CSE 373: Data Structures and Algorithms

Lecture 6: Finishing Amortized Analysis; Dictionaries ADT; Introduction to Hash Tables

Instructor: Lilian de Greef
Quarter: Summer 2017

Today:

- Finish up Amortized Analysis
- Dictionary ADT
- Introduce Hash Tables

Reminder: No class on Monday!

Unofficial holiday – have a good 4-day weekend! ☺

- Will ask you to do a ~30 minute activity to make up for last class time
- Remember that homework 2 is also due the day after we're back

Amortized Analysis

How we calculate the average time!

Amortized Cost

The **amortized cost** of *n* operations is the worst-case total cost of the operations divided by *n*.

Shorthand:

If T(n) = worst-case (upper bound) of total cost

for n = number of operations

$\Rightarrow$ **Amortized Cost = T(n) / n**

Example: Array Stack

What's the amortized cost of calling `push()` *n* times if we double the array size when it's full?

n operations:

- *n* pushes at O(1) each -> total cost = *n*
- cost of resizing = $n + n/2 + n/4 + n/8 + \ldots \leq 2n$ -> total cost $\leq 2n$

-> $T(n) = n + 2n = 3n$

-> Amortized cost = $T(n)/n = 3n/n = 3$

-> Amortized Running time = O(1)

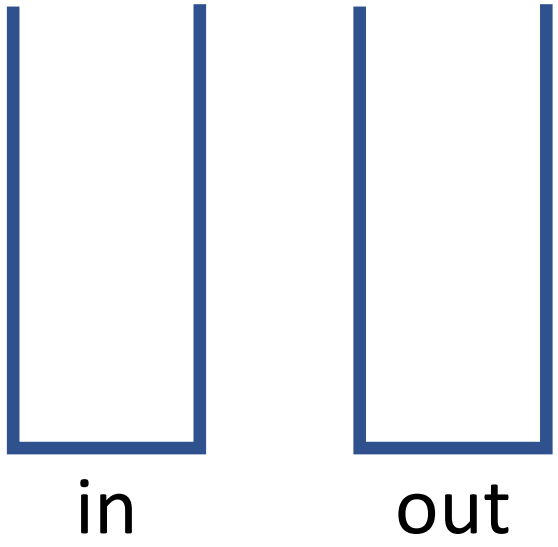
The **amortized cost** of *n* operations is the worst-case total cost of the operations divided by *n*.

Example #2: Queue made of Stacks

A sneaky way to implement Queue using two Stacks

Example walk-through:

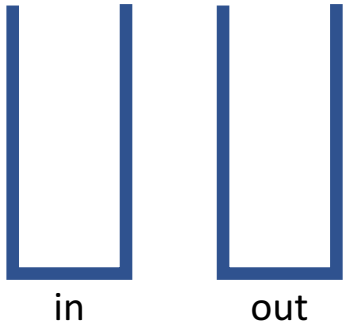
- enqueue A
- enqueue B
- enqueue C
- dequeue
- enqueue D
- enqueue E
- dequeue
- dequeue
- dequeue



Example #2: Queue made of Stacks

A sneaky way to implement Queue using two Stacks

```
class Queue<E> {
  Stack<E> in  = new Stack<E>();
  Stack<E> out = new Stack<E>();
  void enqueue(E x){ in.push(x); }
  E dequeue(){
    if(out.isEmpty()) {
      while(!in.isEmpty()) {
        out.push(in.pop());
      }
    }
    return out.pop();
  }
}
```

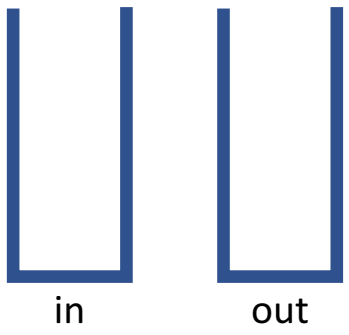


Wouldn't it be nice to have a queue of t-shirts to wear instead of a stack (like in your dresser)?
So have two stacks

- *in*: stack of t-shirts go after you wash them
- *out*: stack of t-shirts to wear
- if *out* is empty, reverse *in* into *out*

Example #2: Queue made of Stacks (Analysis)

```
class Queue<E> {
  Stack<E> in  = new Stack<E>();
  Stack<E> out = new Stack<E>();
  void enqueue(E x){ in.push(x); }
  E dequeue(){
    if(out.isEmpty()) {
      while(!in.isEmpty()) {
        out.push(in.pop());
      }
    }
    return out.pop();
  }
}
```



Assume stack operations are (amortized) O(1).
What's the worst-case for `dequeue()`?

What operations did we need to do to reach that condition (starting with an empty Queue)?

Hence, what is the amortized cost?

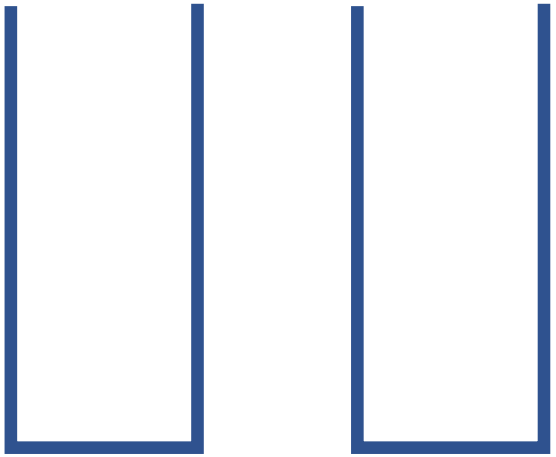
So the average time for `dequeue()` is:

Example #2: Using “Currency” Analogy

“Spend” $2 for every `enqueue` – $1 to the “computer”, $1 to the “bank”.

Example walk-through:

- enqueue A
- enqueue B
- enqueue C
- enqueue D
- enqueue E
- dequeue



Potential Function

Example #3: (Parody / Joke Example)

Lectures are 1 hour long, 3 times per week, so I'm supposed to lecture for 27 hours this quarter.

If I end the first 26 lectures 5 minutes early, then I'd have "saved up" 130 minutes worth of extra lecture time.

Then I could spend it all on the last lecture and can keep you here for 3 hours (bwahahahaha)!

(After all, each lecture would still be 1 hour amortized time)

Wrapping up Amortized Analysis

- In what cases do we care more about the average / amortized run time?

- In what cases do we care more about the worst-case run time?

What have we covered so far?

- Abstract Data Types (**ADTs**)
- Two data structures
  - **Stacks** (both using arrays and linked-lists)
  - **Queues** (including **circular queues**)
- Asymptotic Analysis
  - Intuition for **Big-O**
  - Formally proving Big-O using **Inductive Proofs**
  - Calculating Big-O for recursive methods using **Recurrence Relations**
  - Big-O's cousins: **Big-Ω, Big-θ, little-o, little-ω**
  - Average running time using **Asymptotic Analysis**

Whew!

That was a *lot* of algorithm analysis.

Now shifting gears completely…
on to some new data structures!

Dictionary ADT

Dictionary ADT

Uses of Dictionary ADT

Used to store information with some key and retrieve it efficiently – lots of programs do that!

Examples:

- Contacts in a phone (name: number, email)
- Orca/Husky cards (account number: balance)
- Genome maps (DNA sequence: location on genome)
- Lilian's database of your grades (student ID: assignments, grades)
- Networks (router tables), Operating Systems (page tables), Compilers (symbol tables), Databases
- … and so much more!

Possibly the most widely used ADT!

Motivating Hash Tables

Creative thinking time: how could you implement a dictionary using what you know so far (namely, linked-lists and arrays)?

e.g. map names (key) to phone numbers (value)

(extra space for notes)

Motivating Hash Tables

Running times for Dictionary operations with *n* (key, value) pairs:

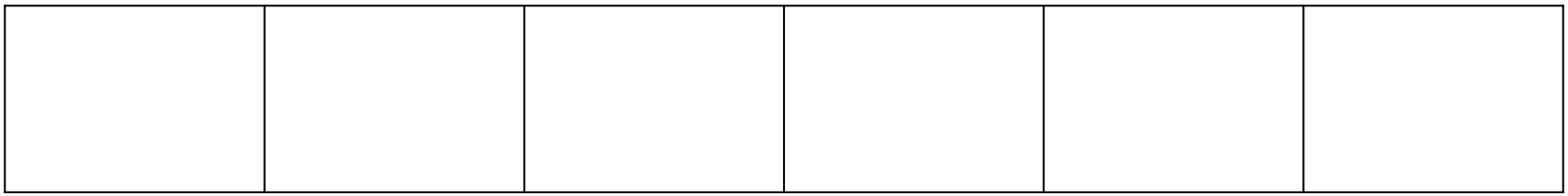
	`insert`	`find`	`delete`
“Magic Array”			

“Magic Array”

Use key to compute array index for an item in O(1) time

Example: phone contacts (name, number)

name → `index` = `computeIndex(name)` → `array[index]` = (name, number)



“Magic Array”

Use key to compute array index for an item in O(1) time

Example: phone contacts (name, number)

name → `index` = `computeIndex(name)` → `array[index]` = (name, number)

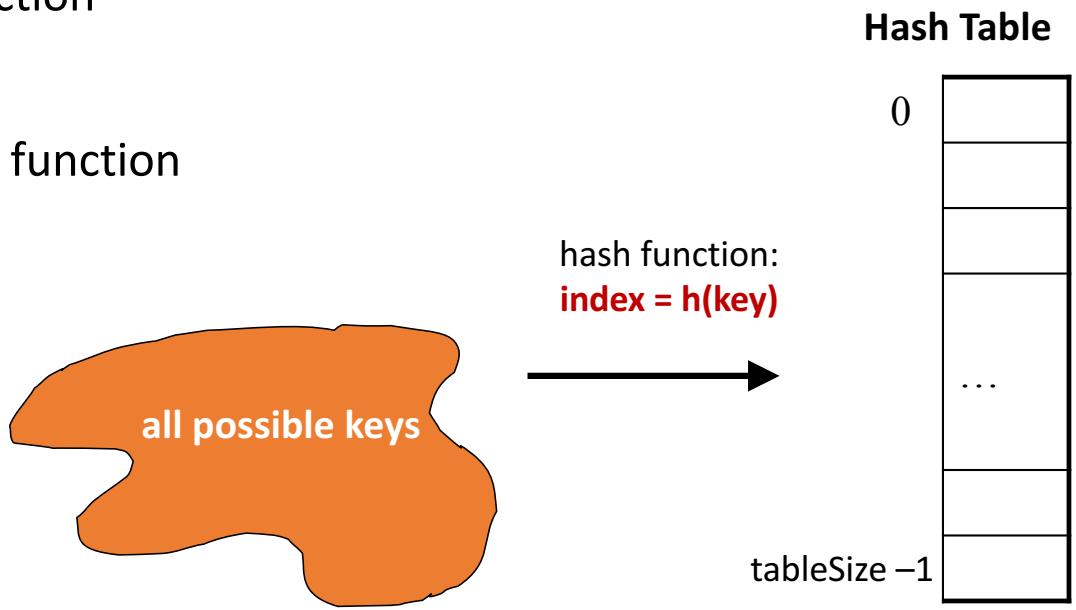
What would be important about the indices from `computeIndex`?

Introducing… Hash Tables!

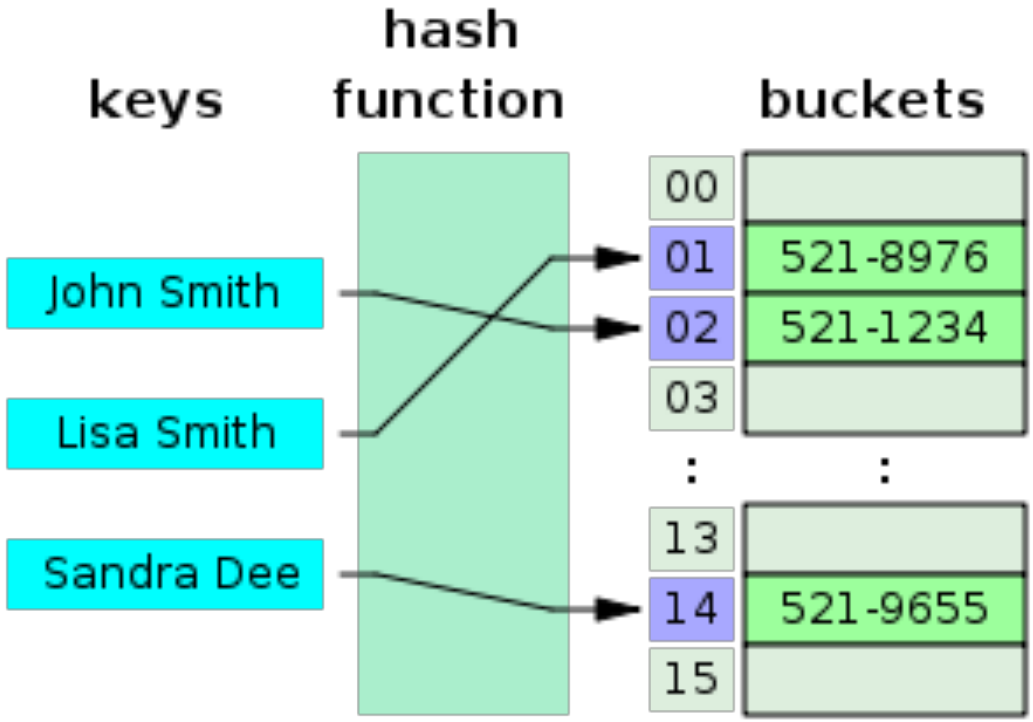
Closest thing to our “magic array”

Hash Tables: closest thing to our "Magic Array"

- *Average case* O(1) `find`, `insert`, and `delete`
  (when under some often-reasonable *assumptions*)
- Our `computeIndex` function is called a
- The `index` from the hash function is called a



Hash Tables: Example Illustration



Hash Functions

Hash functions need to…

-
-
-

For a person's name, would it be a good hash function to…

- Use the ASCII values of first and second letter?
- Use the number of letters in the name?

Example Hash Function

Hash function “djb2”:

```
unsigned long
hash(unsigned char *str)
{
    unsigned long hash = 5381;
    int c;

    while (c = *str++)
        hash = ((hash << 5) + hash) + c; /* hash * 33 + c */

    return hash;
}
```

Hash Functions

- Many datatypes and Objects are hashable
- When writing a class, can make it hashable!

  Do so by implementing `hashCode` method

- We'll focus on `int`s and `String`s in this class.

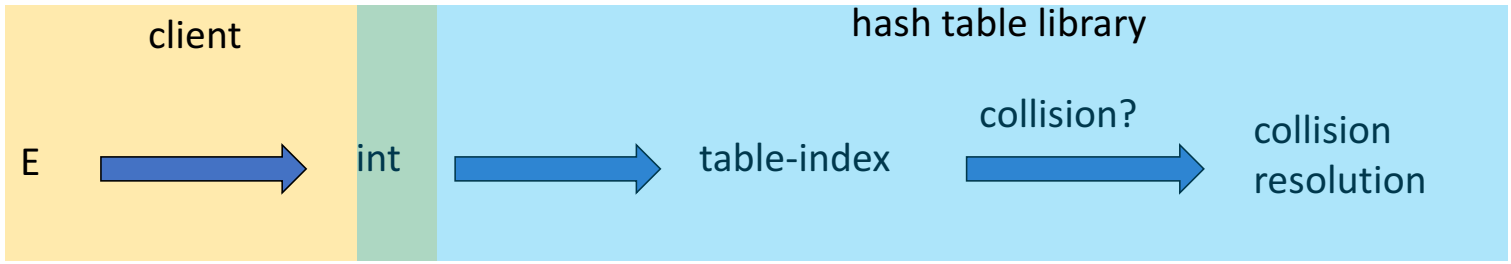
Collisions

Happens when two elements get the same index (unavoidable in practice)

Homework: come up with a strategy, write it down on paper, and bring it to class on Weds

Hash Table roles

When hash tables are a reusable library, the division of responsibility generally breaks down into two roles:



We will learn both roles, but most programmers “in the real world” spend more time as clients while understanding the library

Hash Tables

- There are ***m*** possible keys (*m* typically large, even infinite)
- We expect our table to have only ***n*** items
- ***n*** is much less than ***m*** (often written ***n << m***)

Many dictionaries have this property

- Compiler: All possible identifiers allowed by the language vs. those used in some file of one program
- Database: All possible student names vs. students enrolled
- AI: All possible chess-board configurations vs. those considered by the current player
- …

Hash Table Size

- How can we keep hash values (i.e. the indices) within the table size?

-

- Table size usually prime
  - Real-life data tends to have a pattern
  - "Multiples of 61" probably less likely than "multiples of 60"
  - Helpful for a collision-handling strategy we'll see next week

Review: Hash Tables thus far…

- The hash table is one of the most important data structures – Supports only `find`, `insert`, and `delete` efficiently
  - Have to search entire table for other operations
- Important to use a good hash function
- Important to keep hash table at a good size
- Side-comment: hash functions have uses beyond hash tables – Examples: Cryptography, check-sums
- Big remaining topic: Handling collision

Homework

Come up with a collision-resolution strategy, write it down on paper, and bring it to class on Wednesday

Goal: prime our brains for learning the most common collision-resolution strategies.