CSE 373

APRIL 19TH – AVL OPERATIONS

ASSORTED MINUTIAE

- **Exam review**
  - Wednesday evening (Canvas announcement)
- **Regrade requests for HW2 by end of day Monday**

TODAY'S LECTURE

- **Finish AVL Trees**
  - Proof
- **Memory analysis**
  - Framework and concept

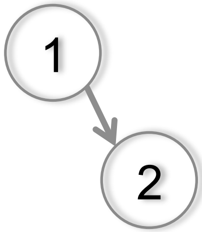
REVIEW

- **AVL Trees**
  - BST trees with AVL property
  - Abs(height(left) – height(right)) <= 1
  - Heights of subtrees can differ by at most one
  - This property must be preserved throughout the tree

REVIEW

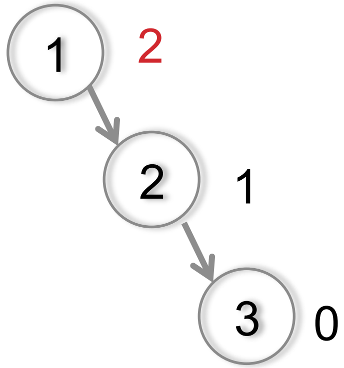
- **Add the following into an AVL Tree**
  - {1,2,3,5,4}

REVIEW



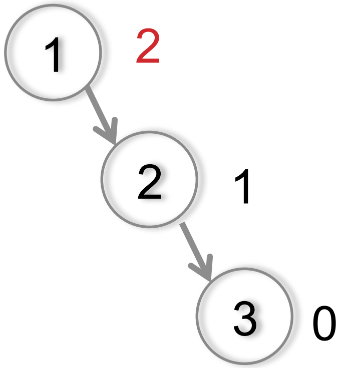
- **Add 2, then verify balance**

REVIEW



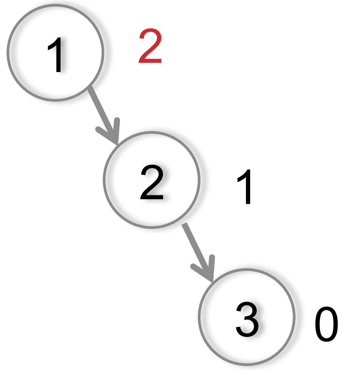
- **Add three, observe that the balance of ‘1’ is off.**
  - What case is this?

REVIEW



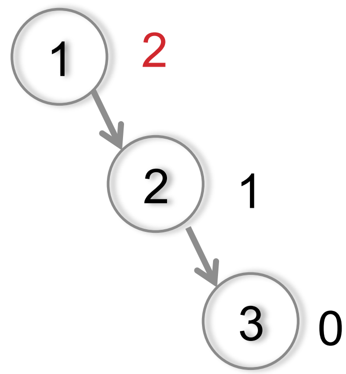
- **Add three, observe that the balance of '1' is off.**
    - What case is this? *Right-right*

REVIEW



- **Rotate the tree to preserve balance**

REVIEW



- **Rotate the tree to preserve balance**
  - What is the new root?

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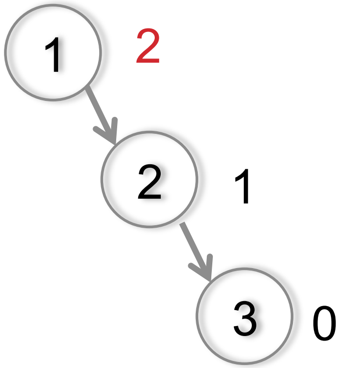
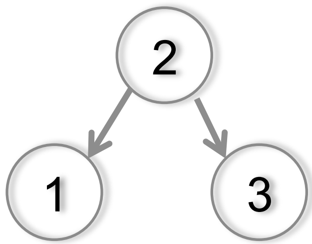


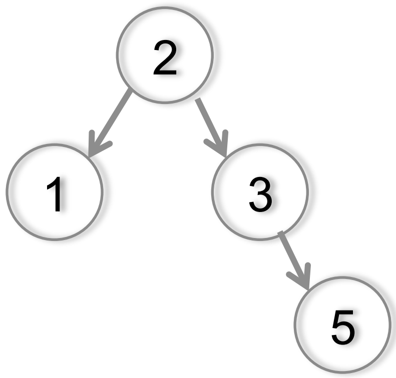
- **Rotate the tree to preserve balance**
  - What is the new root? **2**

REVIEW



- **Perform the ‘left’ rotation which brings two into the root position**

REVIEW



- **Add the 5**

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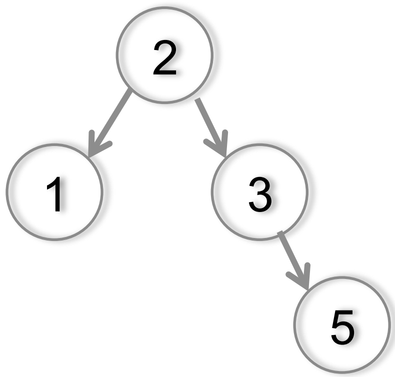
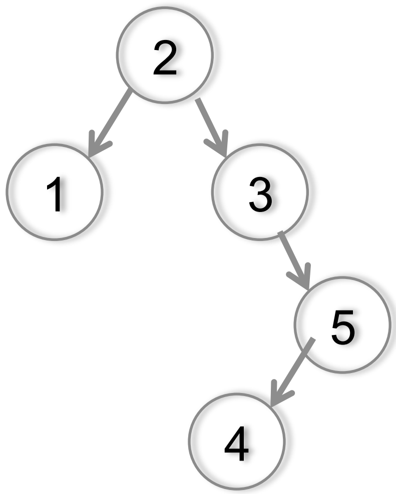


- **Add the 5**
  - Verify balance

REVIEW



- **Add the 4**

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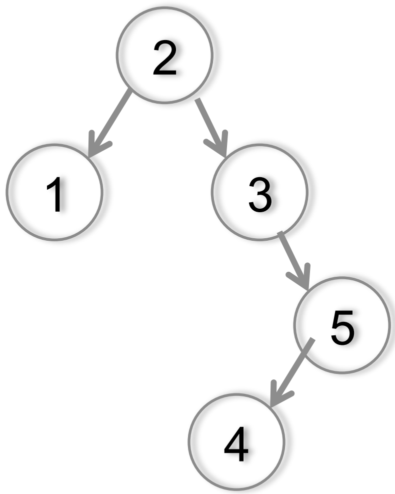

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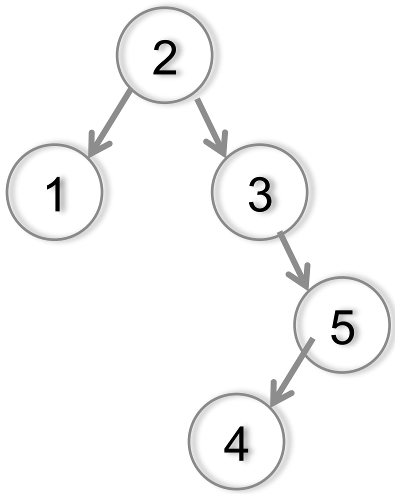

- **Add the 4**
  - Verify balance. Which node(s) are out of balance?

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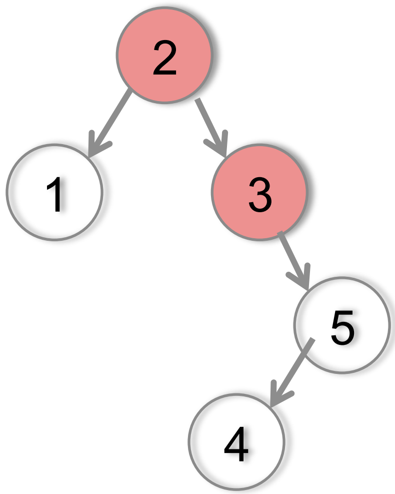

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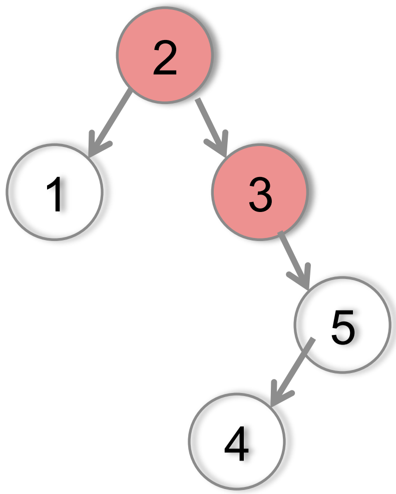
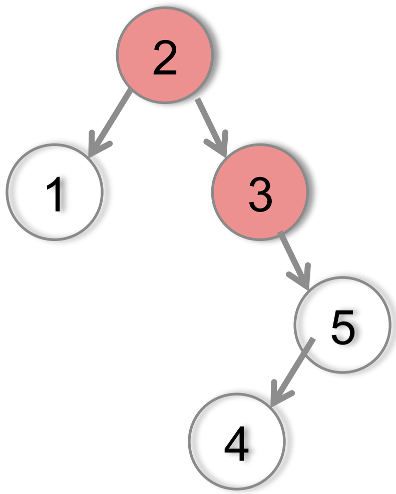


- **Which rotation will fix the tree?**

REVIEW



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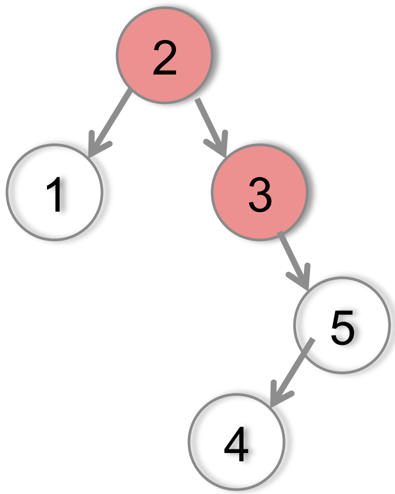
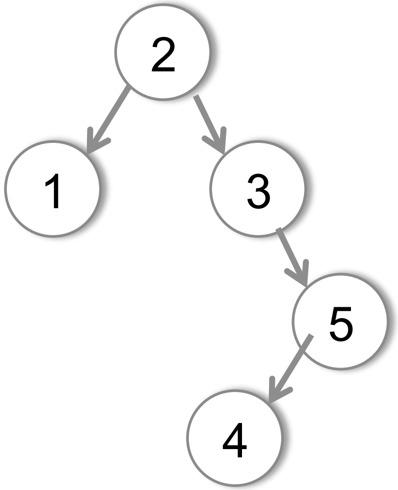


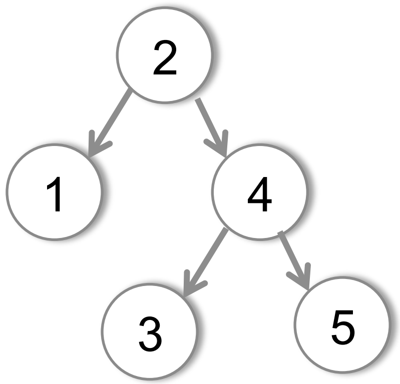
- **Which rotation will fix the tree?**
  - Select the lowest out-of-balance node (right-left case)

REVIEW



- **What does the final tree look like?**

REVIEW



- **The grandchild (4) moves up to the unbalanced position**

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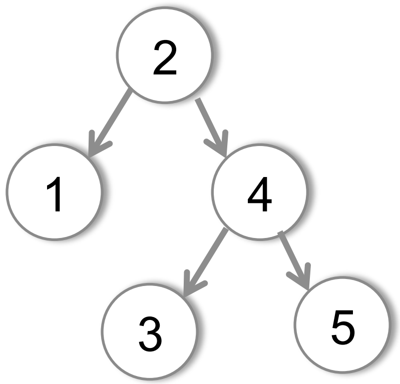
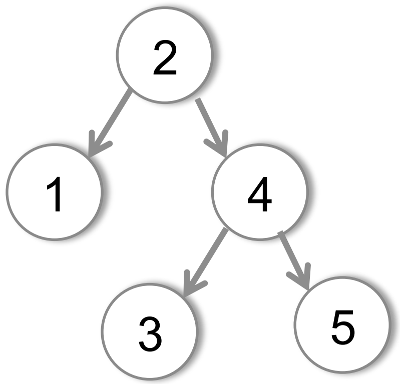


- **The grandchild (4) moves up to the unbalanced position**
  - Observe the tree is balanced

REVIEW



- **Work among yourselves, create an AVL tree from the input sequence**

REVIEW

- On your own or in small groups, produce the AVL tree from the following sequence of inputs.

  {10,20,15,5,0,-5}

REVIEW

- On your own or in small groups, produce the AVL tree from the following sequence of inputs.

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- Once you’ve finished this, think about why this balance condition is enough to give us a tree height in O(log n)

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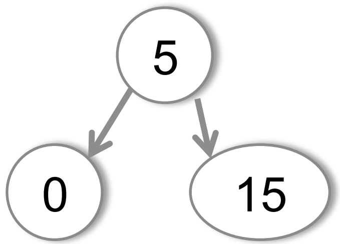

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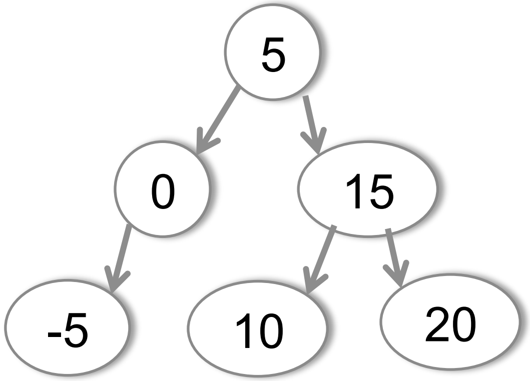

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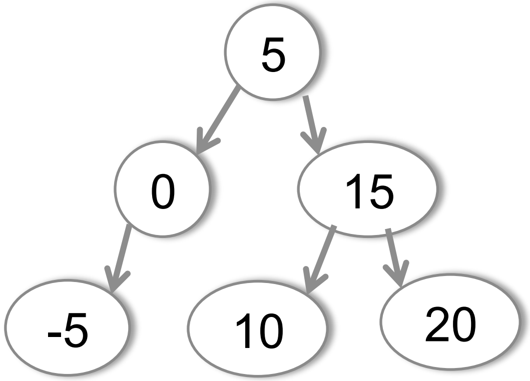

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  - We can get somewhat unbalanced trees
  - Are the balanced enough?

AVL HEIGHT (PROOF)

- You do not need to memorize this proof, but it is interesting to think about

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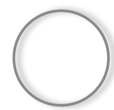
- **You do not need to memorize this proof, but it is interesting to think about**
  - Let’s consider the most “unbalanced” AVL tree, that is: the tree for each height that has the fewest nodes

AVL HEIGHT (PROOF)

- For height 1, there is only one possible tree.



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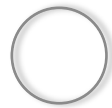

- For height 2, there are two possible trees, each with two nodes.

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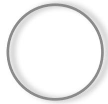

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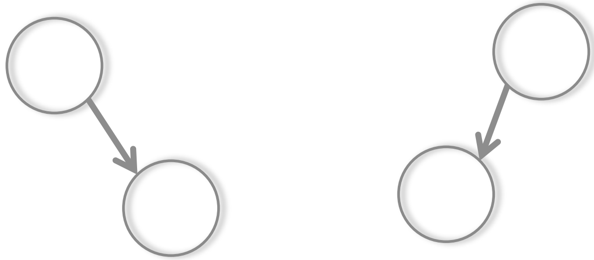

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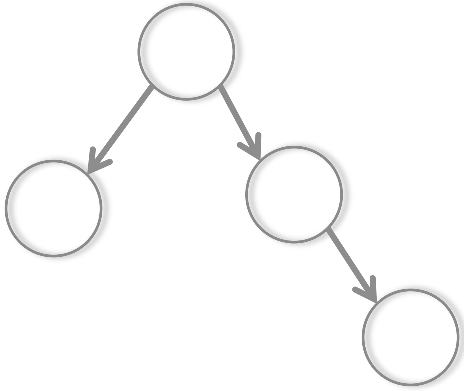

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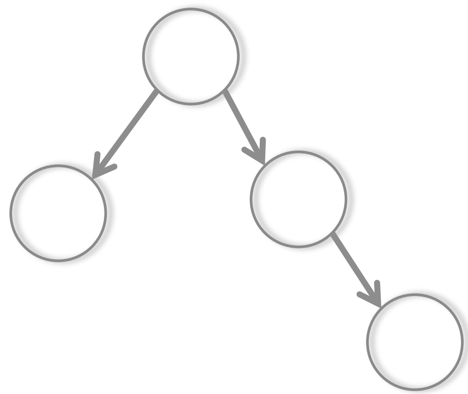
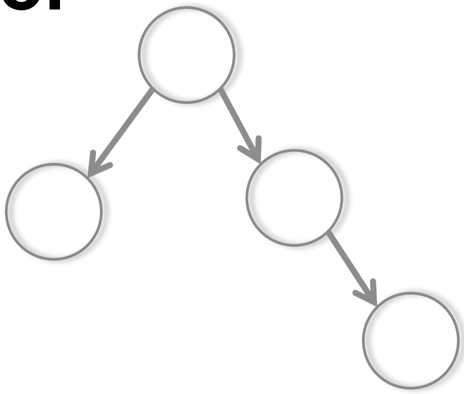


There are multiple of these trees, but what's special about it?

AVL HEIGHT (PROOF)

- The smallest tree of size three is a node where one child is the smallest tree of size one and the other one is the smallest tree of size two.



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  - **$N_4 = 7$, how do I know?**

AVL HEIGHT (PROOF)

- **In general then, if $N_1 = 1$ and $N_2 = 2$ and $N_3 = 4$, what is N_k?**
  - **$N_k = 1 + N_{k-1} + N_{k-2}$**
    Because the smallest AVL tree is a node (1) with a child that is the smallest AVL tree of height k-1 (N_{k-1}) and the other child is the smallest AVL tree of height k-2 (N_{k-2}).

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  - **This means every non-leaf has balance 1**
  - **Nothing in the tree is perfectly balanced.**

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AVL HEIGHT (PROOF)

Substitute the k-1 into the original equation

$$N_k = 1 + N_{k-1} + N_{k-2}$$
$$N_{k-1} = 1 + N_{k-2} + N_{k-3}$$

AVL HEIGHT (PROOF)

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This means the tree doubles in size after every two height (compared to a perfect tree which doubles with every added height)

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- An AVL tree has O(log n) height
- This does not come at an increased asymptotic runtime for insert.
- Rotations take a constant time.

MEMORY ANALYSIS

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  - Rather than counting the number of operations, we count the amount of memory needed
  - During the operation, when does the algorithm need to "keep track" of the most number of things?

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  - **N/2:** half the nodes of a tree are leaves

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  - **AVL Insert?** No, we only need to keep track of the parent and grandparent.
  - **DFS?** Yes, we need to keep track of all the elements leading back up to the root

NEXT WEEK

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  - The O(1) holy grail!

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- **Exam on Friday!**