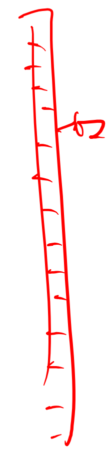


To choose a good load factor, what are our goals?

- short chains (not too high)
- efficient use of table space (not too low)

So for separate chaining, a good load factor is 1, 1.5, or 2

# Open Addressing / Probing

Another family of collision resolution schemes

# Idea: use empty space in the table

| | |
|---|---|
| 0 | 8 |
| 1 | 109 |
| 2 | 10 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | 38 |
| 9 | 19 |

- If `h(key)` is already full,
  - try `(h(key) + 1) % TableSize`. If full,
  - try `(h(key) + 2) % TableSize`. If full,
  - try `(h(key) + 3) % TableSize`. If full…

- Example: insert 38, 19, 8, 109, 10

8 9

$38 \% 10 = 8$

index

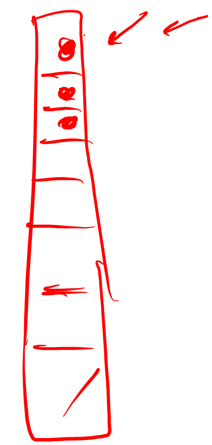
# Open Addressing Terminology

Trying the next spot is called probing (also called open addressing )

- We just did linear probing
  i$^{th}$ probe was `(h(key) + i) % TableSize`
- In general have some probe function `f` and use
  `(h(key) + f(i)) % TableSize`

# Dictionary Operations with Open Addressing

`insert` finds an open table position using a probe function

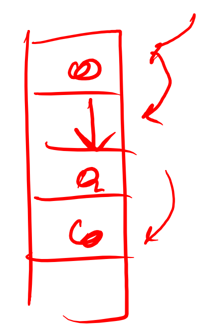


What about `find`?

- must use same probe function to "retrace the trail"
- unsuccessful search when reach empty bucket

What about `delete`?

- use "lazy" deletion
  ↳ replace element with marker/flag to say "no data here, but keep probing"



- Note: `delete` with separate chaining is plain-old list-remove

Practice:

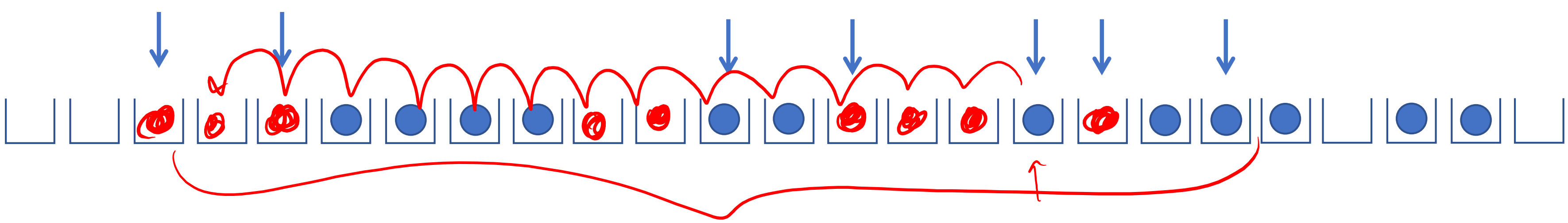
The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

| | (A) | (B) | (C) | (D) |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | 2 | 12 | 12 | 12, 2 |
| 3 | 23 | 13 | 13 | 13, 3, 23 |
| 4 | | | 2 | |
| 5 | 15 | 5 | 3 | 5, 15 |
| 6 | | | 23 | |
| 7 | | | 5 | |
| 8 | 18 | 18 | 18 | 18 |
| 9 | | | 15 | |

# Open Addressing: Linear Probing

$\lambda > 1$ (#elements > table size)

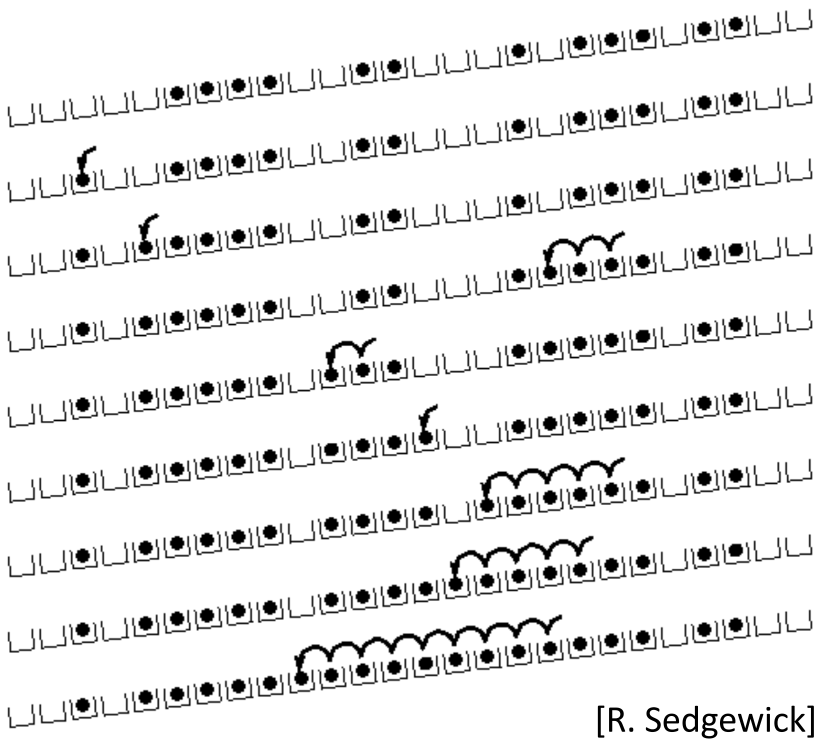
- Quick to compute! ☺
- But mostly a *bad idea*. Why?



# (Primary) Clustering

Linear probing tends to produce ***clusters***, which lead to long probing sequences

- Called primary clustering
- Saw this starting in our example



[R. Sedgewick]

# Analysis of Linear Probing

- For any $\lambda < 1$, linear probing will find an empty slot
  - It is "safe" in this sense: no infinite loop unless table is full
- Non-trivial facts we won't prove:
  Average # of probes given $\lambda$ (in the limit as **`TableSize`** $\rightarrow \infty$)
  - Unsuccessful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$
  - Successful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$

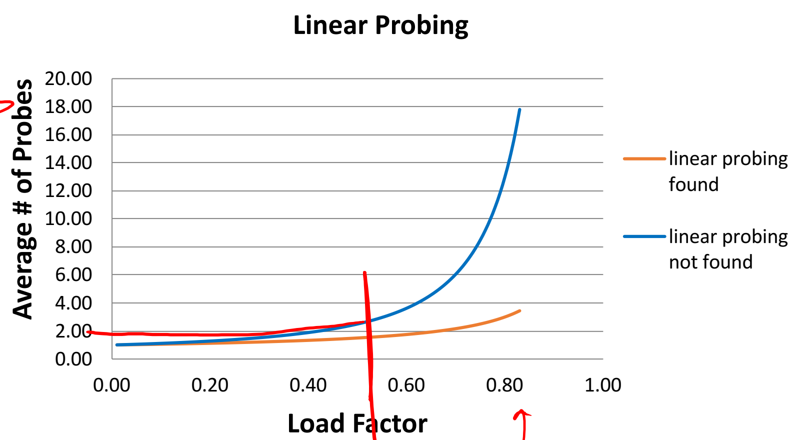
As lambda increases (gets closer to 1), so does the number of probes

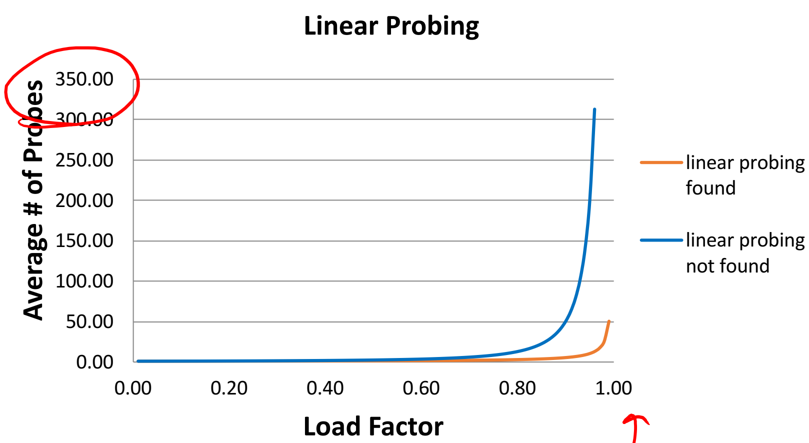
$\lambda = 0.75 \rightarrow$ expect ~8.5 probes
$\lambda = 0.9 \rightarrow$ expect ~50 probes

- This is pretty bad: need to leave sufficient empty space in the table to get decent performance (see chart)

# Analysis: Linear Probing

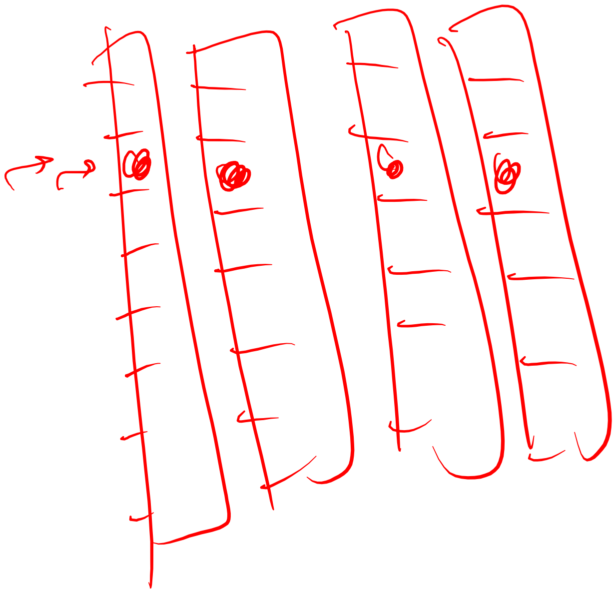
- Linear-probing performance degrades rapidly as table gets full
  (Formula assumes "large table" but point remains)




- By comparison, chaining performance is linear in $\lambda$ and has no trouble with $\lambda > 1$

# Any ideas for alternatives?



Different probe function!

# Open Addressing: Quadratic Probing

- We can avoid primary clustering by changing the probe function

  `(h(key) + f(i)) % TableSize`

(linear probing: $f(i) = i$)

- A common technique is quadratic probing: $\mathbf{f(i) = i^2}$
  - So probe sequence is:
    - $0^{th}$ probe: `h(key) % TableSize`
    - $1^{st}$ probe: (h(key) + 1) % table size
    - $2^{nd}$ probe: (h(key) + 4) % table size
    - $3^{rd}$ probe: (h(key) + 9) % table
    - …
    - $i^{th}$ probe: `(h(key) + i`$^2$`) % TableSize`

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

- Intuition: Probes quickly "leave the neighborhood"

# Quadratic Probing Example #1

| | |
|---|---|
| 0 | 49 |
| 1 | |
| 2 | 58 |
| 3 | 79 |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | 18 |
| 9 | 89 |

TableSize = 10

Insert:

89

18

49

58

79

9+1 % 10 = 0

(8+1)%10 = 9

(8+4)%10 =

$i^{th}$ probe: `(h(key) + i²) % TableSize`

# Quadratic Probing Example #2

| | |
|---|---|
| 0 | 48 |
| 1 | |
| 2 | 5 |
| 3 | 55 |
| 4 | |
| 5 | 40 |
| 6 | 76 |

Yikes! For all n, $(n^2+5)\%7$ is 0, 2, 5, or 6!

47

TableSize = 7

Insert:

| | |
|---|---|
| 76 | (76 % 7 = 6) |
| 40 | (40 % 7 = 5) |
| 48 | (48 % 7 = 6) |
| 5 | ( 5 % 7 = 5) |
| 55 | (55 % 7 = 6) |
| 47 | (47 % 7 = 5) |

$i^{th}$ probe: `(h(key) + i`$^2$`) % TableSize`