CSE 373

MAY 26TH –NON-COMPARISON SORTING

ASSORTED MINUTIAE

- **HW6 Out – Due next Wednesday**
  - No Java Libraries

ASSORTED MINUTIAE

- **HW6 Out – Due next Wednesday**
  - No Java Libraries
- **Two exam review sessions**
  - Wednesday: 1:00 – 2:20 – CMU 120
  - Friday: 4:30 – 6:20 – EEB 105

TODAY

- Non-comparison sorts

SORTING

- "Slow" sorts

SORTING

- **"Slow" sorts**
  - Insertion
  - Selection

SORTING

- **“Slow” sorts**
  - Insertion
  - Selection
- **“Fast” sorts**

SORTING

- **“Slow” sorts**
  - Insertion
  - Selection
- **“Fast” sorts**
  - Quick
  - Merge
  - Heap

SORTING

- **“Slow” sorts**
  - Insertion
  - Selection
- **“Fast” sorts**
  - Quick
  - Merge
  - Heap
- **These are all comparison sorts, can’t do better than O(n log n)**

SORTING

- **Non-comparison sorts**

SORTING

- **Non-comparison sorts**
  - If we know something about the data, we don't strictly need to compare objects to each other

SORTING

- **Non-comparison sorts**
  - If we know something about the data, we don't strictly need to compare objects to each other
  - If there are only a few possible values and we know what they are, we can just sort by identifying the value

SORTING

- **Non-comparison sorts**
  - If we know something about the data, we don't strictly need to compare objects to each other
  - If there are only a few possible values and we know what they are, we can just sort by identifying the value
  - If the data are strings and ints of finite length, then we can take advantage of their sorted order.

SORTING

- Two sorting techniques we use to this end

SORTING

- **Two sorting techniques we use to this end**
  - Bucket sort

SORTING

- **Two sorting techniques we use to this end**
  - Bucket sort
  - Radix sort

SORTING

- **Two sorting techniques we use to this end**
  - Bucket sort
  - Radix sort
- **If the data is sufficiently structured, we can get O(n) runtimes**

BUCKETSORT

If all values to be sorted are known to be integers between 1 and *K* (or any small range):

- Create an array of size *K*
- Put each element in its proper bucket (a.k.a. bin)
- *If* data is only integers, no need to store more than a *count* of how times that bucket has been used

Output result via linear pass through array of buckets

`count` array	
1	3
2	1
3	2
4	2
5	3

- Example:
  K=5
  input (5,1,3,4,3,2,1,1,5,4,5)
  output: 1,1,1,2,3,3,4,4,5,5,5

ANALYZING BUCKET SORT

Overall: $O(n+K)$

- Linear in n, but also linear in K

Good when K is smaller (or not much larger) than n

- We don't spend time doing comparisons of duplicates

Bad when K is much larger than n

- Wasted space; wasted time during linear $O(K)$ pass

For data in addition to integer keys, use list at each bucket

BUCKET SORT

Most real lists aren't just keys; we have data

Each bucket is a list (say, linked list)

To add to a bucket, insert in *O*(1) (at beginning, or keep pointer to last element)

count array	
1	→ Rocky V
2	
3	→ Harry Potter
4	
5	→ Casablanca → Star Wars

- Example: Movie ratings; scale 1-5

  Input:

  5: Casablanca

  3: Harry Potter movies

  5: Star Wars Original Trilogy

  1: Rocky V

- Result: 1: Rocky V, 3: Harry Potter, 5: Casablanca, 5: Star Wars
- Easy to keep 'stable'; Casablanca still before Star Wars

RADIX SORT

Radix = "the base of a number system"

- Examples will use base 10 because we are used to that
- In implementations use larger numbers
    - For example, for ASCII strings, might use 128

Idea:

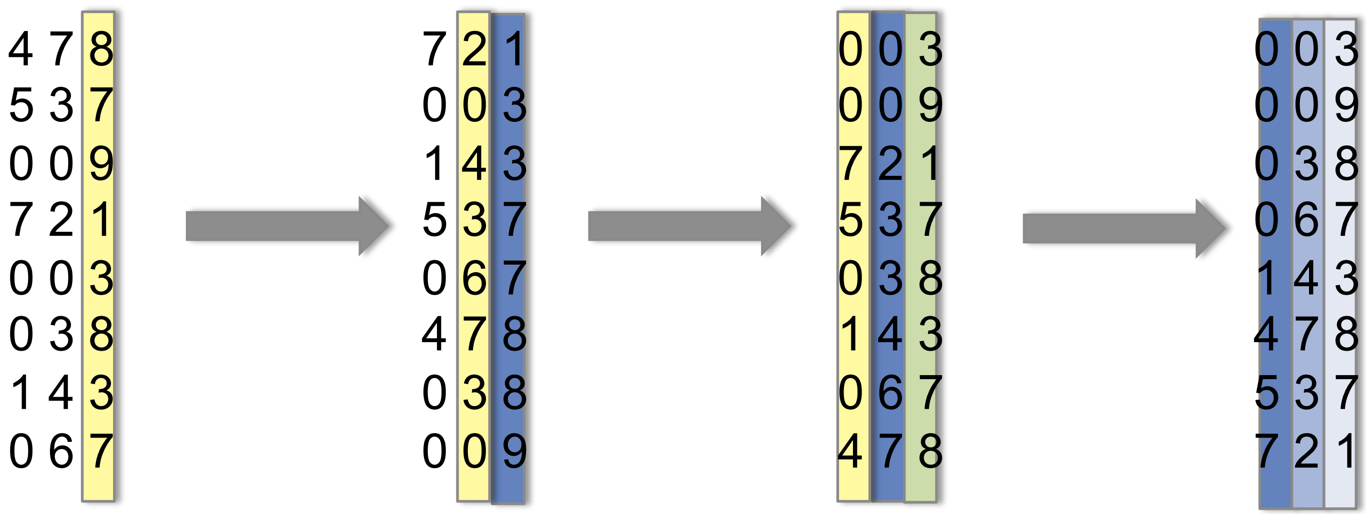
- Bucket sort on one digit at a time
    - Number of buckets = radix
    - Starting with *least* significant digit
    - Keeping sort *stable*
- Do one pass per digit
- **Invariant**: After *k* passes (digits), the last *k* digits are sorted

RADIX SORT EXAMPLE

Radix = 10

Input: 478, 537, 9, 721, 3, 38, 143, 67

3 passes (input is 3 digits at max), on each pass, stable sort the input highlighted in yellow



ANALYSIS

Input size: *n*

Number of buckets = Radix: *B*

Number of passes = "Digits": *P*

Work per pass is 1 bucket sort: $O(B+n)$

Total work is $O(P(B+n))$

Compared to comparison sorts, sometimes a win, but often not

- Example: Strings of English letters up to length 15
  - Run-time proportional to: $15*(52 + n)$
  - This is less than $n \log n$ only if $n > 33{,}000$
  - Of course, cross-over point depends on constant factors of the implementations

SORTING TAKEAWAYS

Simple $O(n^2)$ sorts can be fastest for small n

- Selection sort, Insertion sort (latter linear for mostly-sorted)
- Good for “below a cut-off” to help divide-and-conquer sorts

SORTING TAKEAWAYS

Simple $O(n^2)$ sorts can be fastest for small n

- Selection sort, Insertion sort (latter linear for mostly-sorted)
- Good for "below a cut-off" to help divide-and-conquer sorts

$O(n \texttt{ log } n)$ sorts

- Heap sort, in-place but not stable nor parallelizable
- Merge sort, not in place but stable and works as external sort
- Quick sort, in place but not stable and $O(n^2)$ in worst-case
  - Often fastest, but depends on costs of comparisons/copies

SORTING TAKEAWAYS

Simple $O(n^2)$ sorts can be fastest for small n

- Selection sort, Insertion sort (latter linear for mostly-sorted)
- Good for "below a cut-off" to help divide-and-conquer sorts

$O(n \log n)$ sorts

- Heap sort, in-place but not stable nor parallelizable
- Merge sort, not in place but stable and works as external sort
- Quick sort, in place but not stable and $O(n^2)$ in worst-case
    - Often fastest, but depends on costs of comparisons/copies

$\Omega(n \log n)$ is worst-case and average lower-bound for sorting by comparisons

SORTING TAKEAWAYS

Simple $O(n^2)$ sorts can be fastest for small *n*

- Selection sort, Insertion sort (latter linear for mostly-sorted)
- Good for "below a cut-off" to help divide-and-conquer sorts

$O(n \log n)$ sorts

- Heap sort, in-place but not stable nor parallelizable
- Merge sort, not in place but stable and works as external sort
- Quick sort, in place but not stable and $O(n^2)$ in worst-case
    - Often fastest, but depends on costs of comparisons/copies

$\Omega(n \log n)$ is worst-case and average lower-bound for sorting by comparisons

Non-comparison sorts

- Bucket sort good for small number of possible key values
- Radix sort uses fewer buckets and more phases

Best way to sort? It depends!

SORTING TAKEAWAYS

Simple $O(n^2)$ sorts can be fastest for small n

- Selection sort, Insertion sort (latter linear for mostly-sorted)
- Good for "below a cut-off" to help divide-and-conquer sorts

$O(n \log n)$ sorts

- Heap sort, in-place but not stable nor parallelizable
- Merge sort, not in place but stable and works as external sort
- Quick sort, in place but not stable and $O(n^2)$ in worst-case
    - Often fastest, but depends on costs of comparisons/copies

$\Omega(n \log n)$ is worst-case and average lower-bound for sorting by comparisons

Non-comparison sorts

- Bucket sort good for small number of possible key values
- Radix sort uses fewer buckets and more phases

Best way to sort? It depends!

ALGORITHM DESIGN

- Solving well known problems is great, but how can we use these lessons to approach new problems?

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)
  - Linear Solving

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)
  - Linear Solving
  - Divide and Conquer

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)
  - Linear Solving
  - Divide and Conquer
  - *Randomization and Approximation*

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)
  - Linear Solving
  - Divide and Conquer
  - *Randomization and Approximation*
  - *Dynamic Programming*

LINEAR SOLVING

- **Basic linear approach to problem solving**

LINEAR SOLVING

- Basic linear approach to problem solving
- If the decider creates a set of correct answers, find one at a time

LINEAR SOLVING

- **Basic linear approach to problem solving**
- **If the decider creates a set of correct answers, find one at a time**
  - Selection sort: find the lowest element at each run through
- **Sometimes, the best solution**
  - Find the smallest element of an unsorted array

ALGORITHM DESIGN

- Which approach should be used comes down to how difficult the problem is

ALGORITHM DESIGN

- **Which approach should be used comes down to how difficult the problem is**
- **How do we describe problem difficulty?**
  - P : Set of problems that can be solved in polynomial time

ALGORITHM DESIGN

- **Which approach should be used comes down to how difficult the problem is**
- **How do we describe problem difficulty?**
  - P : Set of problems that can be solved in polynomial time
  - NP : Set of problems that can be verified in polynomial time

ALGORITHM DESIGN

- **Which approach should be used comes down to how difficult the problem is**
- **How do we describe problem difficulty?**
  - P : Set of problems that can be solved in polynomial time
  - NP : Set of problems that can be verified in polynomial time
  - EXP: Set of problems that can be solved in exponential time

ALGORITHM DESIGN

- Some problems are provably difficult

ALGORITHM DESIGN

- **Some problems are provably difficult**
  - Humans haven't beaten a computer in chess in years, but computers are still far away from "solving" chess

ALGORITHM DESIGN

- **Some problems are provably difficult**
  - Humans haven’t beaten a computer in chess in years, but computers are still far away from “solving” chess
  - At each move, the computer needs to approximate the best move

ALGORITHM DESIGN

- **Some problems are provably difficult**
  - Humans haven’t beaten a computer in chess in years, but computers are still far away from “solving” chess
  - At each move, the computer needs to approximate the best move
  - Certainty always comes at a price

APPROXIMATION DESIGN

- What is approximated in the chess game?

APPROXIMATION DESIGN

- **What is approximated in the chess game?**
  - Board quality – If you could easily rank which board layout in order of quality, chess is simply choosing the best board

APPROXIMATION DESIGN

- **What is approximated in the chess game?**
  - Board quality – If you could easily rank which board layout in order of quality, chess is simply choosing the best board
  - It is very difficult, branching factor for chess is ~35

APPROXIMATION DESIGN

- **What is approximated in the chess game?**
  - Board quality – If you could easily rank which board layout in order of quality, chess is simply choosing the best board
  - It is very difficult, branching factor for chess is ~35
  - Look as many moves into the future as time allows to see which move yields the best outcome

APPROXIMATION DESIGN

- Recognize what piece of information is costly and useful for your algorithm

APPROXIMATION DESIGN

- **Recognize what piece of information is costly and useful for your algorithm**
  - Consider if there is a cheap way to estimate that information

APPROXIMATION DESIGN

- **Recognize what piece of information is costly and useful for your algorithm**
  - Consider if there is a cheap way to estimate that information
  - Does your client have a tolerance for error?
  - Can you map this problem to a similar problem?
  - "Greedy" algorithms are often approximators

RANDOMIZATION DESIGN

- **Randomization is also another approach**

RANDOMIZATION DESIGN

- **Randomization is also another approach**
  - Selecting a random pivot in quicksort gives us more certainty in the runtime

RANDOMIZATION DESIGN

- **Randomization is also another approach**
  - Selecting a random pivot in quicksort gives us more certainty in the runtime
  - This doesn’t impact correctness, a randomized quicksort still returns a sorted list

RANDOMIZATION DESIGN

- **Randomization is also another approach**
  - Selecting a random pivot in quicksort gives us more certainty in the runtime
  - This doesn't impact correctness, a randomized quicksort still returns a sorted list
- **Two types of randomized algorithms**
  - Las Vegas – correct result in random time

RANDOMIZATION DESIGN

- **Randomization is also another approach**
  - Selecting a random pivot in quicksort gives us more certainty in the runtime
  - This doesn’t impact correctness, a randomized quicksort still returns a sorted list
- **Two types of randomized algorithms**
  - Las Vegas – correct result in random time
  - Montecarlo – estimated result in deterministic time

RANDOMIZATION DESIGN

- **Can we make a Montecarlo quicksort?**

RANDOMIZATION DESIGN

- **Can we make a Montecarlo quicksort?**
  - Runs O(n log n) time, but not guaranteed to be correct

RANDOMIZATION DESIGN

- **Can we make a Montecarlo quicksort?**
  - Runs O(n log n) time, but not guaranteed to be correct
  - Terminate a random quicksort early!

RANDOMIZATION DESIGN

- **Can we make a Montecarlo quicksort?**
  - Runs O(n log n) time, but not guaranteed to be correct
  - Terminate a random quicksort early!
  - If you haven't gotten the problem in some constrained time, just return what you have.

RANDOMIZATION DESIGN

- **How *close* is a sort?**
- **If we say a list is 90% sorted, what do we mean?**

RANDOMIZATION DESIGN

- **How *close* is a sort?**
- **If we say a list is 90% sorted, what do we mean?**
  - 90% of elements are smaller than the object to the right of it?

RANDOMIZATION DESIGN

- **How *close* is a sort?**
- **If we say a list is 90% sorted, what do we mean?**
  - 90% of elements are smaller than the object to the right of it?
  - The longest sorted subsequence is 90% of the length?

RANDOMIZATION DESIGN

- **How *close* is a sort?**
- **If we say a list is 90% sorted, what do we mean?**
  - 90% of elements are smaller than the object to the right of it?
  - The longest sorted subsequence is 90% of the length?
- **Analysis for these problems can be very tricky, but it's an important approach**

ALGORITHMS

- There aren't many *easy* problems left!
- Understand the tools for problem solving

ALGORITHMS

- There aren’t many *easy* problems left!
- Understand the tools for problem solving
- Eliminate as many non-feasible solutions as possible

ALGORITHMS

- **There aren't many *easy* problems left!**
- **Understand the tools for problem solving**
- **Eliminate as many non-feasible solutions as possible**
- **Understand, that some problems are too difficult for a fast, elegant solution**

ALGORITHMS

- There aren't many *easy* problems left!
- Understand the tools for problem solving
- Eliminate as many non-feasible solutions as possible
- Understand, that some problems are too difficult for a fast, elegant solution
- Academics are great for providing ideas, but sometimes better asymptotic runtimes don't become apparent until $n > 10^{10}$

HARDWARE CONSTRAINTS

- So far, we've taken for granted that memory access in the computer is constant and easily accessible

HARDWARE CONSTRAINTS

- **So far, we've taken for granted that memory access in the computer is constant and easily accessible**
  - This isn't always true!

HARDWARE CONSTRAINTS

- **So far, we’ve taken for granted that memory access in the computer is constant and easily accessible**
  - This isn’t always true!
  - At any given time, some memory might be cheaper and easier to access than others

HARDWARE CONSTRAINTS

- **So far, we've taken for granted that memory access in the computer is constant and easily accessible**
  - This isn't always true!
  - At any given time, some memory might be cheaper and easier to access than others
  - Memory can't always be accessed easily

HARDWARE CONSTRAINTS

- **So far, we've taken for granted that memory access in the computer is constant and easily accessible**
  - This isn't always true!
  - At any given time, some memory might be cheaper and easier to access than others
  - Memory can't always be accessed easily
  - Sometimes the OS lies, and says an object is "in memory" when it's actually on the disk

HARDWARE CONSTRAINTS

- Back on 32-bit machines, each program had access to 4GB of memory

HARDWARE CONSTRAINTS

- **Back on 32-bit machines, each program had access to 4GB of memory**
  - This isn’t feasible to provide!

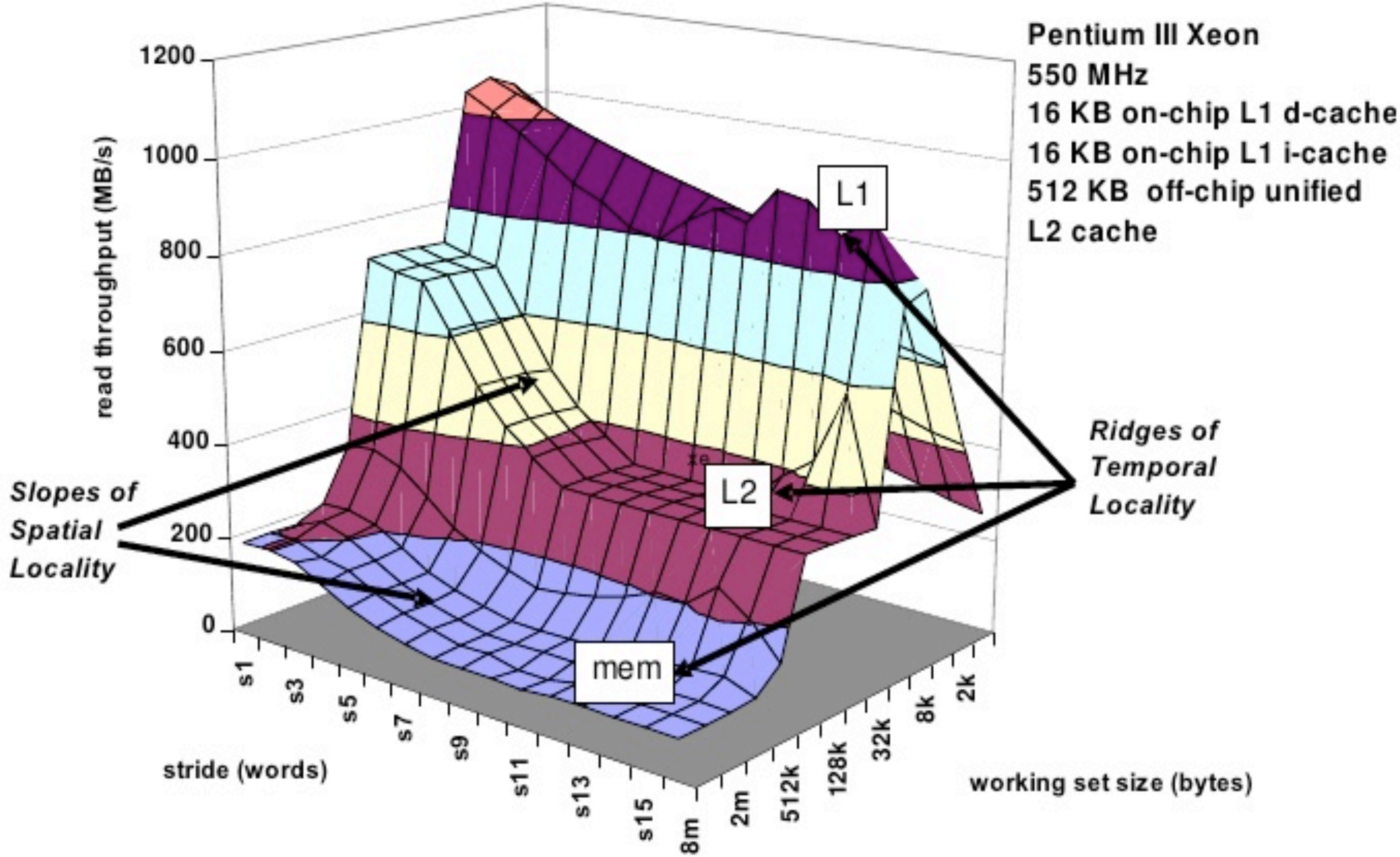
HARDWARE CONSTRAINTS

- **Back on 32-bit machines, each program had access to 4GB of memory**
  - This isn’t feasible to provide!
  - Sometimes there isn’t enough available, and so memory that hasn’t been used in a while gets pushed to the disk

HARDWARE CONSTRAINTS

- **Back on 32-bit machines, each program had access to 4GB of memory**
  - This isn’t feasible to provide!
  - Sometimes there isn’t enough available, and so memory that hasn’t been used in a while gets pushed to the disk
- **Memory that is frequently accessed goes to the cache, which is even faster than RAM**

The Memory Mountain



NEXT WEEK

- No class on Monday – Happy Memorial Day!

NEXT WEEK

- **No class on Monday – Happy Memorial Day!**
- **Formalize discussion of the “memory mountain” and how this should impact your design decisions**