CSE 373

MAY 5TH – MORE GRAPHS

MINUTIAE

- **HW4 is out**
- **Exam regrades until 4:30 after class today**
- **Also available through next week**
- **Exams not picked up are in my office**

EXCEPTIONS

- **HW4 requires exception throwing**
  - https://docs.oracle.com/javase/tutorial/essential/exceptions/throwing.html
  - Here’s a good tutorial
  - But, here are the basics

EXCEPTIONS

- **What to do during unacceptable behavior?**
  - Crashing isn’t ideal
  - Exiting doesn’t give the client much information on why the crash occurred
  - Throwing an exception allows the user to understand exactly what went wrong.

EXCEPTIONS

- **You may use any exception that you want, throwing the default `Exception()` is fine, but you should get in a habit of throwing informative errors**
  - DuplicateEdgeException on an edge insertion is much more useful than a crash or terminate
  - Null Pointer Exception, Array Index Out of Bounds Exception, **Illegal Argument Exception (good for much of HW4)**

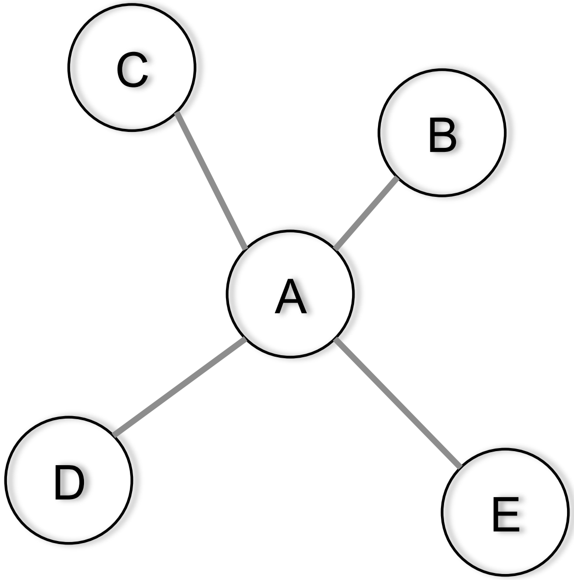
GRAPHS

- **Graphs are not an ADT**
  - There is no “functions” that a graph supports
  - Rather, graphs are a theoretical framework for understanding certain types of problems.
  - Travelling salesman, path finding, resource allocating

GRAPHS

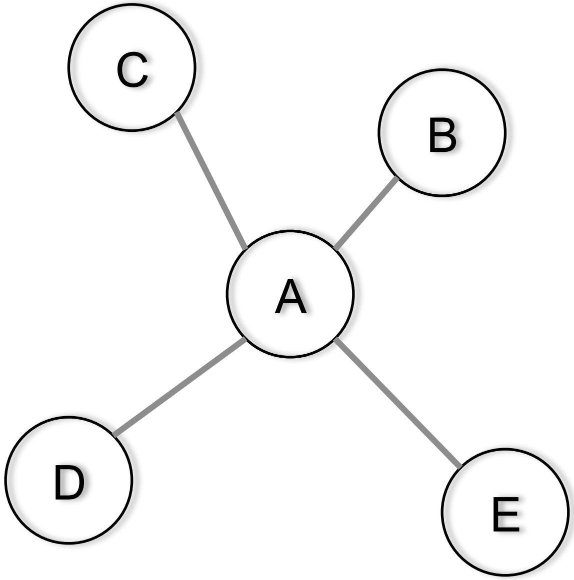
- **A graph is composed of two things**
  - A set of vertices
  - A set of edges (which are vertex tuples)
- **Trees are types of graphs**
  - Each of the nodes is a vertex
  - Each pointer from parent to child is an edge
- **Represented as G(V,E) to indicate that V is the set of vertices and E is the set of edges**

GRAPHS



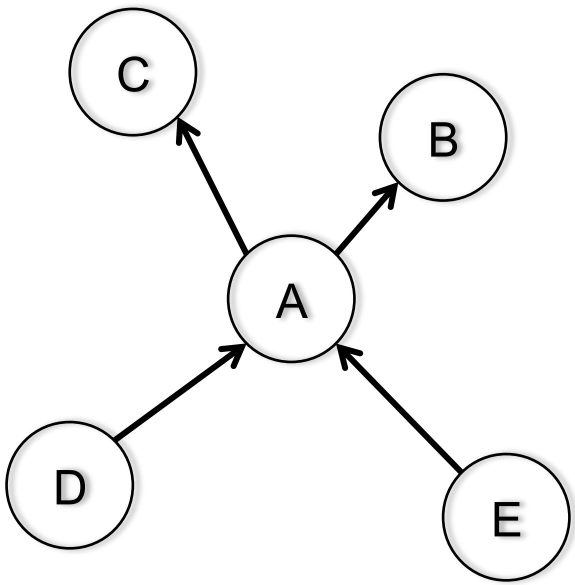
- **What this graphs vertices and edges?**

GRAPHS



- **What this graphs vertices and edges?**
  - V = {A, B, C, D, E}
  - E = {(A,B) , (A,C), (A,D), (A,E)}

GRAPHS



- **What this graphs vertices and edges?**
  - V = {A, B, C, D, E}
  - E = {(A,B) , (A,C), (D,A), (E,A)}

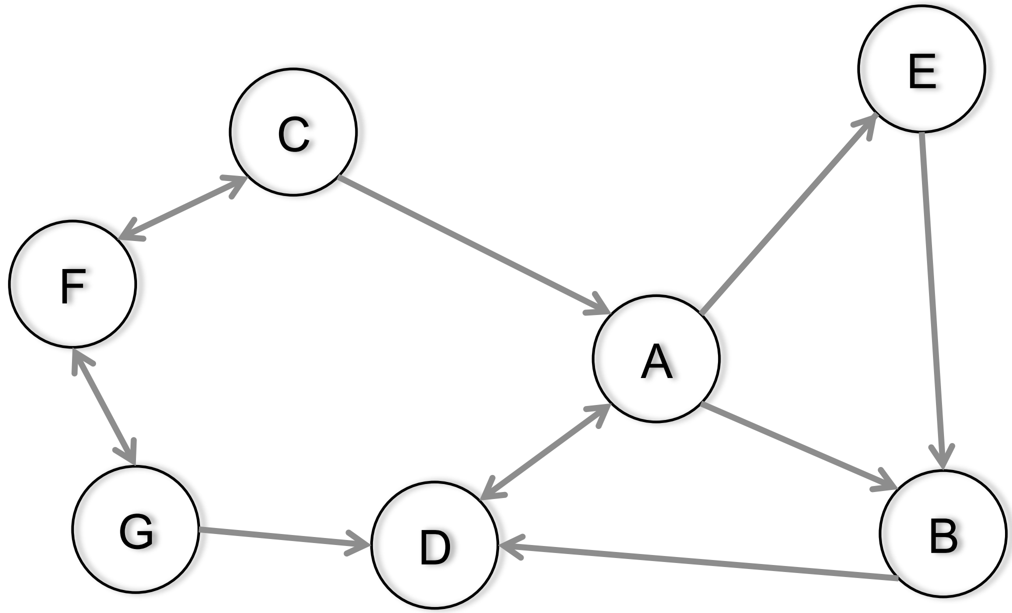
GRAPHS

- **Graphs can be either directed or undirected**
  - Undirected graph, if (A,B) is in the set of edges, (B,A) must be in the set of edges
  - Directed graphs, both can be in the set of edges, but those graphs have different connectivity
- **We call a graph *connected* if there is a path between every pair of vertices**

GRAPHS

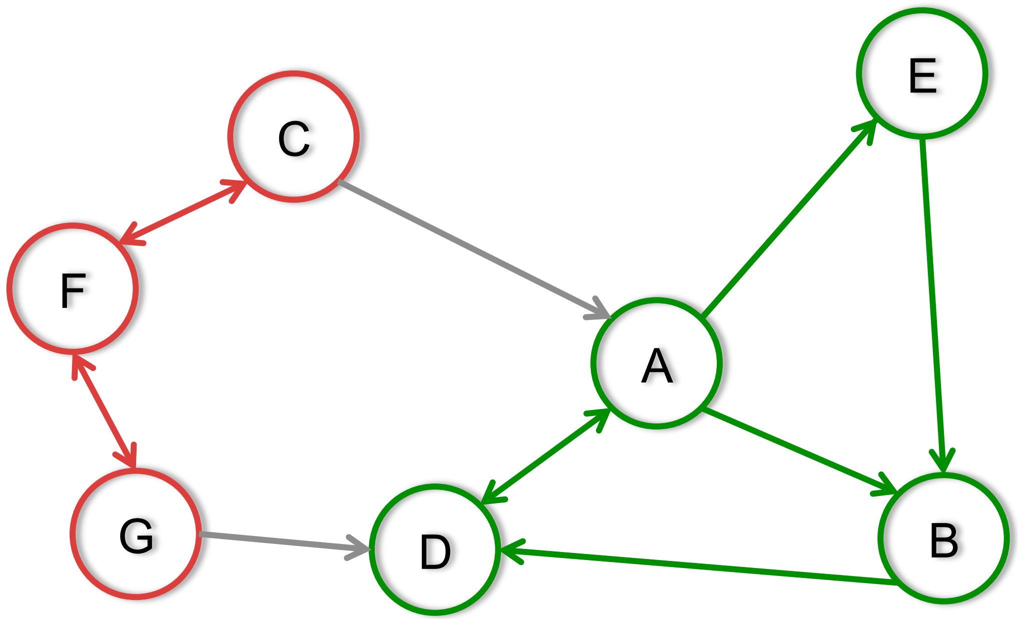
- **Paths and Cycles**
  - A path: a set of edges connecting two vertices where all of the edges are connected and neither edges nor vertices are repeated
  - A cycle: a path that starts and ends on the same

GRAPHS



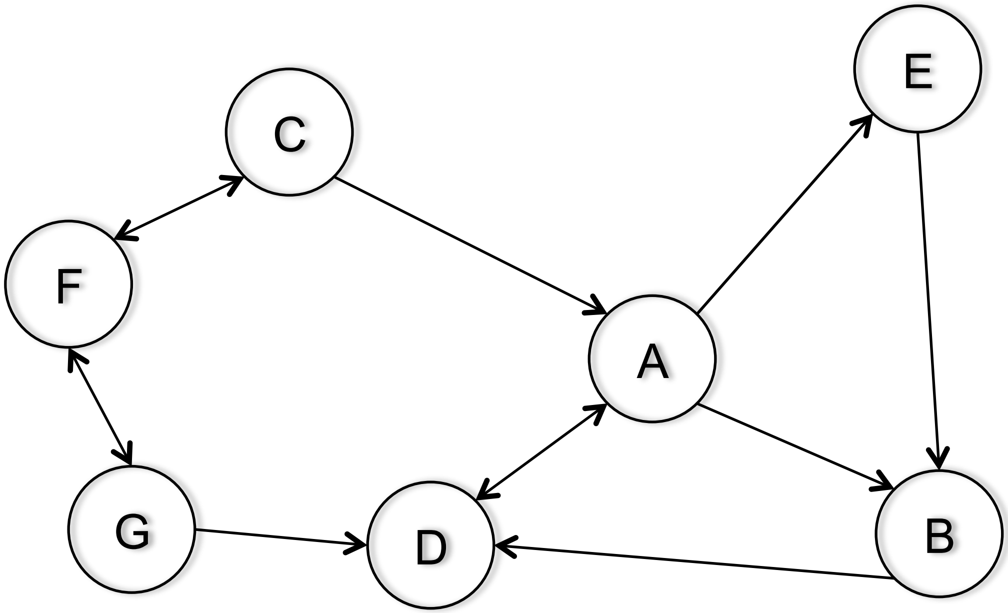
- **Is this graph connected?**
  - Is there a path between every pair of vertices?

GRAPHS



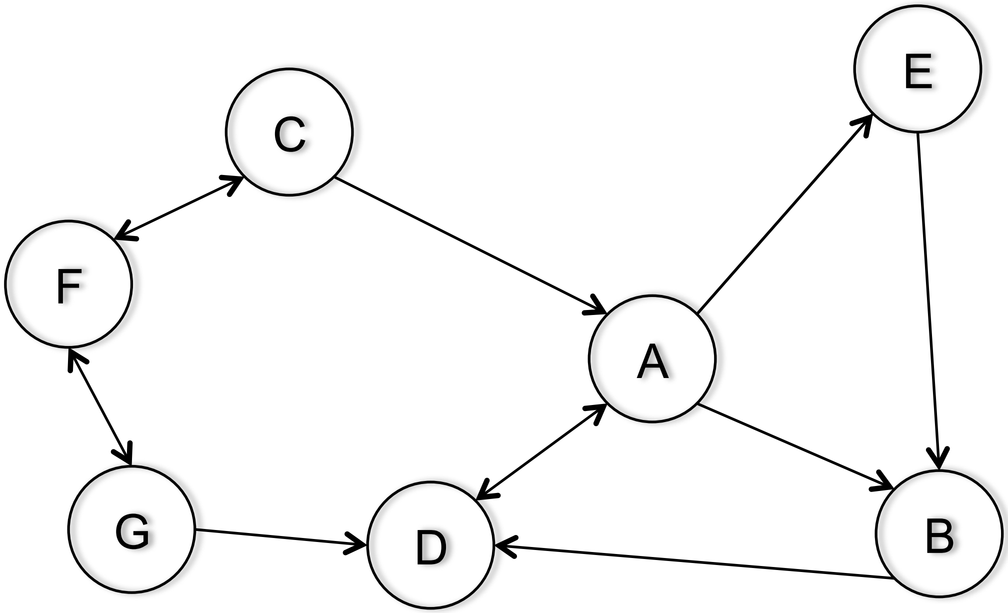
- **Is this graph connected?**
  - There's no way to get from the green graph to the red

GRAPHS



- **Does this graph have a cycle?**
  - How many does it have?

GRAPHS



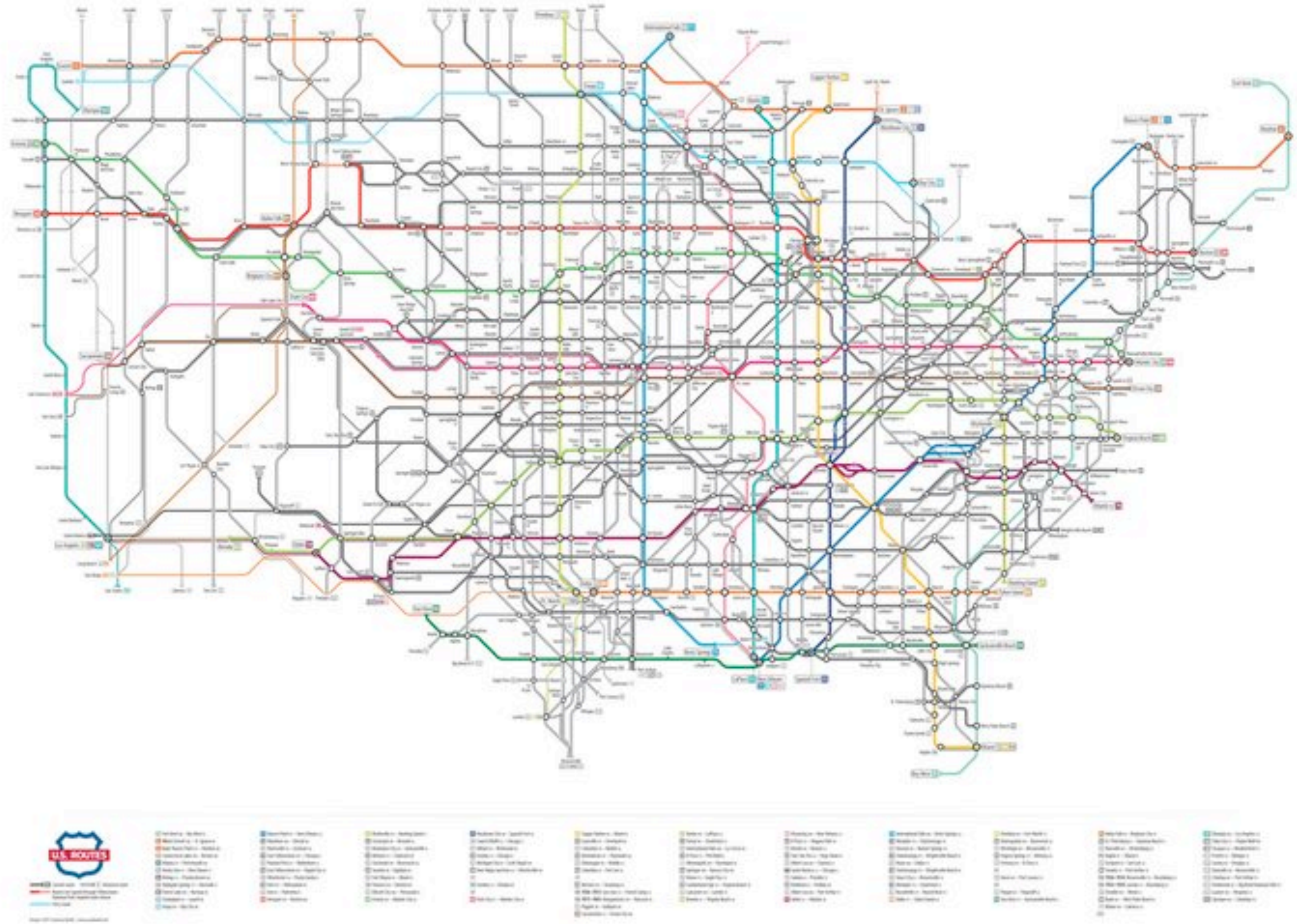
- **Does this graph have a cycle?**
  - {(A,E),(E,B),(B,D),(D,A)}
  - {(A,B),(B,D),(D,A)}

GRAPHS

- **Paths and cycles can not have repeated vertices or edges**
  - A path that can repeat vertices or edges is called a walk
  - A path that can repeat vertices but not edges is called a trail
  - A circuit is a trail that starts and ends at the same vertex

GRAPHS

- **Edges can have weights**
  - This becomes important when we consider path finding algorithms
  - Usually, we consider the weights to be the costs of using a particular edge.
  - In a graph representation of the US interstate system, the I-90 edge between Seattle and Spokane may have weight 270 for miles or 4 for hours, depending on what we want to minimize!


U.S. ROUTES

GRAPHS

- **When we consider graphs, we determine them to be either dense or sparse**
  - Dense graphs are very connected, each vertex is connected to a fraction of the total vertices
  - Sparse graphs are less connected and can be more clustered, each vertex is connected to some constant number of vertices

GRAPHS

- **When graphs are small, it is difficult to distinguish between the two**
  - If we represent Facebook as a graph, where users are vertices and “friendships” are edges, what can we say about the graph?
    - Directed? **No, (A,B) means (B,A)**
    - Connected? **Very probably**
    - Cyclic? **Yes, mutual friends**
    - Sparse/Dense? **Sparse! 338 average!**

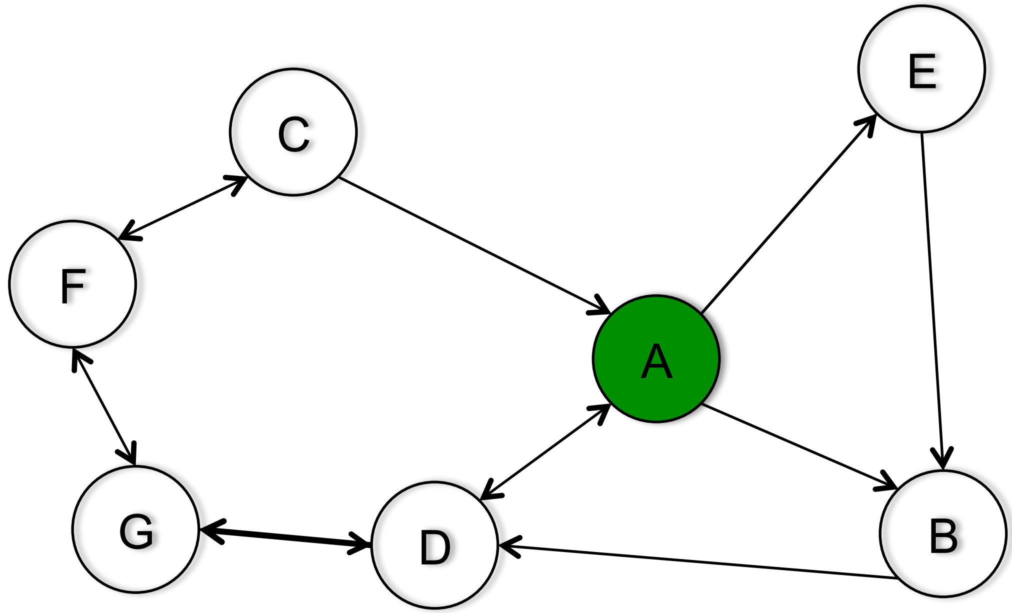
GRAPHS

- **This “value” is called the degree of the vertex**
  - If you have 338 friends, then that vertex has degree 338.
- **In directed graph, we separate this into in-degree and out-degree**
  - Consider Twitter, where friendship isn’t symmetric. The number of followers you have is your in-degree and the number of people you follow is your out degree

TRAVERSALS

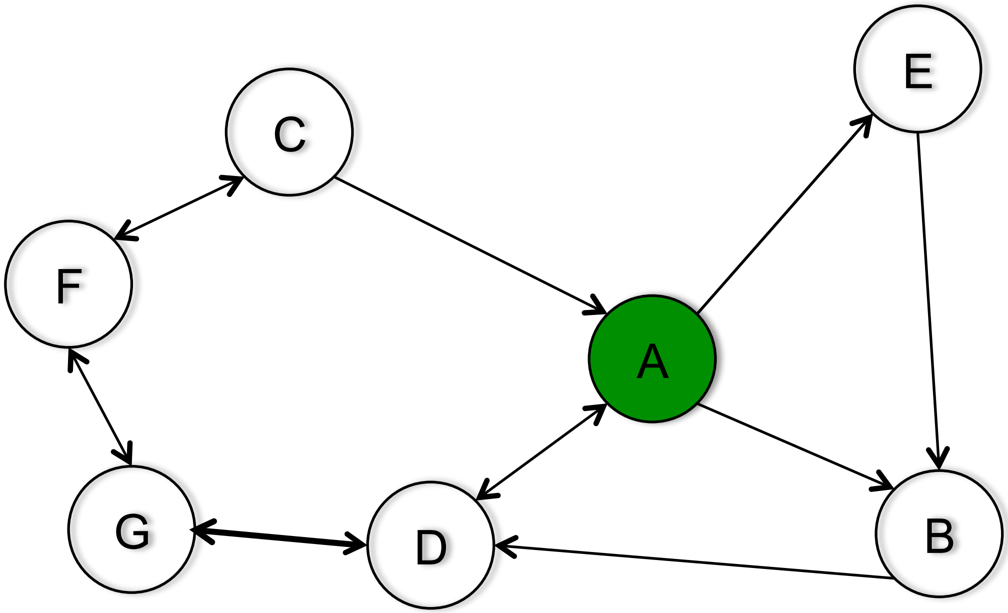
- **Since graphs are abstractions similar to trees, we can also perform traversals.**
  - If a graph is connected, i.e. there is a path between all pairs of vertices, then a traversal can output all nodes if you do it cleverly

TRAVERSAL



- **Depth-first search (prev graph with (D,G) added to make it connected**
  - Traverse the tree with DFS, if there are multiple nodes to choose from, go alphabetically. Start at A.

TRAVERSAL

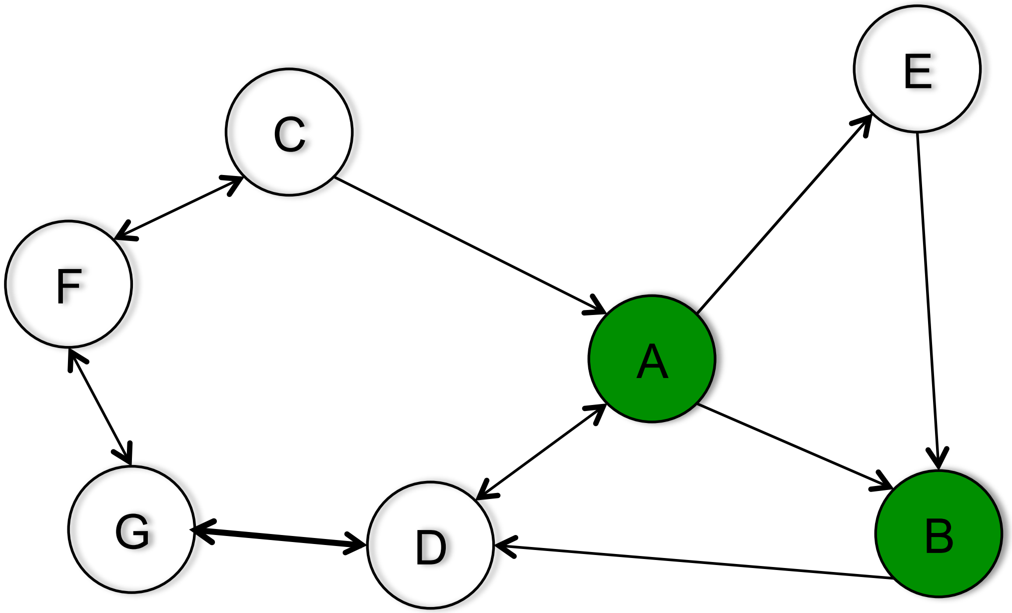


Output: A

Current Node: A

Out-vertices: B, D, E

TRAVERSAL

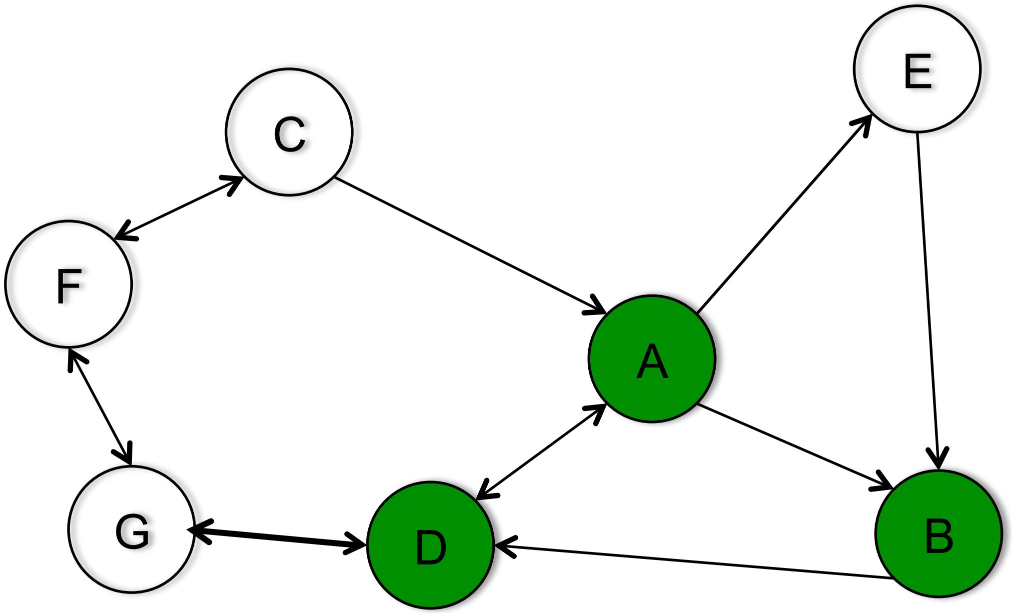


Output: A,B

Current Node: B

Out-vertices: D

TRAVERSAL

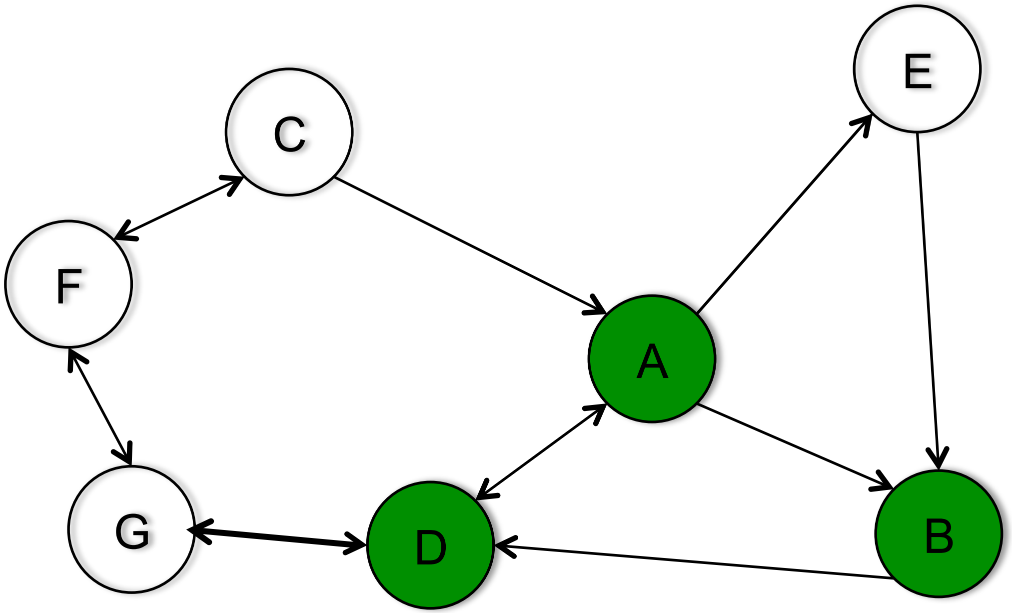


Output: A,B, D

Current Node: D

Out-vertices: A,G

TRAVERSAL

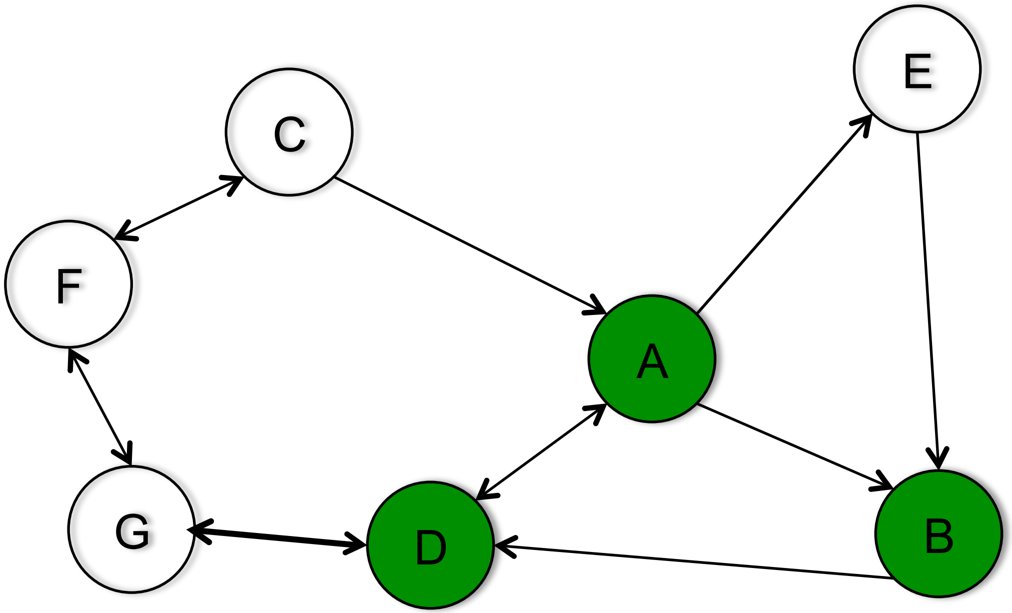


Output: A,B, D, A

Current Node: A

Out-vertices: B,D,E

TRAVERSAL



Output: A,B, D, A

Oh, no! We have repeated output!

Current Node: A

Out-vertices: B,D,E

TRAVERSAL

- **Depth first search needs to check which nodes have been output or else it can get stuck in loops.**
  - This increases the runtime and memory constraints of the traversal
- **In a connected graph, a BFS will print all nodes, but it will repeat if there are cycles and may not terminate**

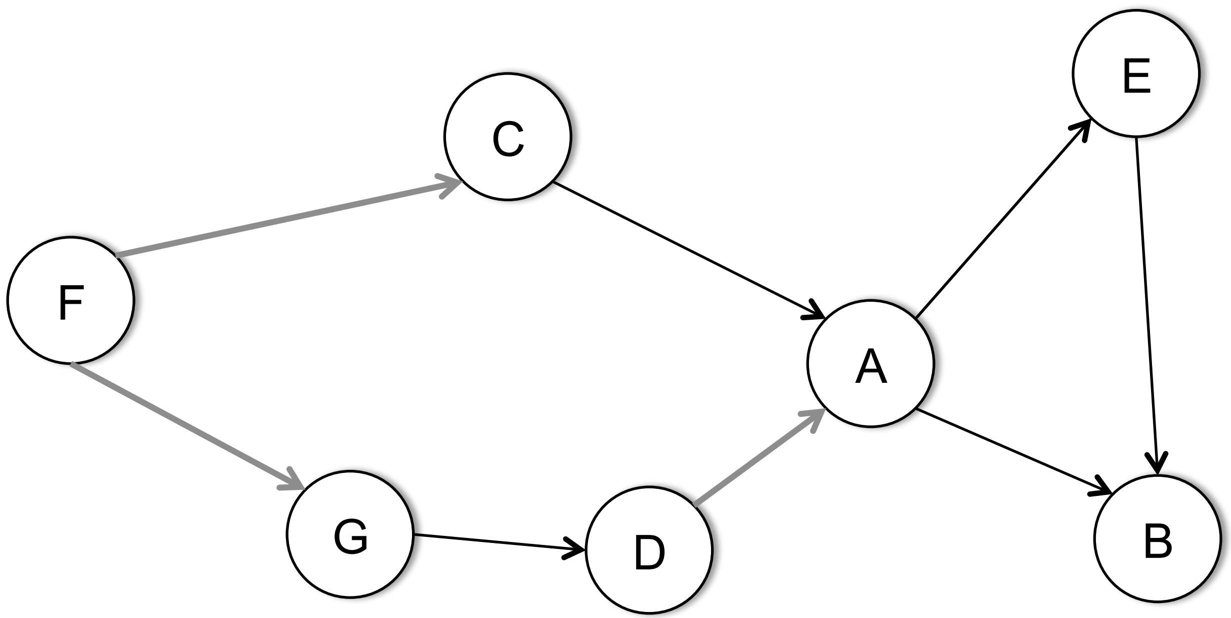
TRAVERSAL

- **As an aside, in-order, pre-order and post-order traversals only make sense in binary trees, so they aren't important for graphs. However, we do need some way to order our out-vertices (left and right in BST).**

TRAVERSAL

- **Topological ordering**
  - One final ordering for graphs
  - Ordering with a focus on dependency resolutions
- **Example, consider a graph where courses are vertices and edges are prerequisites. A topological ordering is any valid class order**

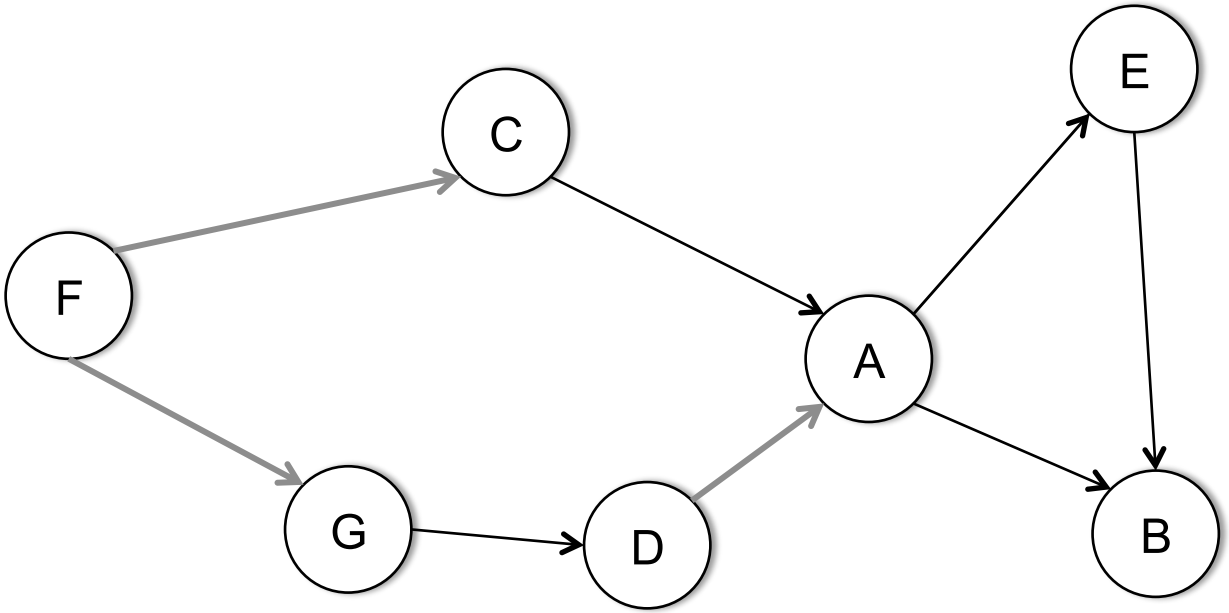
TRAVERSAL



Start with the nodes that have in-degree 0 (no prereqs)

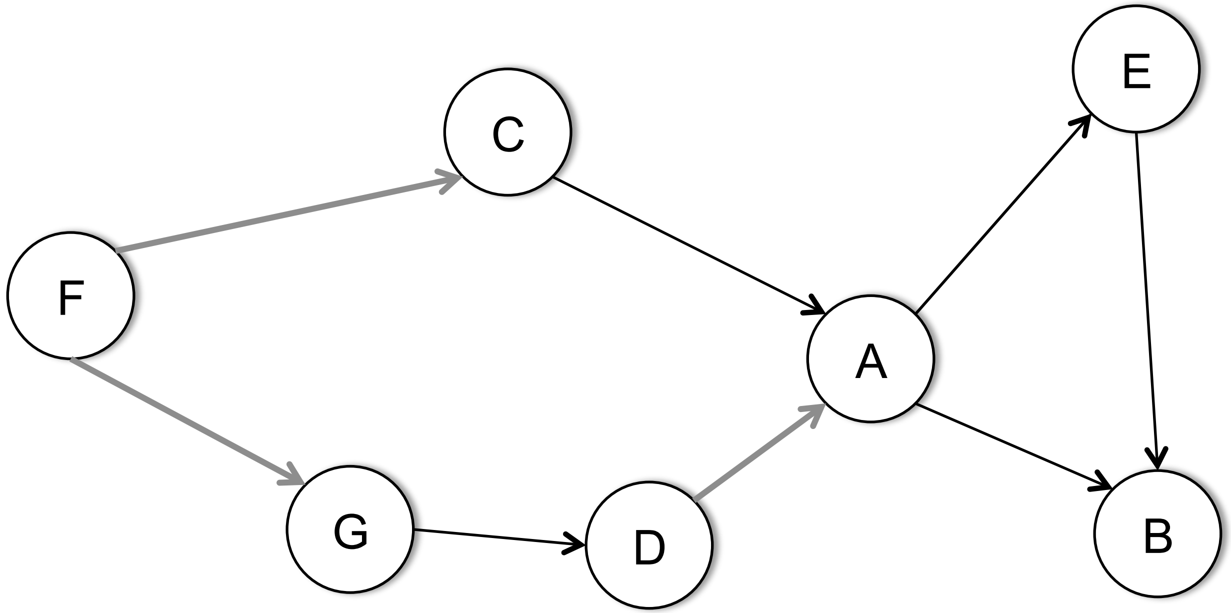
Then eliminate that vertex (print it out) and eliminate its out edges.

TRAVERSAL



What is a valid topological sort of this graph?

TRAVERSAL



What is a valid topological sort of this graph?

F,C,G,D,A,E,B

F,G,D,C,A,E,B

F,G,C,D,A,E,B

Is this all the valid solutions?

NEXT WEEK

- Another topological sort problem
- Weights and pathfinding
- Start Dijkstra’s algorithm