CSE 373: Data Structures and Algorithms

Lecture 2: Wrap up Queues, Asymptotic Analysis, Proof by Induction

Instructor: Lilian de Greef
Quarter: Summer 2017

Today:

- Announcements
- Wrap up Queues
- Begin Asymptotic Analysis: Big-O
- Proof by Induction

Announcement: Office Hours

- Announced! See course webpage for times
- Most held in 3rd floor breakouts in CSE (whiteboards near stairs)
- Lilian's additional "actual office" office hours
  - CSE 220 (a more private environment)
  - During listed times
  - And by appointment! (email me >24 hours ahead of time with several times that work for you)
  - Come talk to me about anything! (feedback, grad school, Ultimate Frisbee, life problems, whatever)

Announcement: Sections

- When & where: listed on course webpage
- What: TA-led…
  - Review sessions of course material
  - Practice problems
  - Question-answering
- Optional, but highly encouraged!

I wouldn't have passed 332 (Data Structures and Parallelism) without regularly going to section! – Vlad (TA)

Other Announcements

- Homework 1 is out
  - On material covered in Lecture 1
  - Go forth!
  - …or at least get Eclipse set up today.
- Only required course reading:
  - 10 pages, easy read on commenting style
  - Due beginning of class on Monday
- July 3rd
  - Not an official UW holiday *(sorry guys)*
  - But I'm declaring it an unofficial holiday! Go enjoy a 4-day July 4th weekend

University Holidays

Classes are not in session on the following holidays:

SUMMER 2017		
Full-term	**A-term**	**B-term**
July 4, 2017 Independence Day	July 4, 2017 Independence Day	

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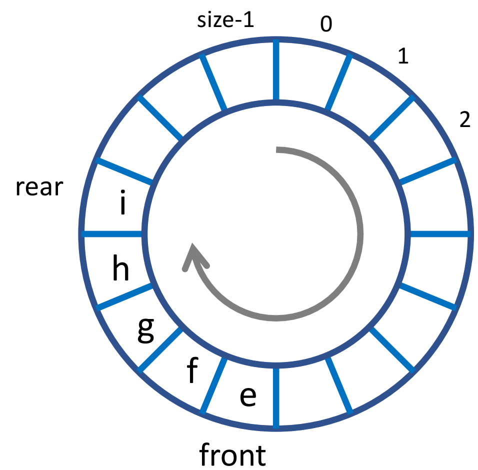
Finishing up Queues

Let’s resolve that cliff-hanger!

If we can assume the queue is not empty, how can we implement dequeue()?

```
Public E dequeue() {
    size--;
    E e = array[front];
    <Your code here!>
    return e;
}
```



A)
```
front++;
if (front == array.length)
    front = 0;
```

B)
```
rear = rear-1;
if (rear < 0)
    rear = array.length-1;
```

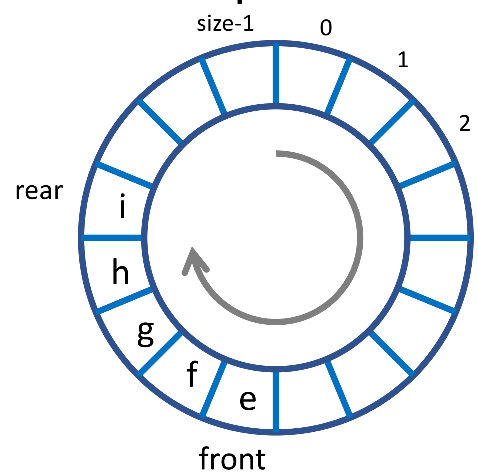
C)
```
for (int i = 0; i < rear; i++) {
    array[i] = array[i+1]
}
front++;
if (front == array.length)
    front = 0;
```

D) None of these are correct

(Notes for yourself)

If we can assume the array is not full, how can we implement enqueue(E e)?

```
Public enqueue(E e) {
   <Your code here!>
   size++;
}
```



A)
```
rear++;
if (rear == array.length)
    rear = 0;
array[rear] = e;
```

B)
```
rear++;
array[rear] = e;
```

C)
```
for (int i=front; i<rear; i++) {
    array[i] = array[i+1]
}
array[rear] = e;
rear++;
```

D) None of these are correct

(Notes for yourself)

Between arrays and linked-lists which one *always* is the fastest at `enqueue`, `dequeue`, and `seeKthElement` operations?

(where `seeKthElement` lets you peek at the kth element in the stack)

Fastest:	`enqueue`	`dequeue`	`seeKthElement`
A)	Arrays	Linked-Lists	Neither
B)	Linked-lists	Neither	Neither
C)	Linked-lists	Neither	Arrays
D)	*They're all the same*		

(Notes for yourself)

Which one’s better?

Arrays

Linked-lists

Trade-offs!

- The ability to choose wisely between trade-offs is why it's important to understand underlying data structures.
- Common Trade-offs
  - Time vs space
  - One operation's efficiency vs another
  - Generality vs simplicity vs performance

Asymptotic Analysis

Oh ho! The Big-O!

Algorithm Analysis

- Why: to help choose the right algorithm or data structure for the job
- Often in **asymptotic** terms

- Most common way: **Big-O Notation**
  - General idea:

  - A common way to describe “worst-case running time”

Example #1:

The `barn` is an array of `Cows`, excitement is an integer, and `Cow.addHat()` runs in constant time.

```
println("The alien is visiting!");
println("Party time!");
excitement++;
for (int i=0; i<barn.length; i++) {
    Cow cow = barn[i];
    cow.addHat();
}
```

Let's assume that one line of code takes 1 "unit of time" to run
This is not always true, i.e. calls to non-constant-time methods)

Important! Always begin by specifying what “n” is!
(or “x”or “y” or whatever letter)

Example #1:

```
println("The alien is visiting!");
println("Party time!");
excitement++;
for (int i=0; i<barn.length; i++) {
    Cow cow = barn[i];
    cow.addHat();
}
```

Example #2: Your turn!

```
for (Person player: sportsTeam) {
  player.smile();
  for (Person teamMate: sportsTeam) {
    player.say("Good game!");
    player.highFive(teamMate);
}
```

Assume that the above `Person` method calls run in constant time

What's the asymptotic runtime of this (semi-)pseudocode?

```
 x := 0;
for i=1 to N do
  for j=1 to i do
     x := x + 3;
return x;
```

A. O(n)
B. O(n^2)
C. O(n + n/2)
D. None of the above

What's the asymptotic runtime of this (semi-)pseudocode?

```
x := 0;
for i=1 to N do
  for j=1 to i do
     x := x + 3;
return x;
```

A. O(n)
B. $O(n^2)$
C. O(n + n/2)
D. None of the above

How do we prove the right answer?

Proof by Induction!

Inductive Proofs

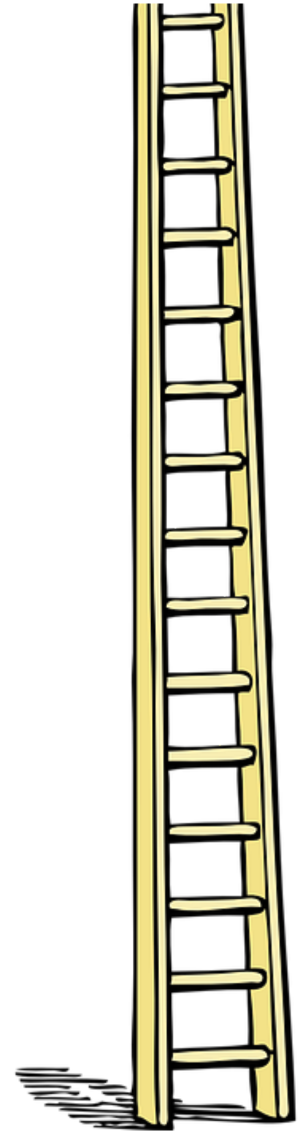
(Interlude from Asymptotic Analysis)

Steps to Inductive Proof

1. If not given, **define n** (or “x” or “t” or whatever letter you use)
2. **Base Case**
3. **Inductive Hypothesis (IHOP):**
   Assume what you want to prove is true for some arbitrary value k
   (or “p” or “d” or whatever letter you choose)
4. **Inductive Step**:
   Use the base case and IHOP to prove it's true for n = k+1

Example #0:
Proof that I can climb any length ladder

1. **Let n =** number of rungs on a ladder.
2. **Base Case:** for n = 1
3. **Inductive Hypothesis (IHOP):**
   Assume true for some arbitrary integer n = k.
4. **Inductive Step:** (aiming to prove it's true for n = k+1)
   - If I climb k steps of the ladder, then I have one step left to go.
   - By IHOP, I can climb k steps of the ladder.
   - By Base Case, I can climb the last step.
   - So I can climb k+1 steps.
   - I can climb forever!



Example #1

Prove that the number of loop iterations is $\frac{n*(n+1)}{2}$

```
x := 0;
for i=1 to N do
  for j=1 to i do
     x := x + 3;
return x;
```

(Extra room for notes)

Example #2:

Prove that $1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 1$

(Extra room for notes)

Useful Mathematical Property!

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

You’ll use it or see it again before the end of CSE 373.

Powers of 2

- A bit is 0 or 1 (just two different "letters" or "symbols")
- A sequence of *n* bits can represent 2^n distinct things
  (For example, the numbers 0 through 2^n-1)
- 2^{10} is 1024 ("about a thousand", kilo in CSE speak)
- 2^{20} is "about a million", mega in CSE speak
- 2^{30} is "about a billion", giga in CSE speak

Java: an `int` is 32 bits and signed, so "max int" is "about 2 billion"
a `long` is 64 bits and signed, so "max long" is 2^{63}-1

Which means…

You could give a unique id to…

- Every person in the U.S. with 29 bits
- Every person in the world with 33 bits
- Every person to have ever lived with 38 bits (estimate)
- Every atom in the universe with 250-300 bits

So if a password is 128 bits long and randomly generated,
do you think you could guess it?

Example #3: (Parody) Reverse Induction!

Proof by Reverse Induction That You Can Always Cage a Lion:

Let n = number of lions

Base Case: There exists some countable, arbitrarily large value of M such that when n = M, the lions are so packed together that it's trivial to cage one.

I**HOP**: Assume this is also true for n = k.

Inductive Step: Then for n = k-1, release a lion to reduce the problem to the case of n = k, which by the IHOP is true.

QED :)

Fun fact: Reverse induction is a thing! The math part of the above is actually correct.