CSE 373

APRIL 19TH – AVL OPERATIONS

ASSORTED MINUTIAE

- **HW2 code grades out tonight**
- **HW3 due tonight**
  - Last HW before midterm
- **Exam review**
  - Next Wednesday (in class)
  - Options for TA review session out tonight

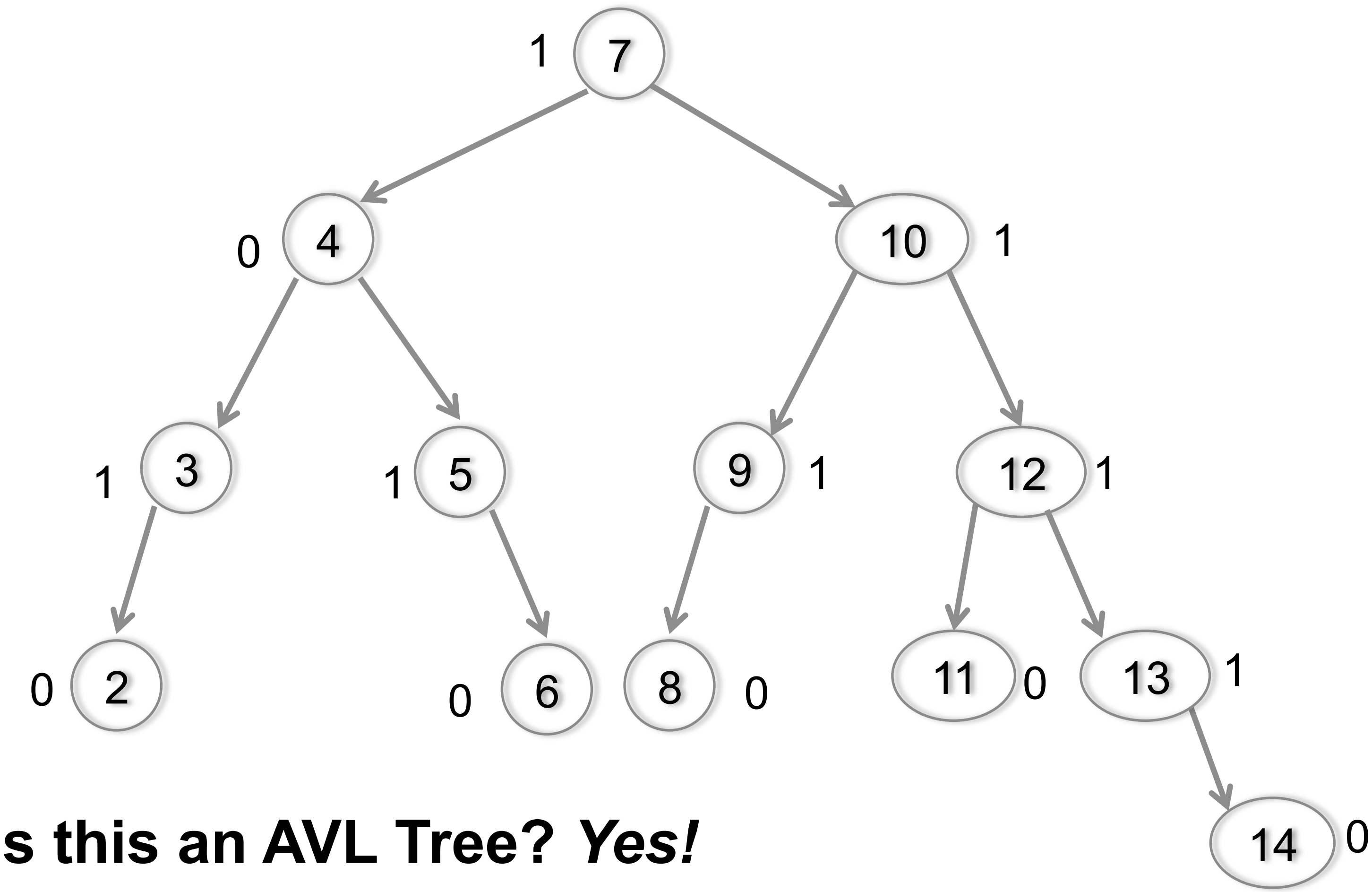
TODAY'S LECTURE

- **AVL Trees**
  - Balance
  - Implementation
- **Memory analysis**
  - Will discuss after AVL on Friday

REVIEW

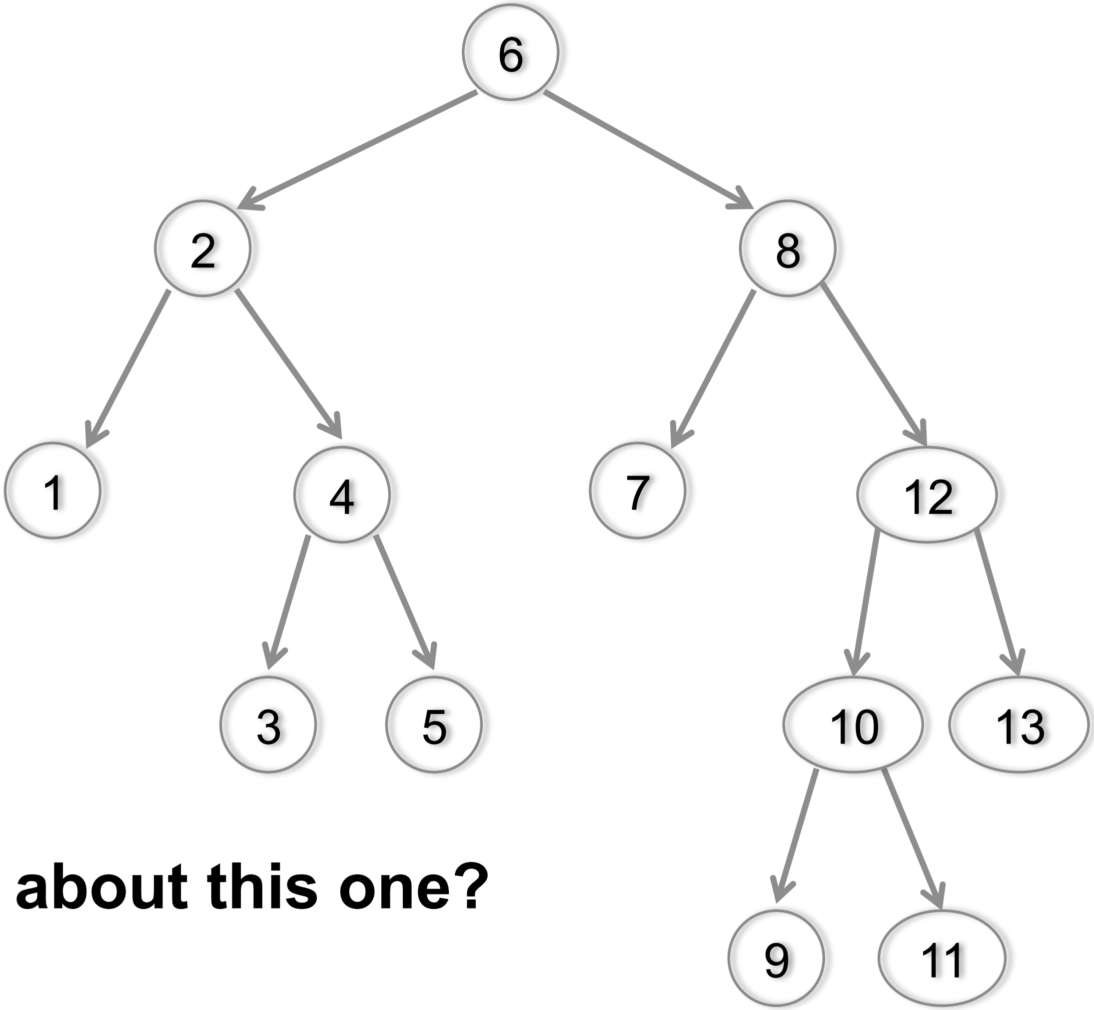
- **AVL Trees**
  - BST trees with AVL property
  - Abs(height(left) – height(right)) <= 1
  - Heights of subtrees can differ by at most one
  - This property must be preserved throughout the tree

REVIEW



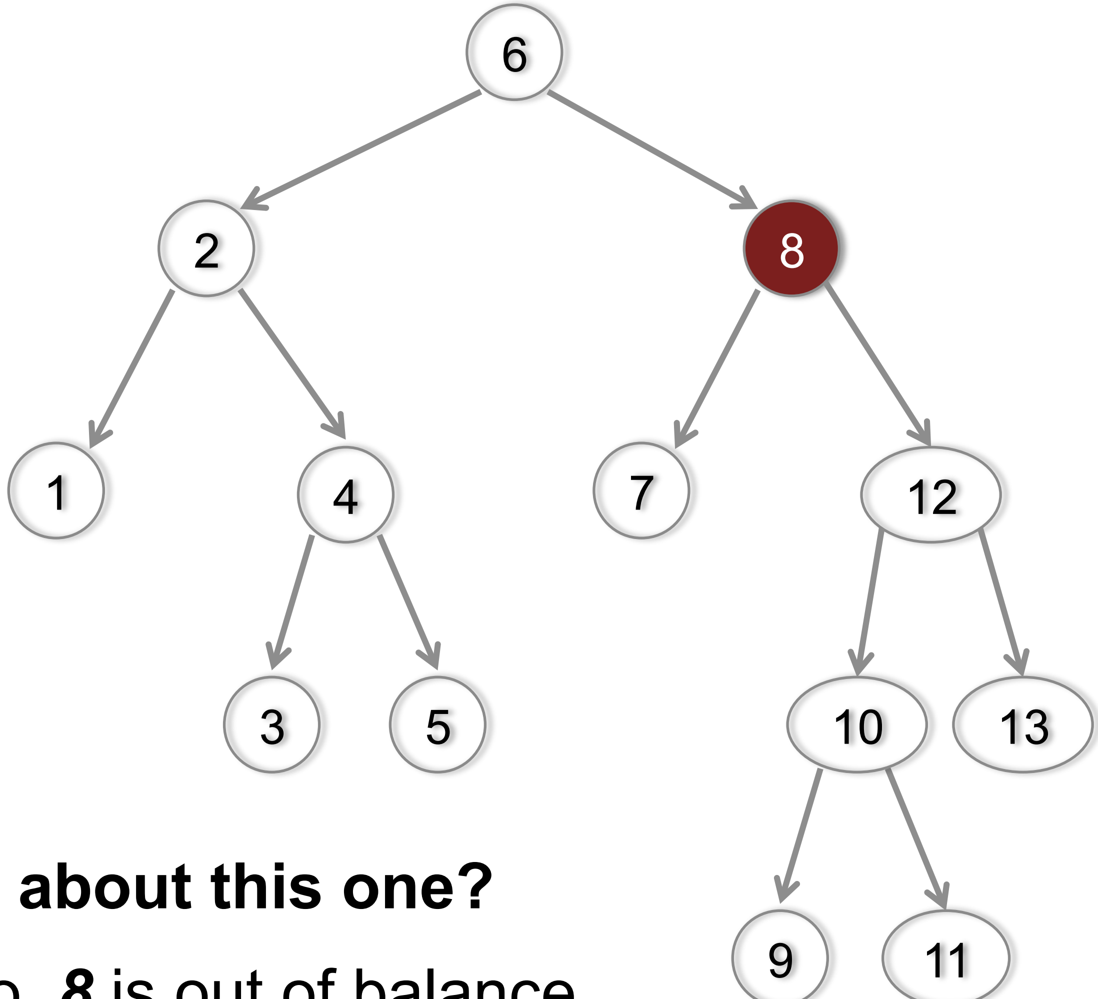
- **Is this an AVL Tree? *Yes!***
  - Calculate balance for each node

REVIEW



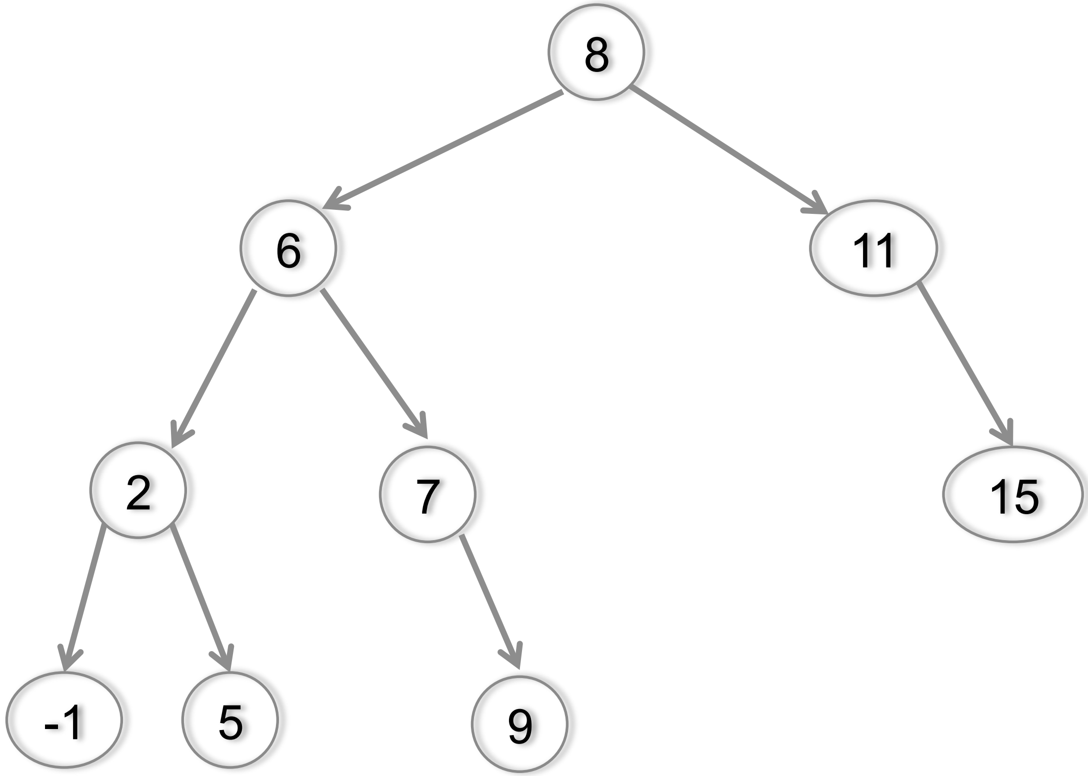
- **What about this one?**

REVIEW



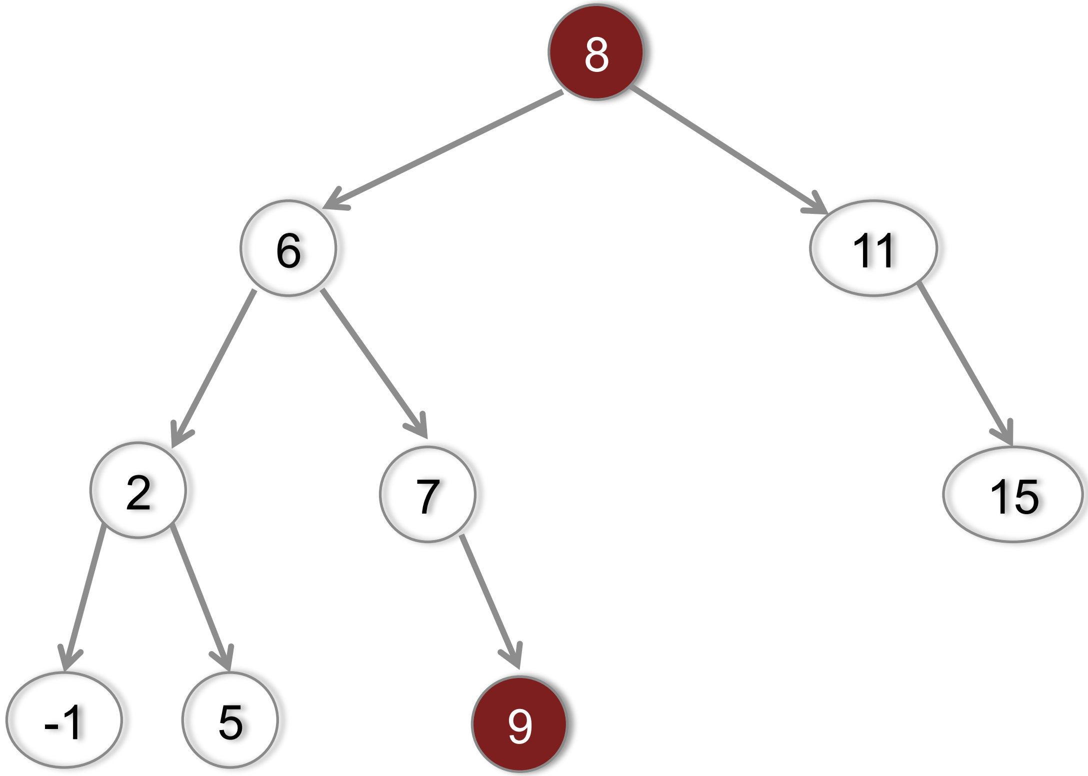
- **What about this one?**
  - No, ***8*** is out of balance

REVIEW



- **Is this an AVL Tree?**

REVIEW



- **Is this an AVL Tree?**
  - No, AVL trees must still maintain Binary Search

AVL OPERATIONS

- **Since AVL trees are also BST trees, they should support the same functionality**
  - Insert(key k, value v)
  - *Find(key k): Same as BST!*
  - *Delete(key k)*: *Not presented in this course*
- **For insert, we should maintain AVL property as we build**

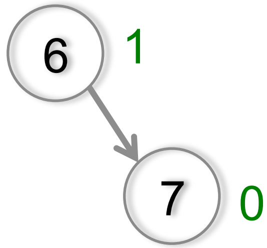
AVL OPERATIONS

- **Insert(key k, value v):**
  - Insert the key value pair into the dictionary
  - Verify that balance is maintained
  - If not, correct the tree
- **How do we correct the tree?**

AVL INSERT

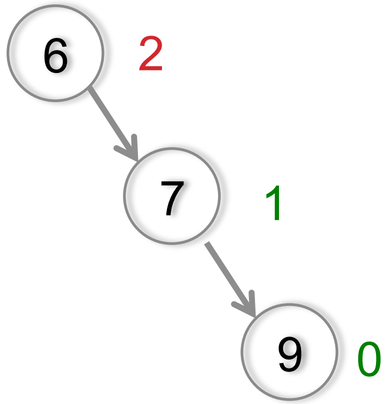
- **Start with the single root**

AVL INSERT



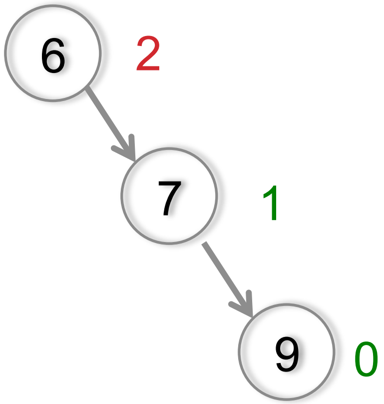
- **Add 7 to the tree. Is balance preserved?**
  - Yes

AVL INSERT



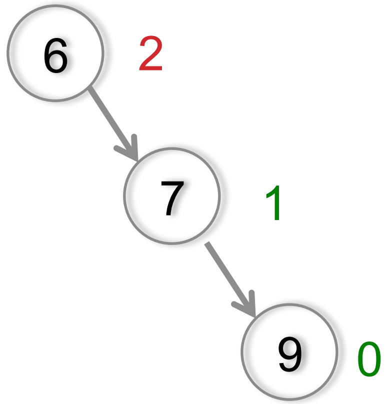
- **Add 9 to the tree. Is balance preserved?**
  - No.

AVL INSERT

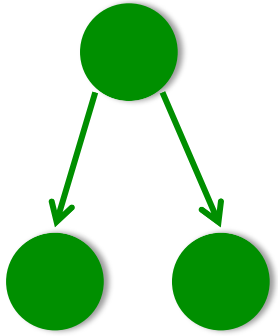


- **How do we correct this imbalance?**
  - Important to preserve binary search

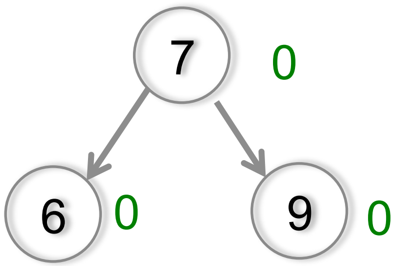
AVL INSERT



- **What shape do we want?**
  - What then do we have as the root?



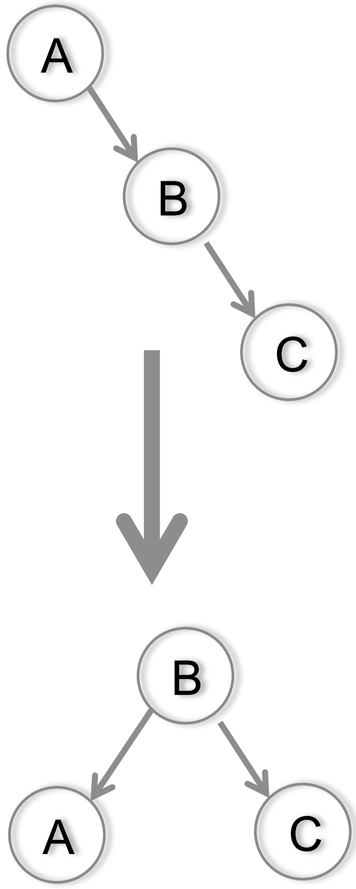
AVL INSERT



- **Since 7 must be the root, we “rotate” that node into position.**

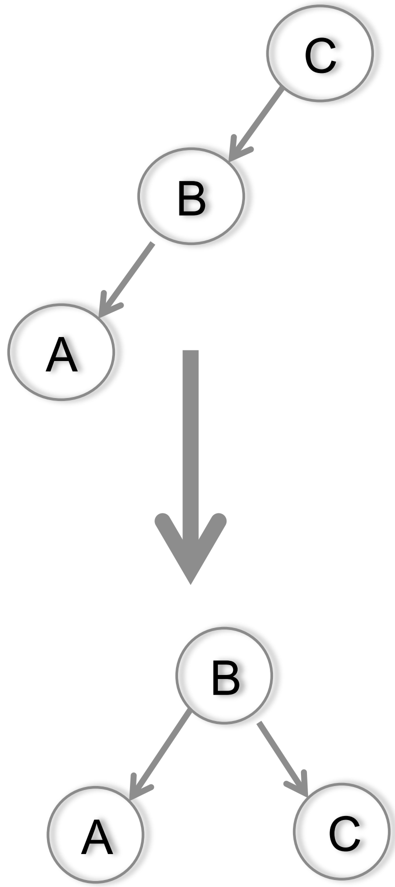
AVL "ROTATION"

- **To correct this case:**
  - B must become the root
  - We rotate B to the root position
  - A becomes the left child of B
  - This is called the "left rotation"



AVL “ROTATION”

- **Right rotation**
  - Symmetric concept
  - B must become the new root

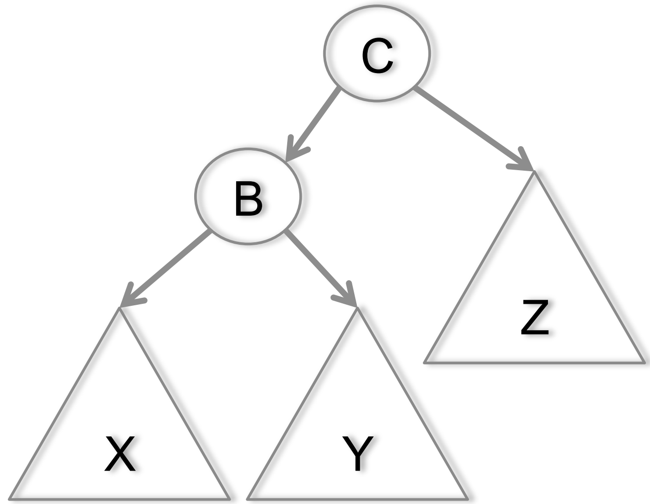


AVL “ROTATION”

- **These are the “single” rotations**
  - In general, this rotation occurs when an addition is made to the right-right or left-left grandchild
  - **The balance might not be off on the parent! An insert might upset balance up the tree**

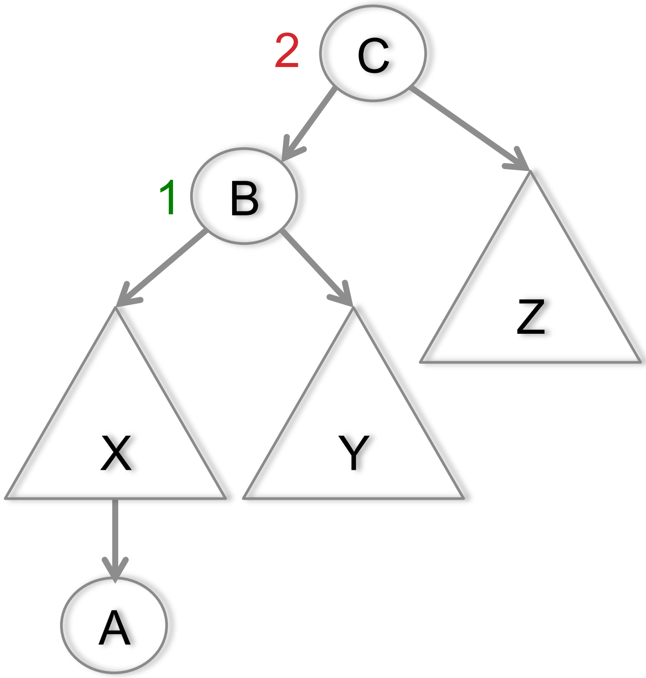
AVL "ROTATION"

- **General case**
  - Suppose this tree is balanced, {X,Y,Z} all have the same height



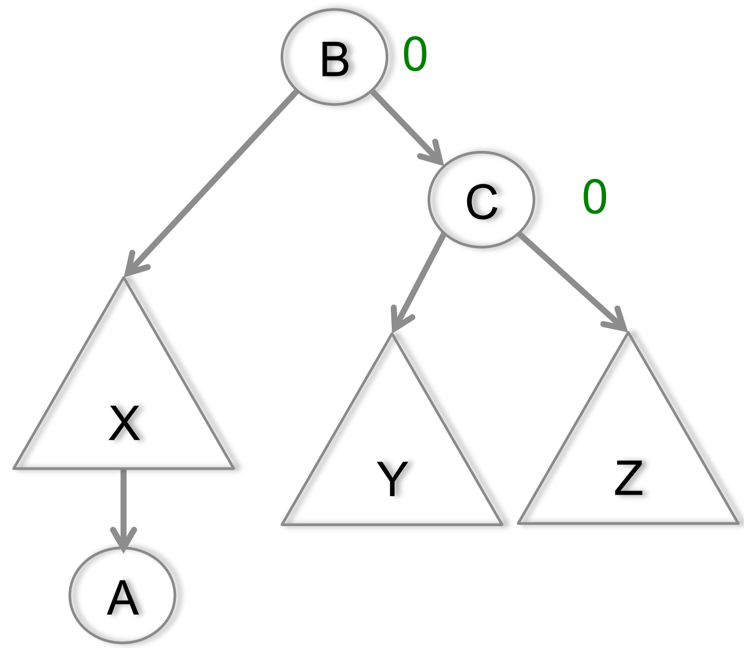
AVL "ROTATION"

- **General case**
  - Suppose this tree is balanced, {X,Y,Z} all have the same height
  - Adding A, puts C out of balance
  - Rotate B up and pass the Y subtree to C

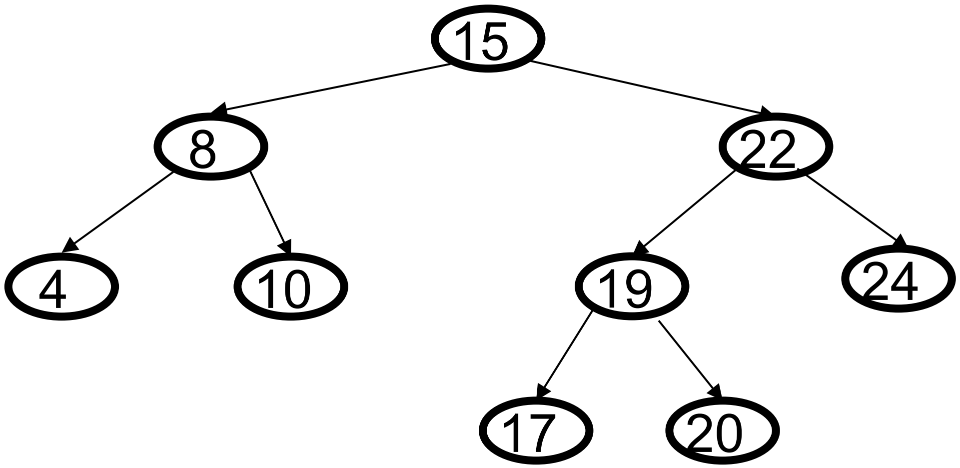


AVL "ROTATION"

- **General case**
  - Suppose this tree is balanced, {X,Y,Z} all have the same height
  - Adding A, puts C out of balance
  - Rotate B up and pass the Y subtree to C
  - **Perform this rotation at the lowest point of imbalance**

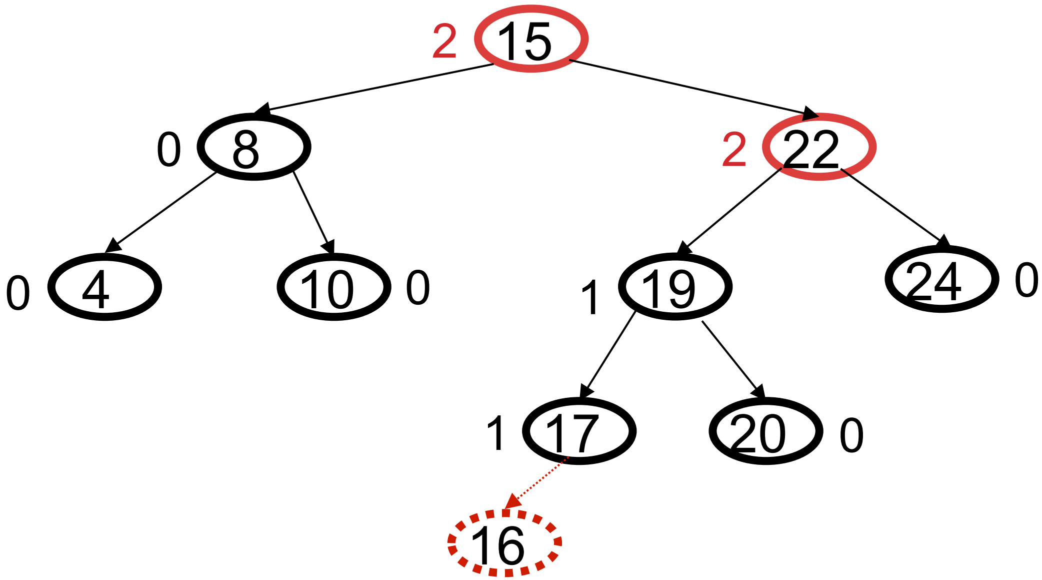


SINGLE ROTATION EXAMPLE



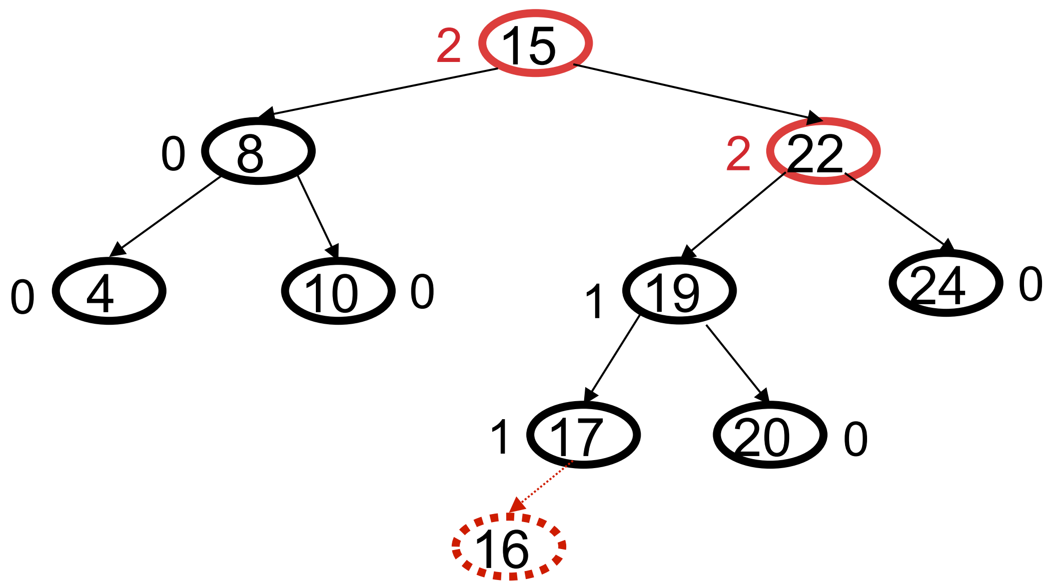
- **Consider the above tree**
  - Is it an AVL tree? *Yes*

SINGLE ROTATION EXAMPLE

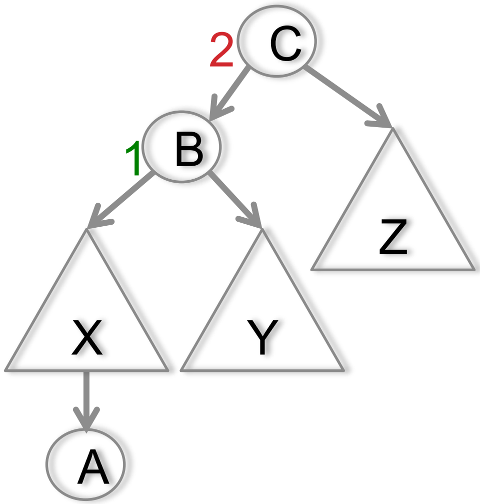


- **Add 16 to the tree**
  - Is it unbalanced now? Where? **22**
  - Also at 15, but we choose the lowest point

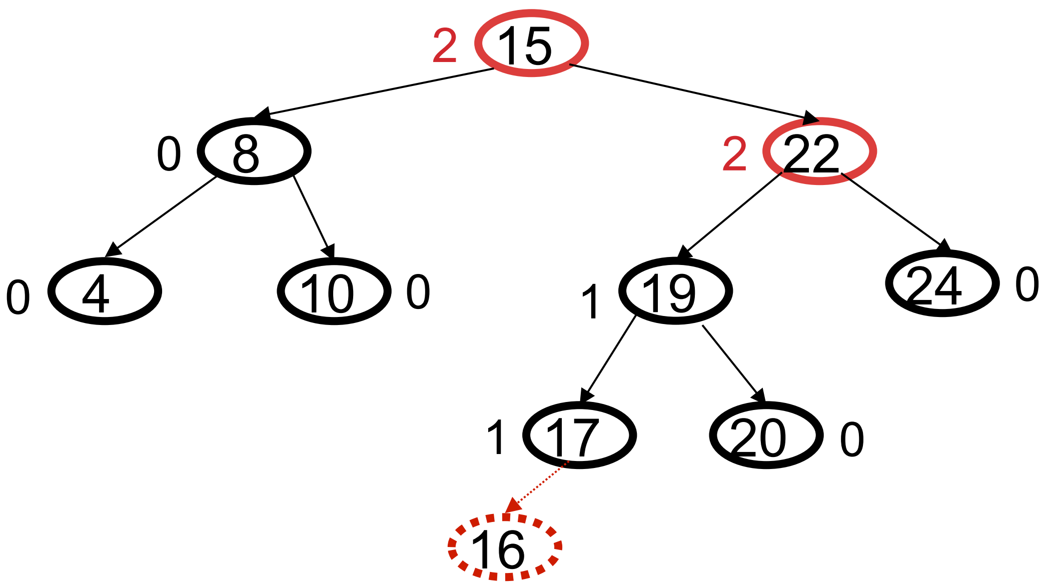
SINGLE ROTATION EXAMPLE



- **Perform the rotation around 22**
  - What rotation takes place?

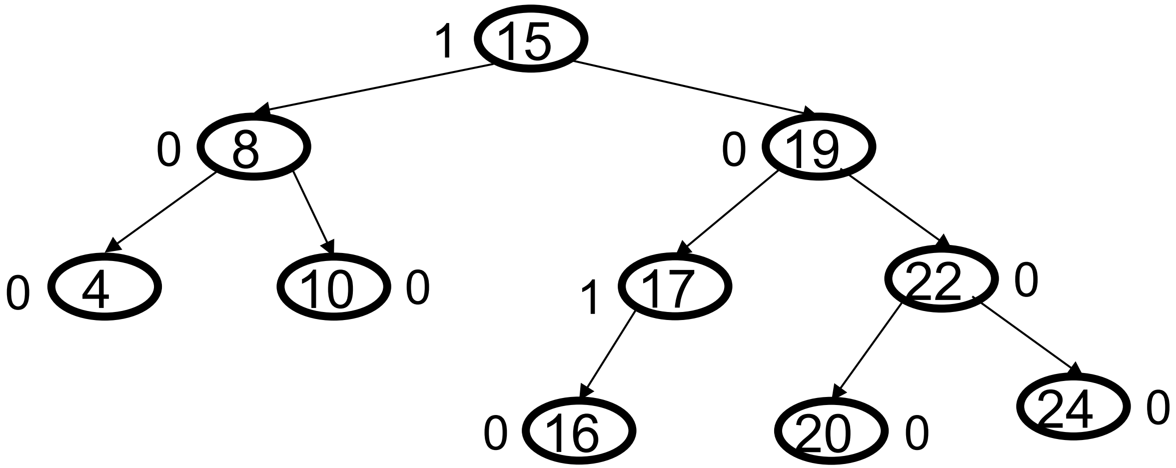


SINGLE ROTATION EXAMPLE



- **Perform the rotation around 22**
  - What rotation takes place?
  - What is the resulting tree?

SINGLE ROTATION EXAMPLE



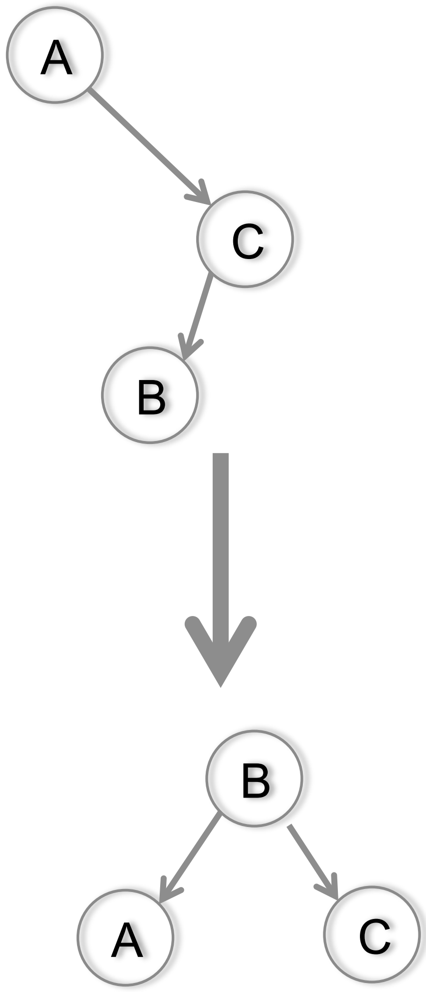
- **19 must move up to where 22 was**
  - 20 changes parents
  - Balances are recomputed throughout the tree

AVL “ROTATION”

- **These two rotations (right-right and left-left) are symmetric and can be solved the same way**
  - Named by the location of the added node relative to the unbalanced node
  - What are the other two cases?

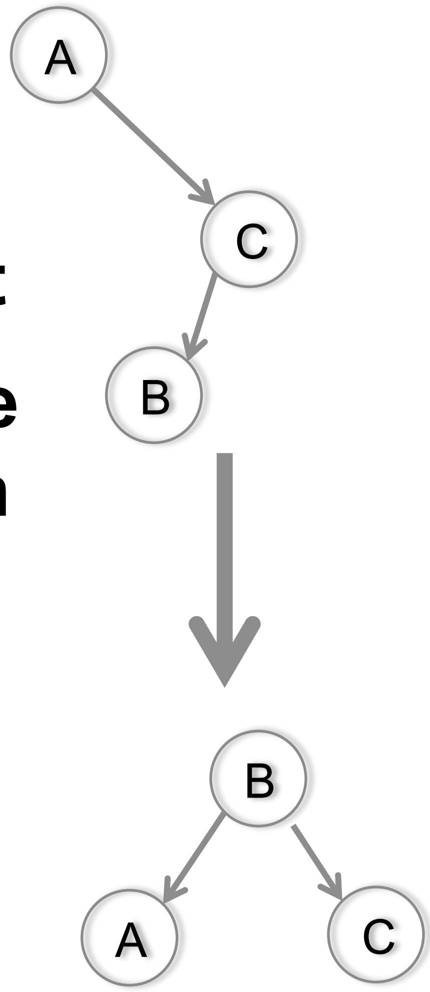
AVL "ROTATION"

- **Right left case**
  - Again, A is out of balance
  - This time, the addition (B) comes between A and C
  - In this case, the grandchild must become the root.



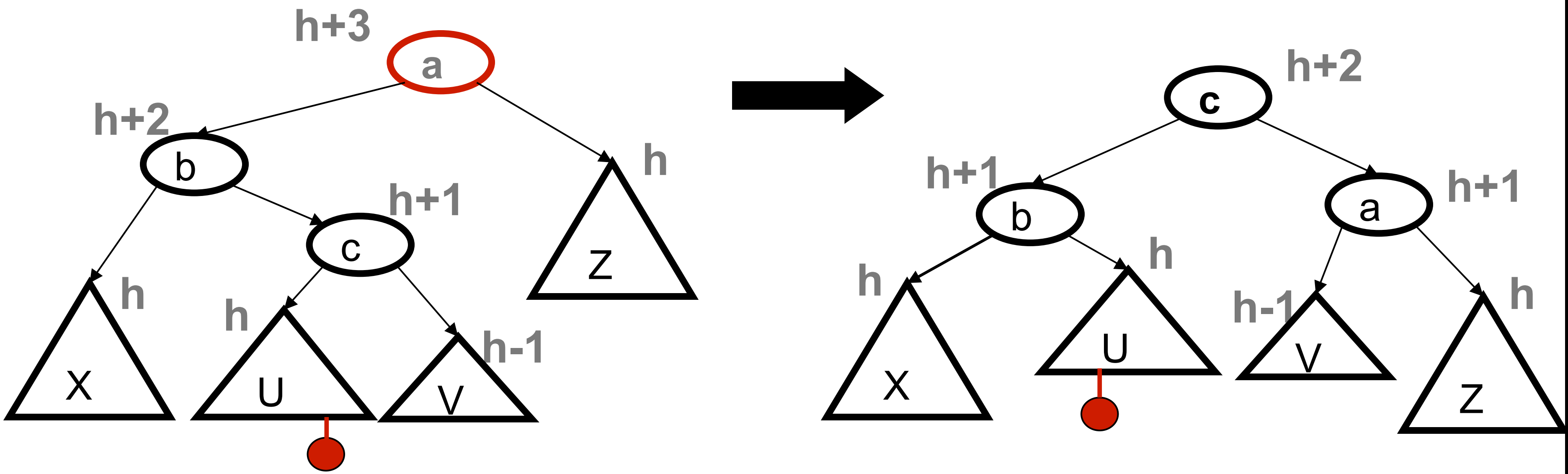
AVL "ROTATION"

- Identifying what should be the new root is key
- Imagine "lifting" up the root
- Where will the children have to go to maintain the search property?

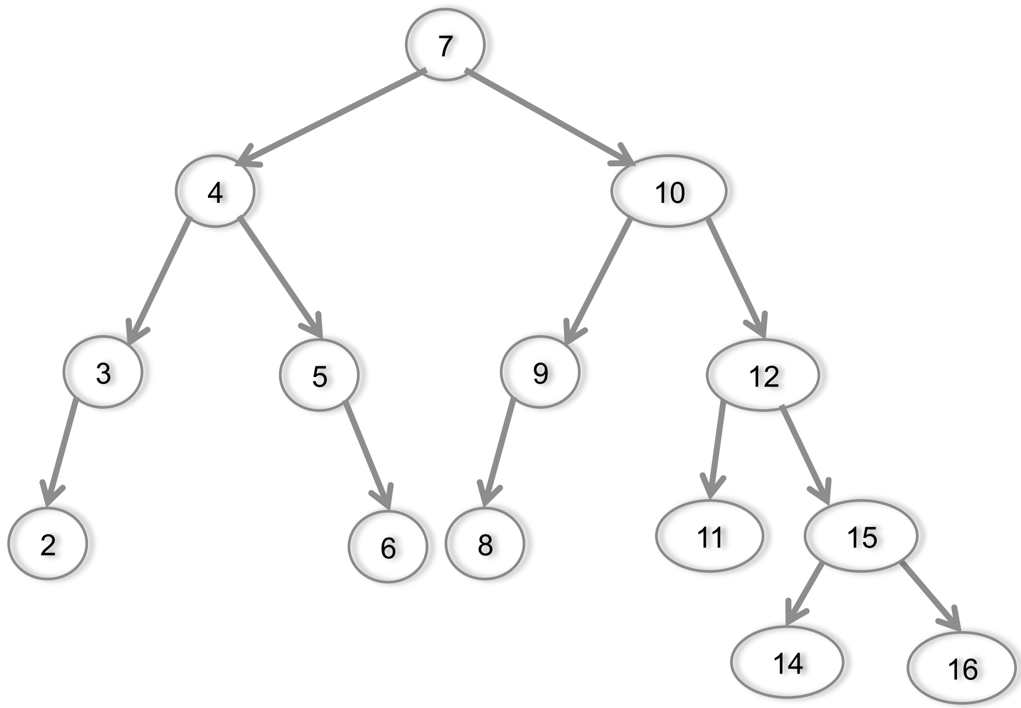


AVL "ROTATION"

- This is for your reference later.

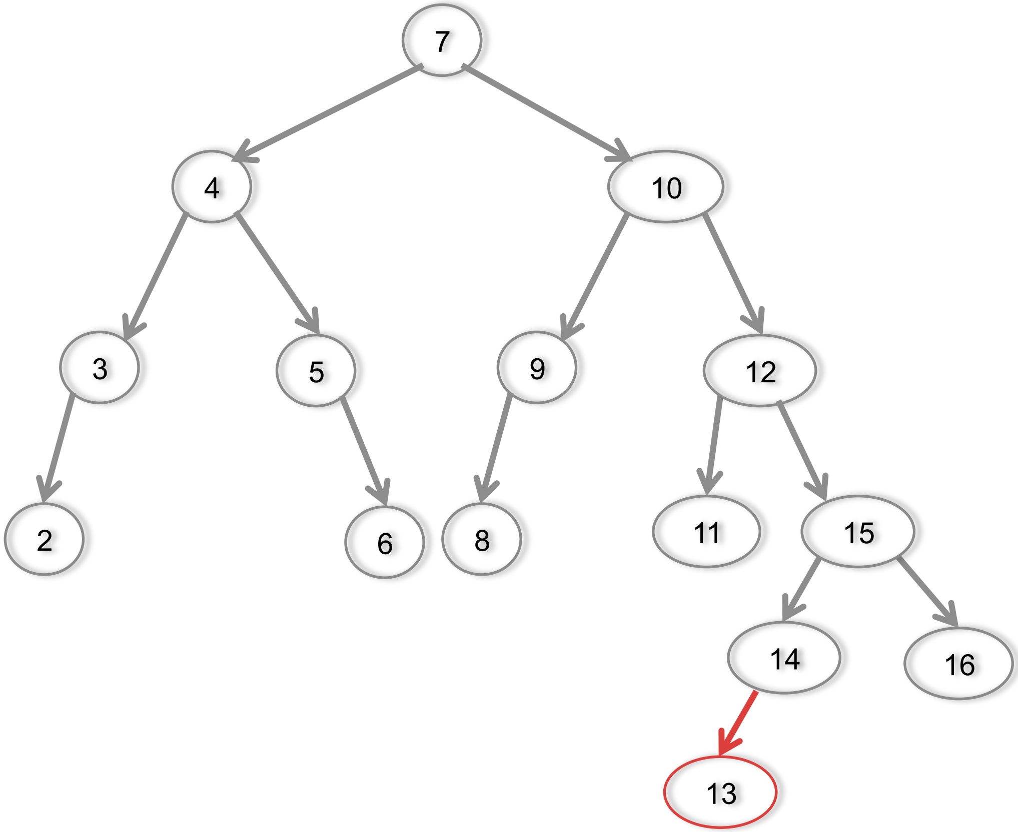


AVL "ROTATION"



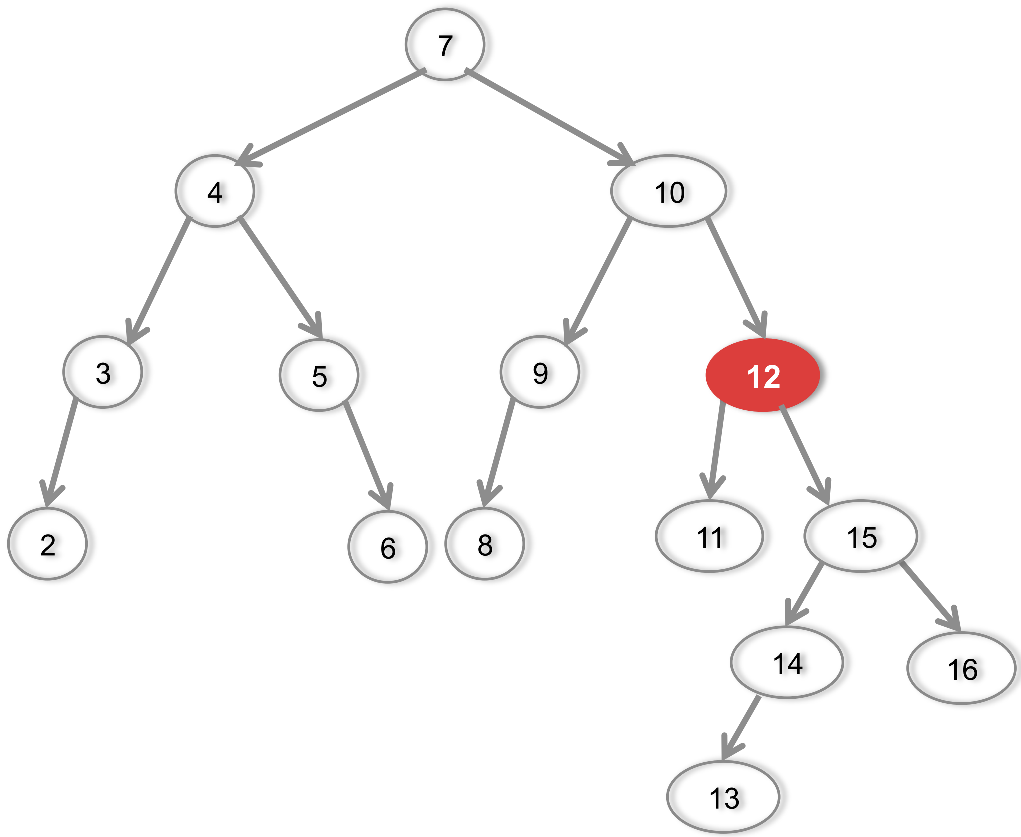
- **Let's do an example. Insert(13)**

AVL "ROTATION"



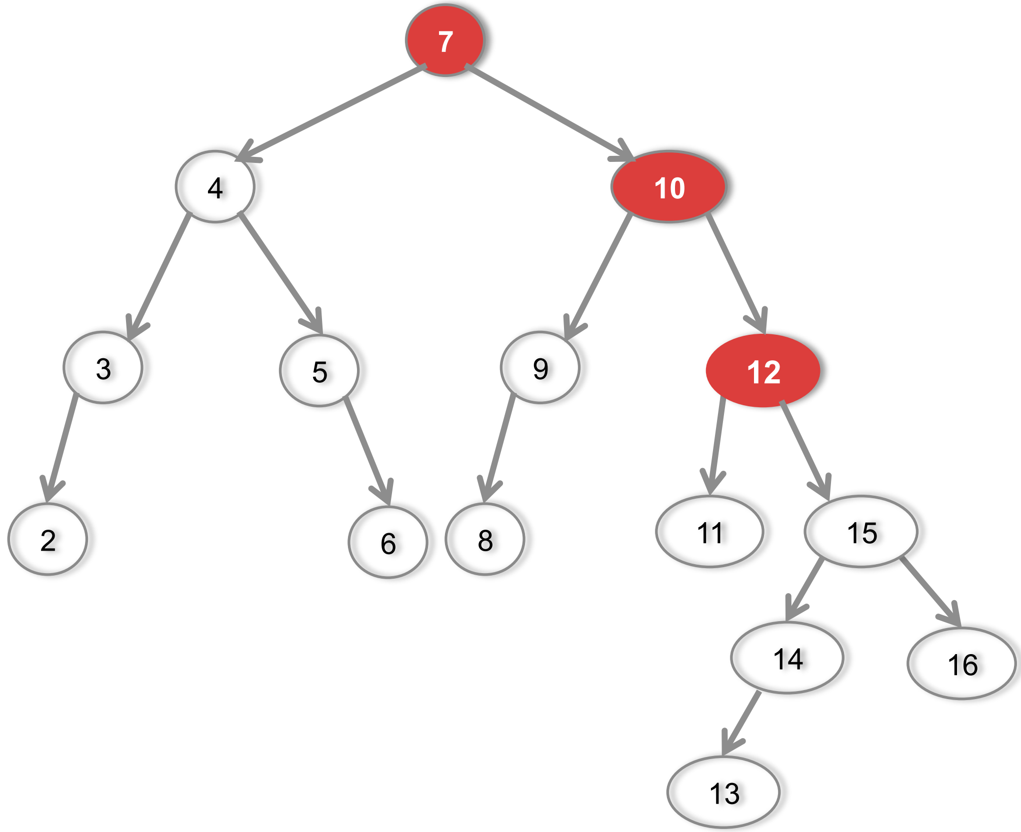
- **Where is the imbalance?**

AVL "ROTATION"



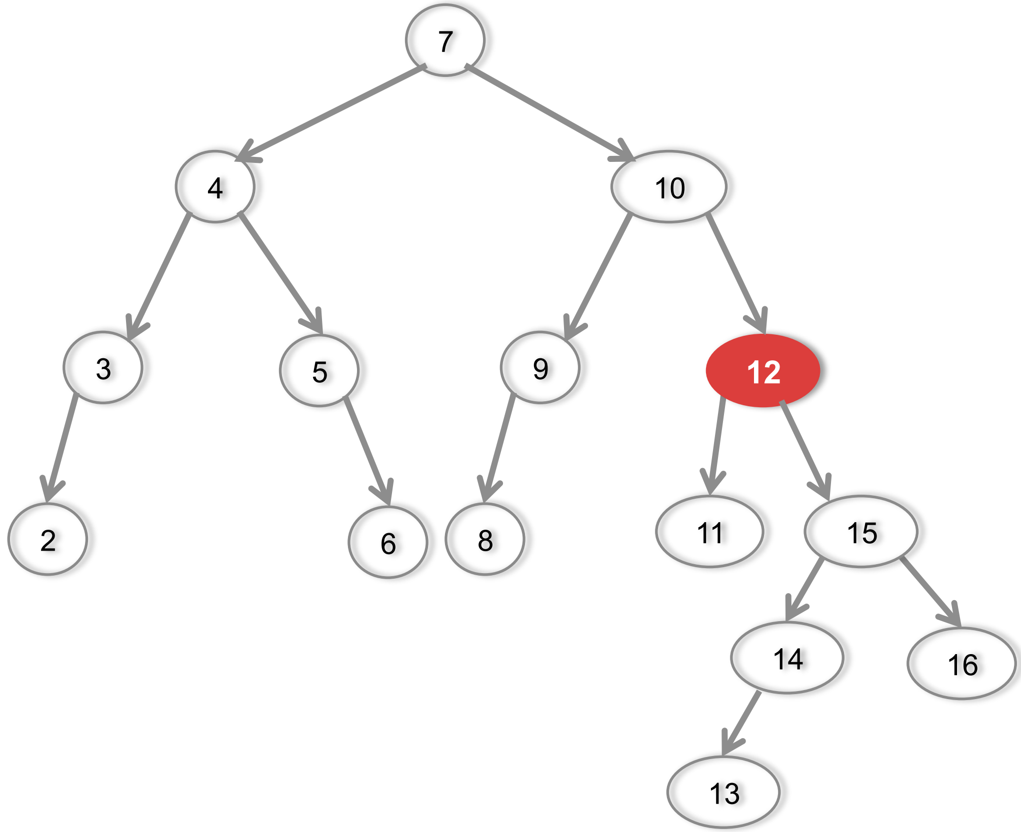
- **Where is the imbalance?**

AVL “ROTATION”



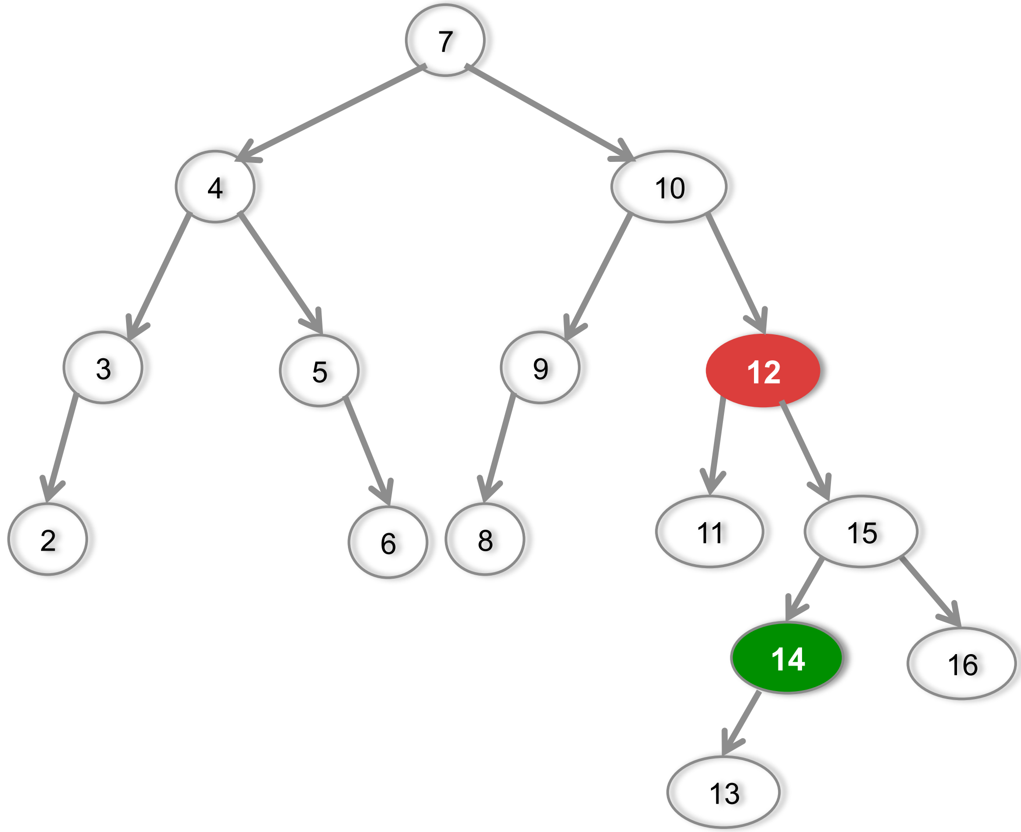
- **Where is the imbalance?** (also 7 and 10)

AVL "ROTATION"



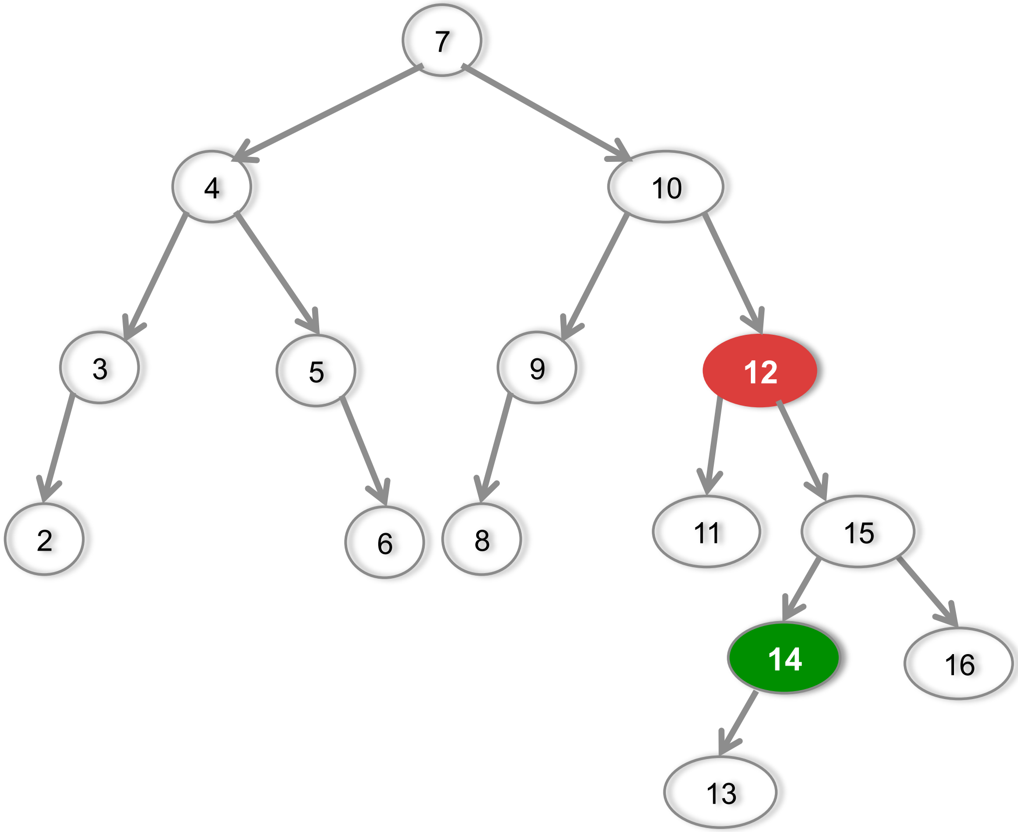
- **What must be the new root?**

AVL "ROTATION"



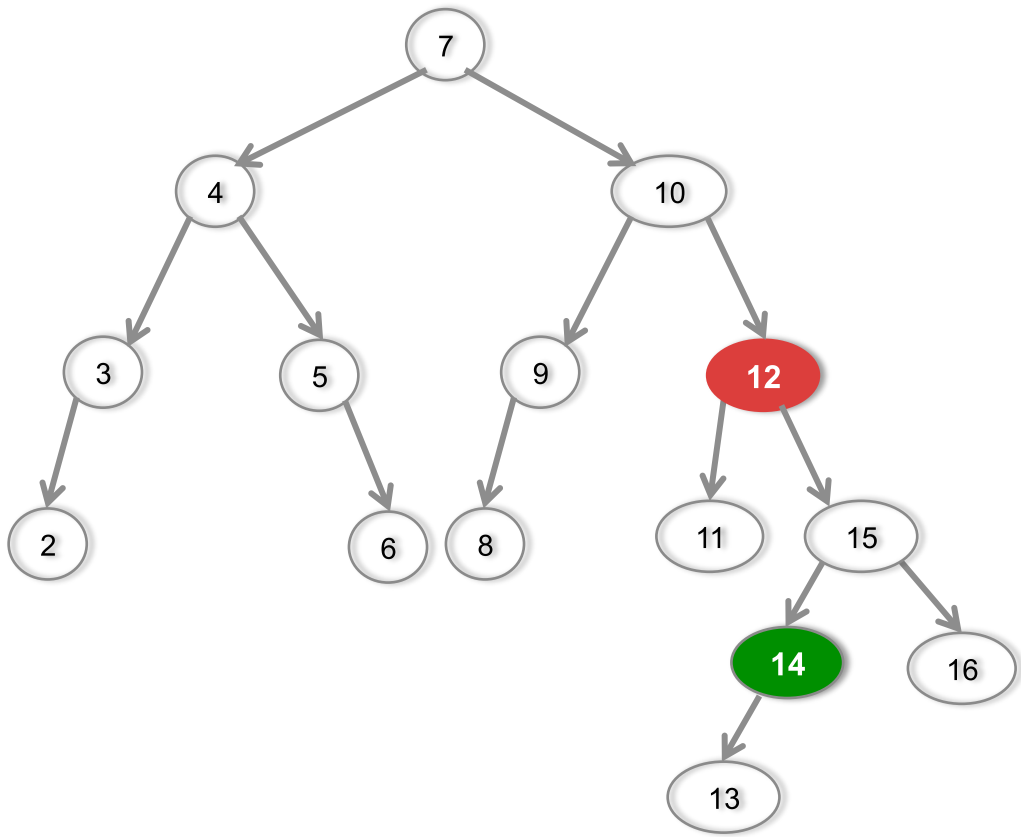
- **What must be the new root?**

AVL "ROTATION"



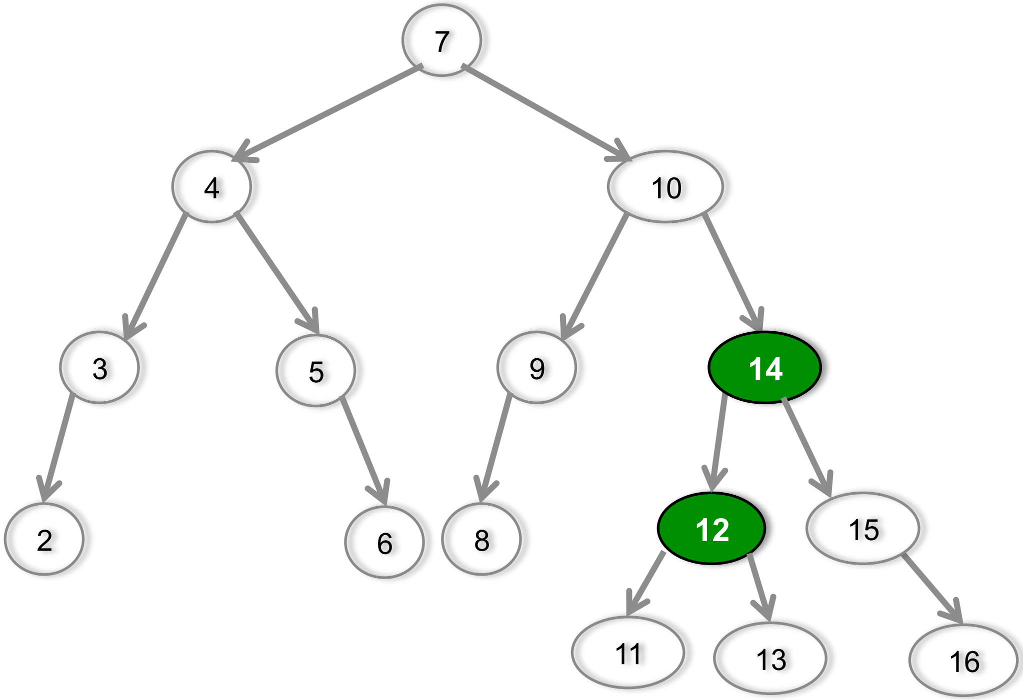
- **What must be the new root? Why?**

AVL "ROTATION"



- What does the new tree look like?

AVL "ROTATION"



- **The replaced root is always a child of the new root!**

NEXT CLASS

- **AVL Trees**
  - Even more examples!
  - Showing that this actually gives us O(log n) height
  - Showing insert is O(1)
- **Memory analysis**
  - Formalization to help with confusion from last week