CSE 373

MAY 31ST – EVAN'S FUN LECTURE

ASSORTED MINUTIAE

- Exam Review – Friday 4:30 – 6:00 EEB 105

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- **Exam: Tue Jun 6, 2:30 – 4:20**

TODAY'S LECTURE

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  - Randomization Rant

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  - Randomization Rant
  - Hardware constraints

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  - How bad is it?
    - Necessary for some hard problems
    - Still can be useful for some easier problems

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  - Suppose we want to have a confidence equal to α, how do we get this?

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$k * \ln(1-p) = \ln \alpha$

$$k = \frac{(\ln \alpha)}{(\ln(1-p))}$$

$k = \log_{(1-p)} \alpha$

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- **Cool, I guess… but what does this mean?**
- **Suppose P = 0.5 (we only have a 50% chance of success on any given run) and α = 0.001, we only tolerate a 0.1% error**
- **How many runs do we need to get this level of confidence?**
  - Only 10! This is a constant multiple

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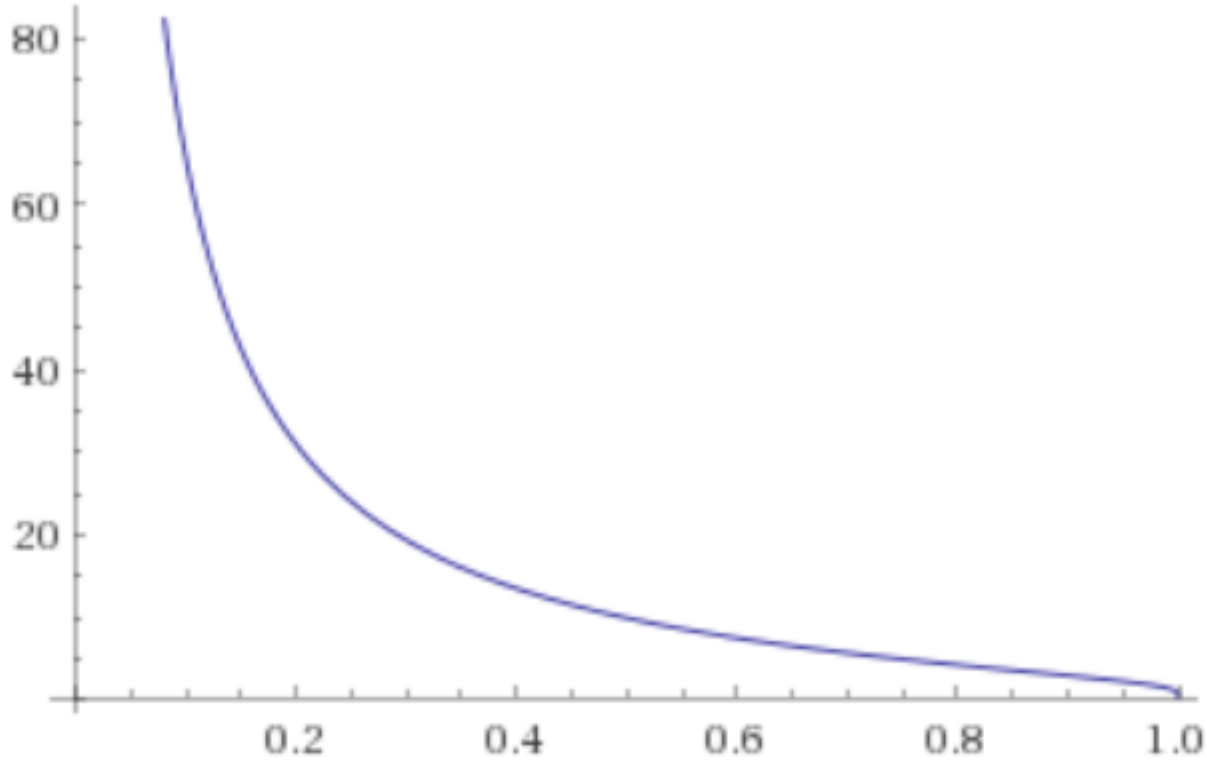

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- **Even if p is 0.1, only a 10% chance of success, we only need to run the algorithm 80 times to get a 0.001 confidence level**
- **What does this mean?**
  - Randomized algorithms don't have to be complicated, if you can create a *reasonable* guess and can verify it in a short amount of time, then you can get good performance just from running repeatedly.

MINCUT

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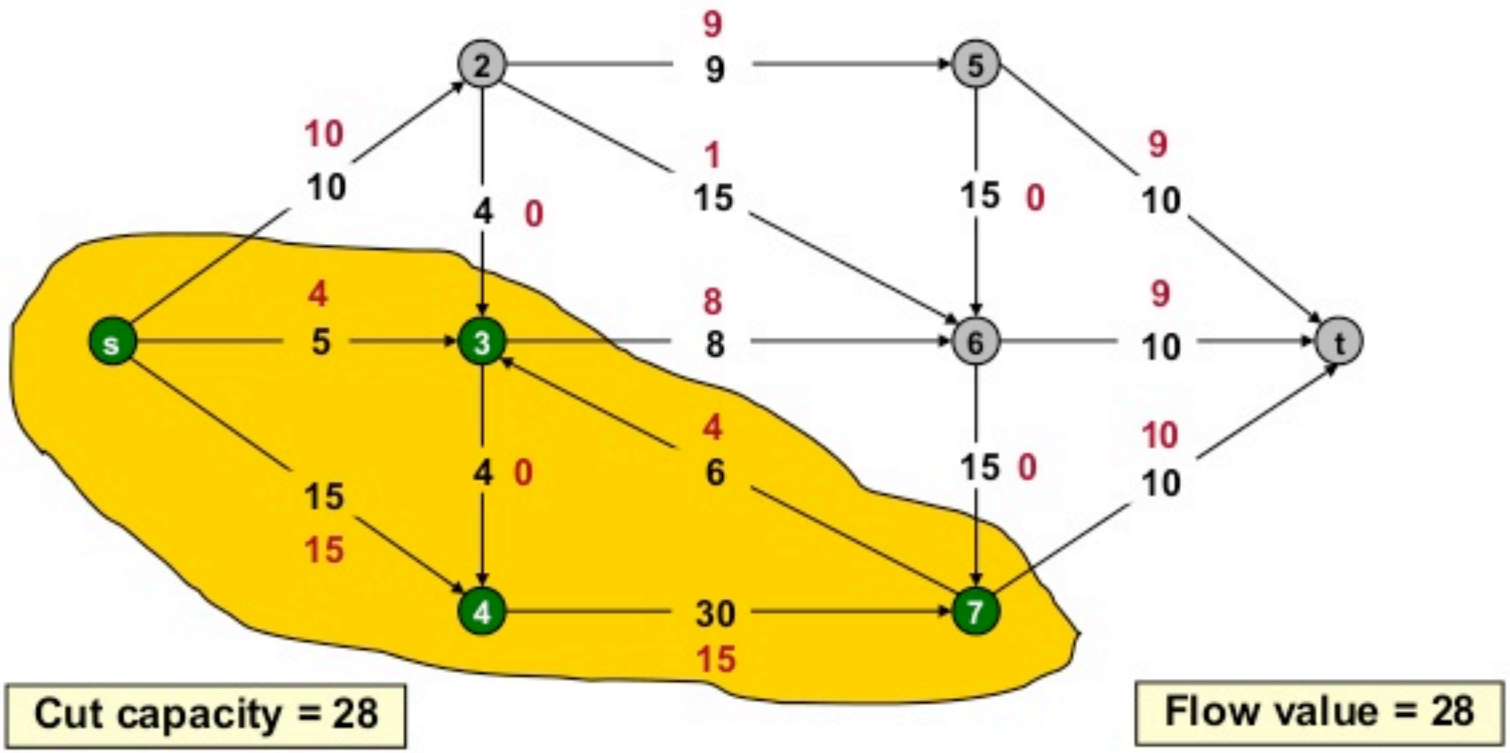
- **Why do we even care?**
  - The min-cut is the maximum flow, if we are trying to connect two cities, the limit of traffic flow between nodes in the network

Max-Flow Min-Cut Theorem

MAX-FLOW MIN-CUT THEOREM (Ford-Fulkerson, 1956): In any network, the value of the max flow is equal to the value of the min cut.

- "Good characterization."
- Proof IOU.



FORD-FULKERSON

Algorithm [edit]

Let $G(V,E)$ be a graph, and for each edge from u to v, let $c(u,v)$ be the capacity and $f(u,v)$ be the flow. We want to find the maximum flow from the source s to the sink t. After every step in the algorithm the following is maintained:

Capacity constraints:	$\forall(u,v)\in E\ f(u,v)\le c(u,v)$	The flow along an edge can not exceed its capacity.
Skew symmetry:	$\forall(u,v)\in E\ f(u,v)=-f(v,u)$	The net flow from u to v must be the opposite of the net flow from v to u (see example).
Flow conservation:	$\forall u\in V: u\neq s \text{ and } u\neq t \Rightarrow \sum_{w\in V} f(u,w)=0$	That is, unless u is s or t. The net flow to a node is zero, except for the source, which "produces" flow, and the sink, which "consumes" flow.
Value(f):	$\sum_{(s,u)\in E} f(s,u)=\sum_{(v,t)\in E} f(v,t)$	That is, the flow leaving from s must be equal to the flow arriving at t.

This means that the flow through the network is a *legal flow* after each round in the algorithm. We define the **residual network** $G_f(V,E_f)$ to be the network with capacity $c_f(u,v)=c(u,v)-f(u,v)$ and no flow. Notice that it can happen that a flow from v to u is allowed in the residual network, though disallowed in the original network: if $f(u,v)>0$ and $c(v,u)=0$ then $c_f(v,u)=c(v,u)-f(v,u)=f(u,v)>0$.

Algorithm Ford–Fulkerson

Inputs Given a Network $G=(V,E)$ with flow capacity c, a source node s, and a sink node t

Output Compute a flow f from s to t of maximum value

1. $f(u,v)\leftarrow 0$ for all edges (u,v)
2. While there is a path p from s to t in G_f, such that $c_f(u,v)>0$ for all edges $(u,v)\in p$:
   1. Find $c_f(p)=\min\{c_f(u,v):(u,v)\in p\}$
   2. For each edge $(u,v)\in p$
      1. $f(u,v)\leftarrow f(u,v)+c_f(p)$ (*Send flow along the path*)
      2. $f(v,u)\leftarrow f(v,u)-c_f(p)$ (*The flow might be "returned" later*)

The path in step 2 can be found with for example a breadth-first search or a depth-first search in $G_f(V,E_f)$. If you use the former, the algorithm is called Edmonds–Karp.

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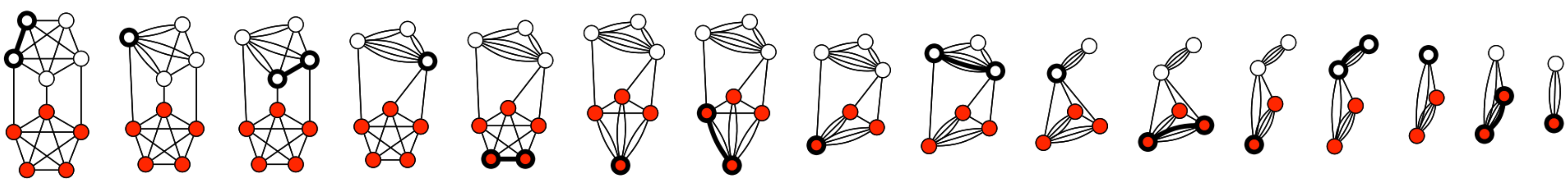
- **Contract edges at random!**
  - How many edges will you contract to get two subgraphs?



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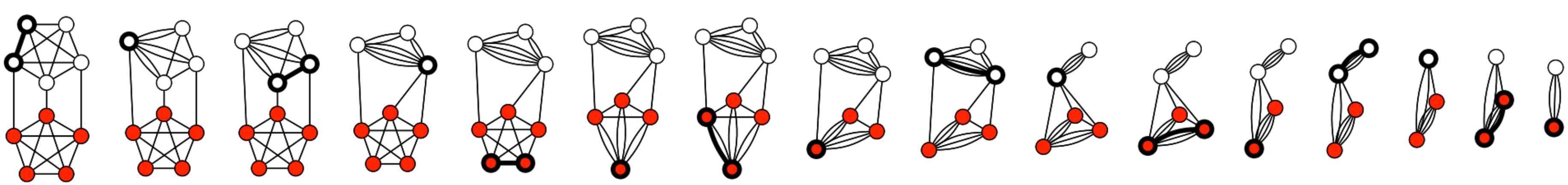

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- **Does this work?**
  - Success probability of 2/|E|
  - Run it O(E) times, and you have a bounded success rate!

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- **So far, we've taken for granted that memory access in the computer is constant and easily accessible**
  - This isn't always true!
  - At any given time, some memory might be cheaper and easier to access than others
  - Memory can't always be accessed easily
  - Sometimes the OS lies, and says an object is "in memory" when it's actually on the disk

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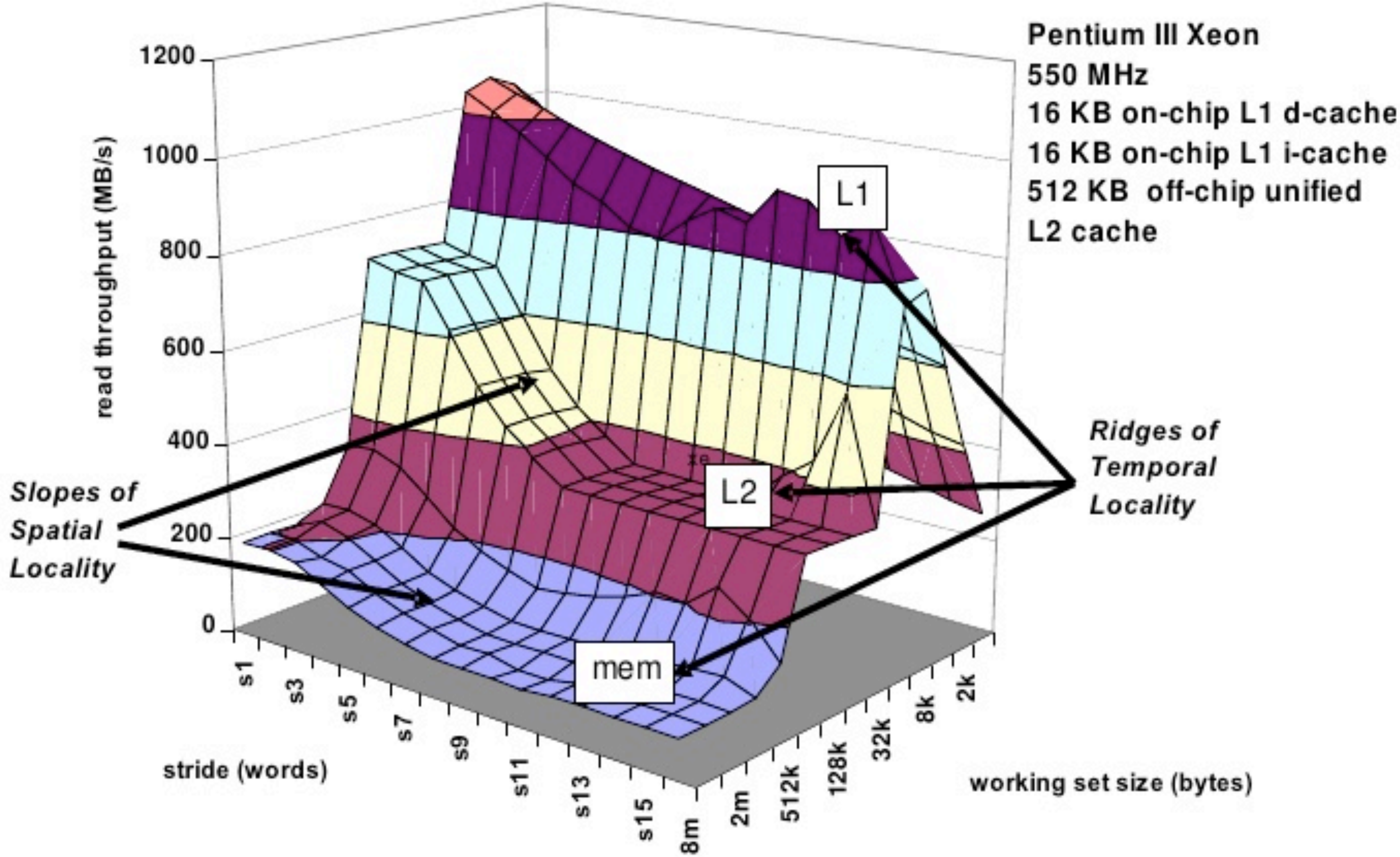
  - This isn't feasible to provide!
  - Sometimes there isn't enough available, and so memory that hasn't been used in a while gets pushed to the disk
- **Memory that is frequently accessed goes to the cache, which is even faster than RAM**

The Memory Mountain



LOCALITY AND PAGES

- **So, the OS does two smart things**
  - Spatial locality – if you use memory index Ox371347AB, you are likely to need Ox371347AC – bring both into cache
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  - Spatial locality – if you use memory index Ox371347AB, you are likely to need Ox371347AC – bring both into cache
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  - All of the processes on your computer have access to pages in memory.

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  - So if you allocate 100Bytes of data, you overallocate to 4kb!
  - But you can use that 4kb if you need more

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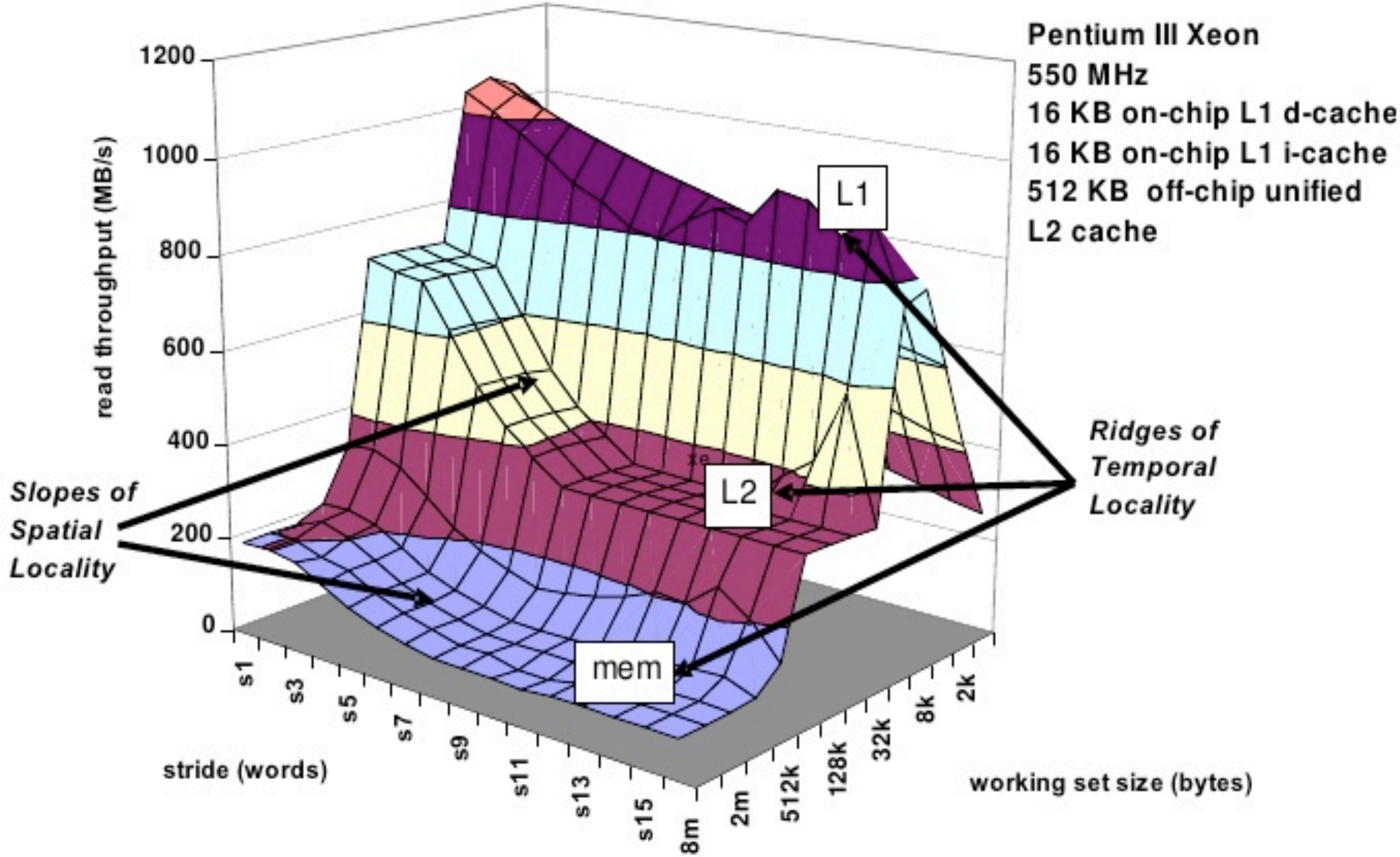

LOCALITY AND PAGES

- **Secondly, the OS uses temporal locality,**
  - Memory recently accessed is likely to be accessed again
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- **Types of memory (by speed)**
  - Register
  - L1,L2,L3
  - Memory
  - Disk
  - The interwebs (the cloud)

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- **Important to consider when designing and explaining design problems.**

FRIDAY

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