CSE 373

APRIL 17TH – TREE BALANCE AND AVL

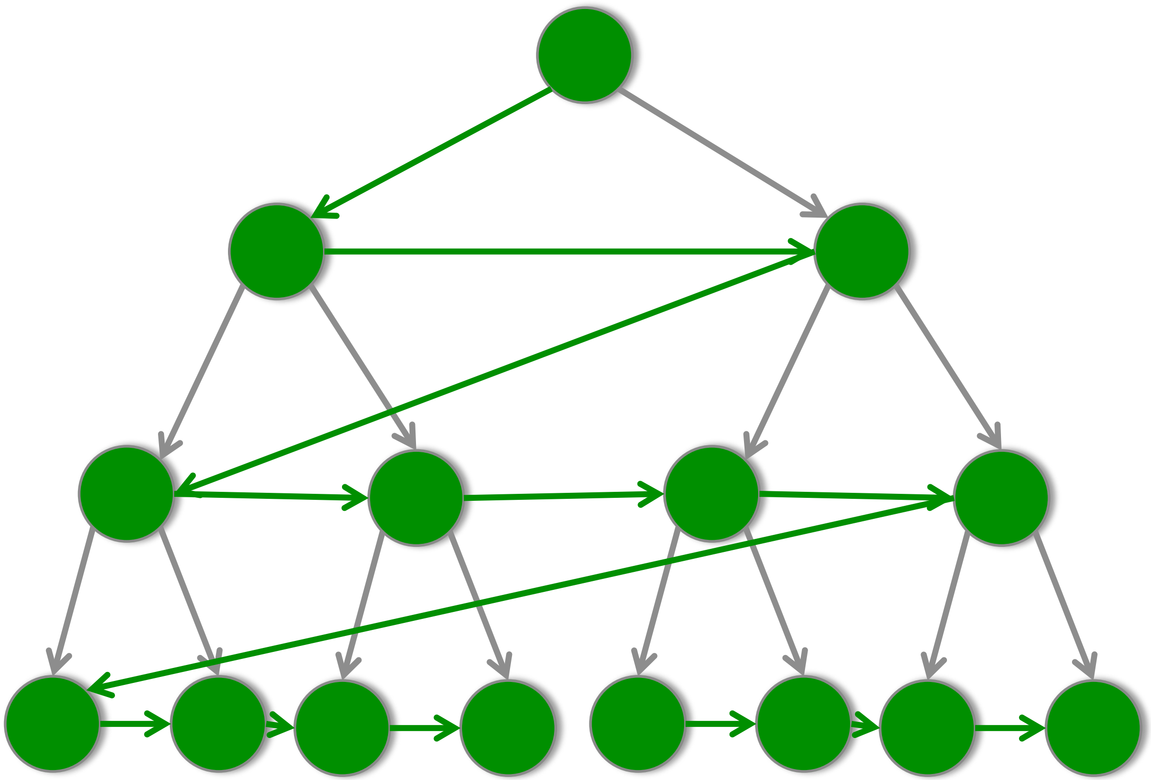
ASSORTED MINUTIAE

- **HW3 due Wednesday**
  - Double check submissions
  - Use binary search for SADict
- **Midterm text Friday**
  - Review in Class on Wednesday
- **Testing Advice**
  - Empty and New are different edge cases
  - HW1 regrade

TODAY'S LECTURE

- **Tree traversals**
  - Memory Allocation
  - Traversal ordering
- **Tree Balance**
  - Improving on worst case time for trees

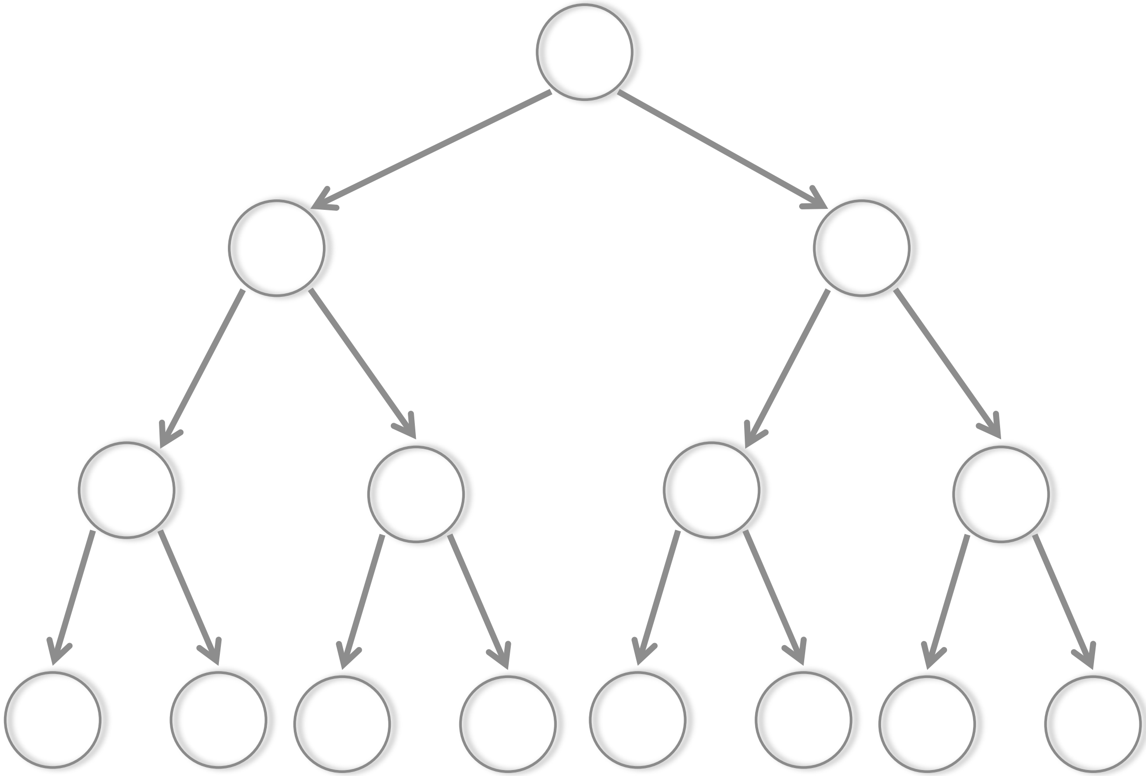
REVIEW



REVIEW

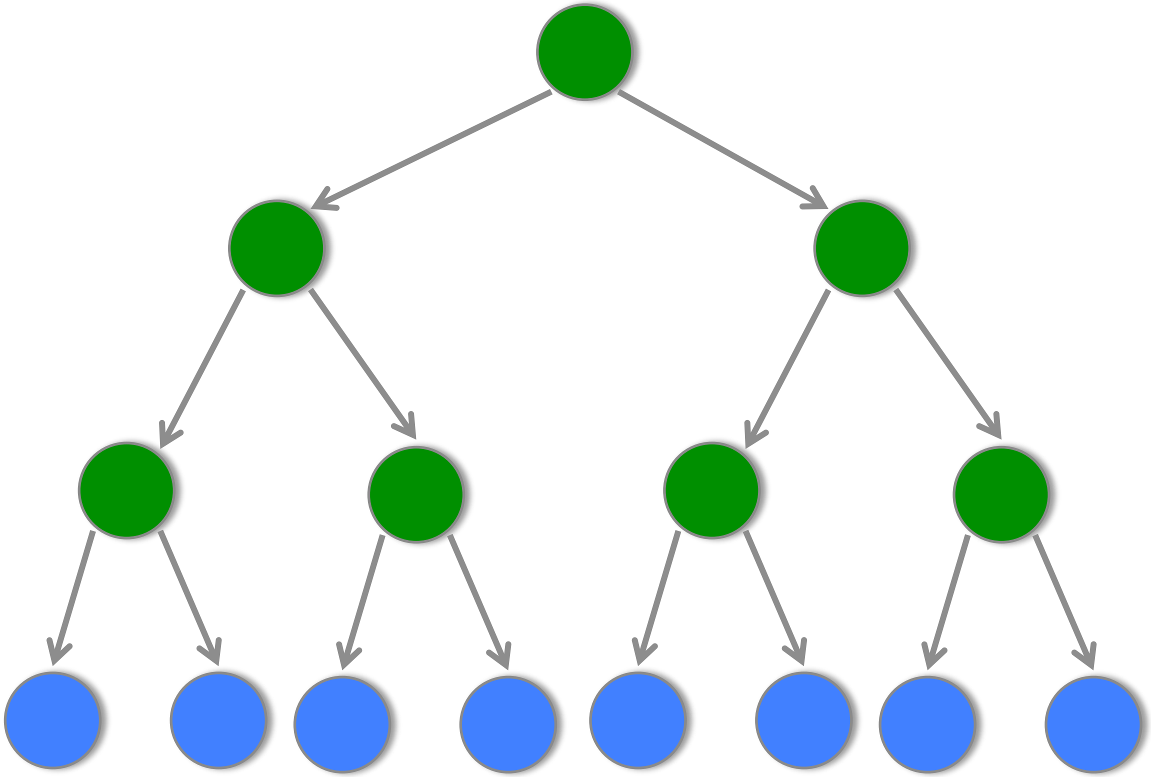
- **Breadth First Search**
  - Enqueue the root
  - While the queue has elements
    - Dequeue
    - Process
    - Enqueue children
  - How much memory does this take?

SEARCH MEMORY USE



- **When does the queue have the most elements?**

SEARCH MEMORY USE



- **At the widest point in the traversal**
  - How many elements is this?

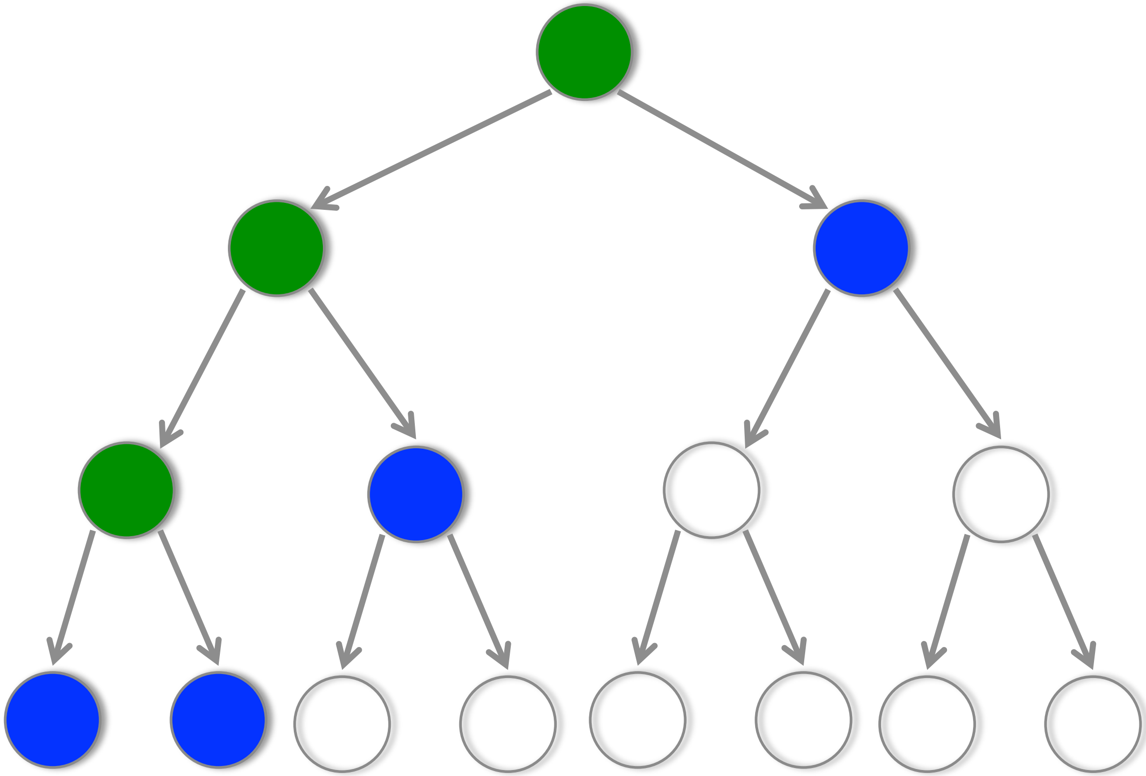
SEARCH MEMORY USE

- **Breadth First Search**
  - In a perfect tree (where every row is complete) of size **n**, how many elements are in the last row?
    - **ceiling(N/2)**, this is important to know!
    - **O(n)** memory usage!

SEARCH MEMORY USE

- **What about depth first search?**
  - When does the stack have the most elements on it?

SEARCH MEMORY USE



- **When does the stack have the most elements?**
  - When it's at the bottom

SEARCH MEMORY USE

- **How many elements are in the stack in this worst case?**
  - The height of the tree, **O(n)** if the tree is one-sided, but **O(log n)** if the tree is balanced
  - We will discuss balance later
  - Classic exam question! Consider memory AND execution times

REVIEW

- **Depth First Search**
  - Iterative and Recursive options
  - Consider the recursive approach we discussed in class

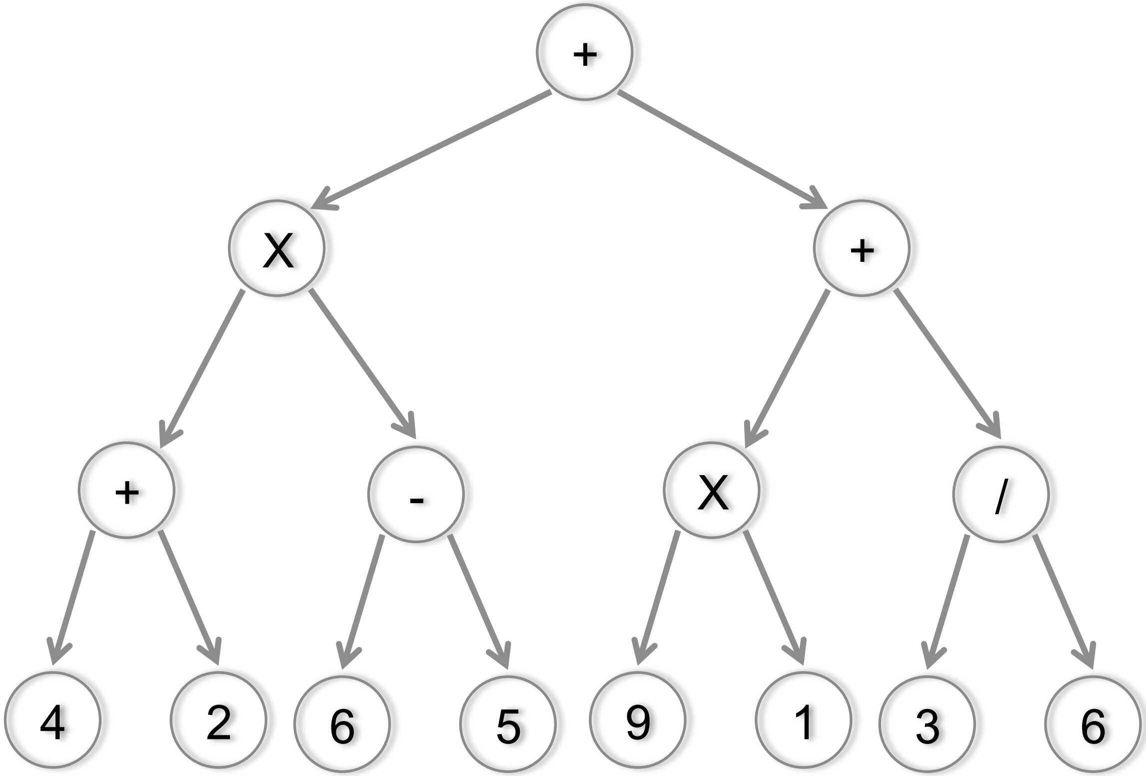
REVIEW

- **Ordering**
  - What is the difference between these three implementations
    - Process; DFS(left); DFS(right)
    - DFS(left); Process; DFS(right)
    - DFS(left); DFS(right); Process
  - *How does this impact the final output?*

REVIEW

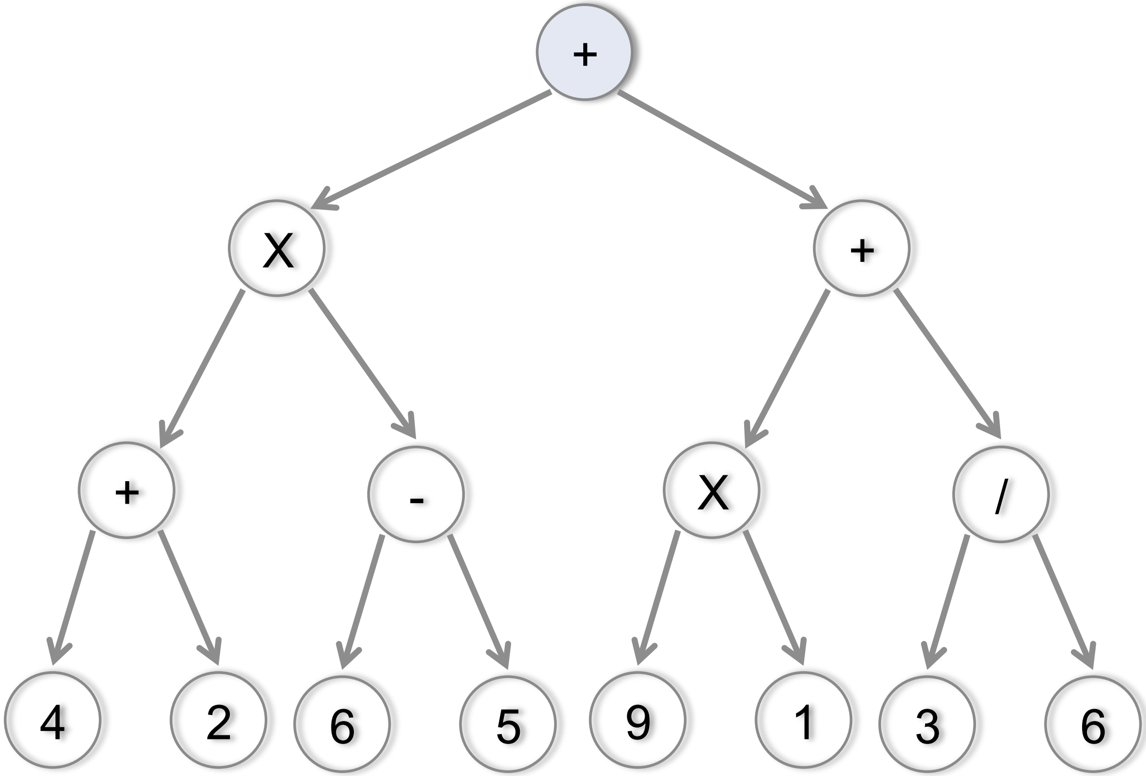
- **Ordering**
  - Three traversal types
    - Pre-order
    - In-order
    - Post-order
- **Instruction (Parse) trees**

PREORDER TRAVERSAL



Stack:	
Output:	

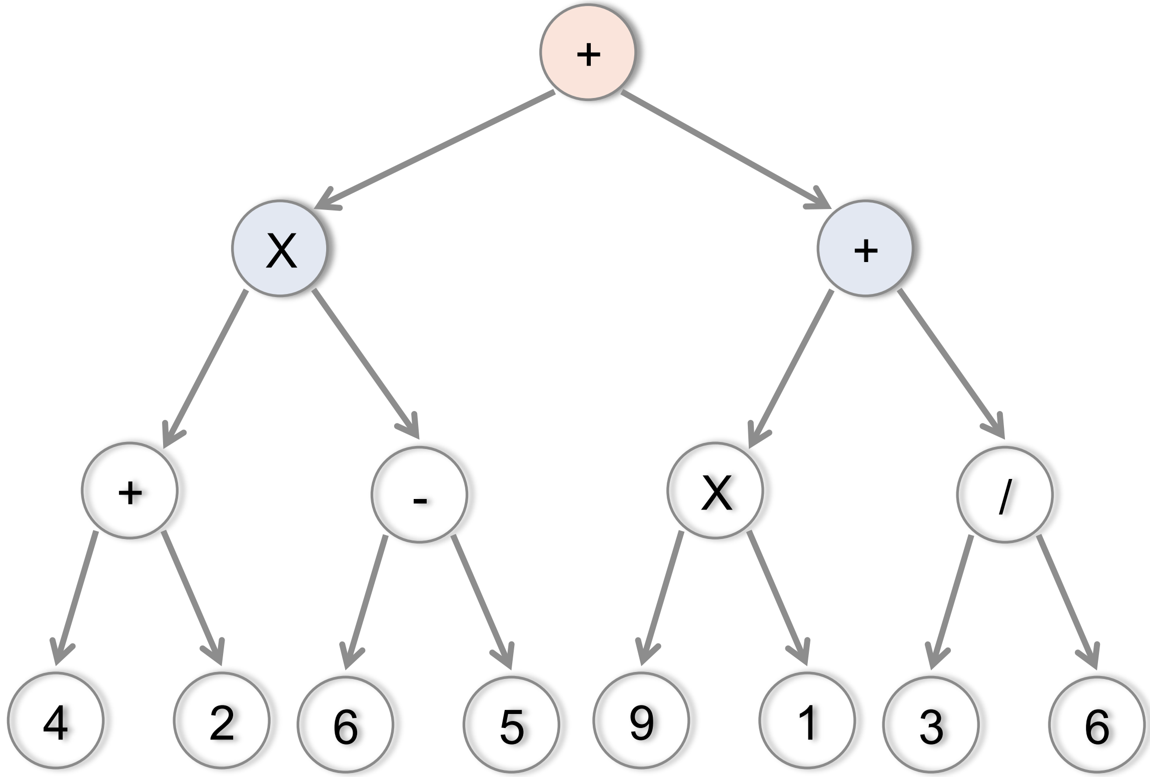
PREORDER TRAVERSAL



Add the root to the stack

Stack:	+ |
Output:	

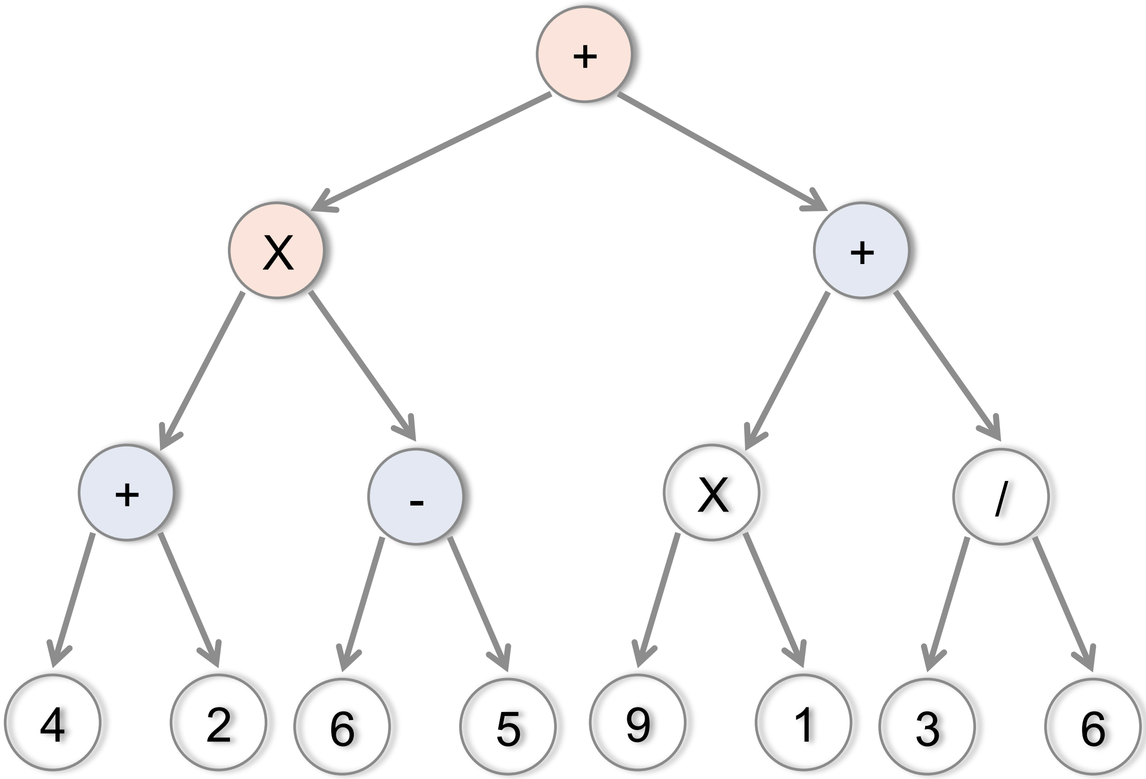
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	X | +
Output:	+

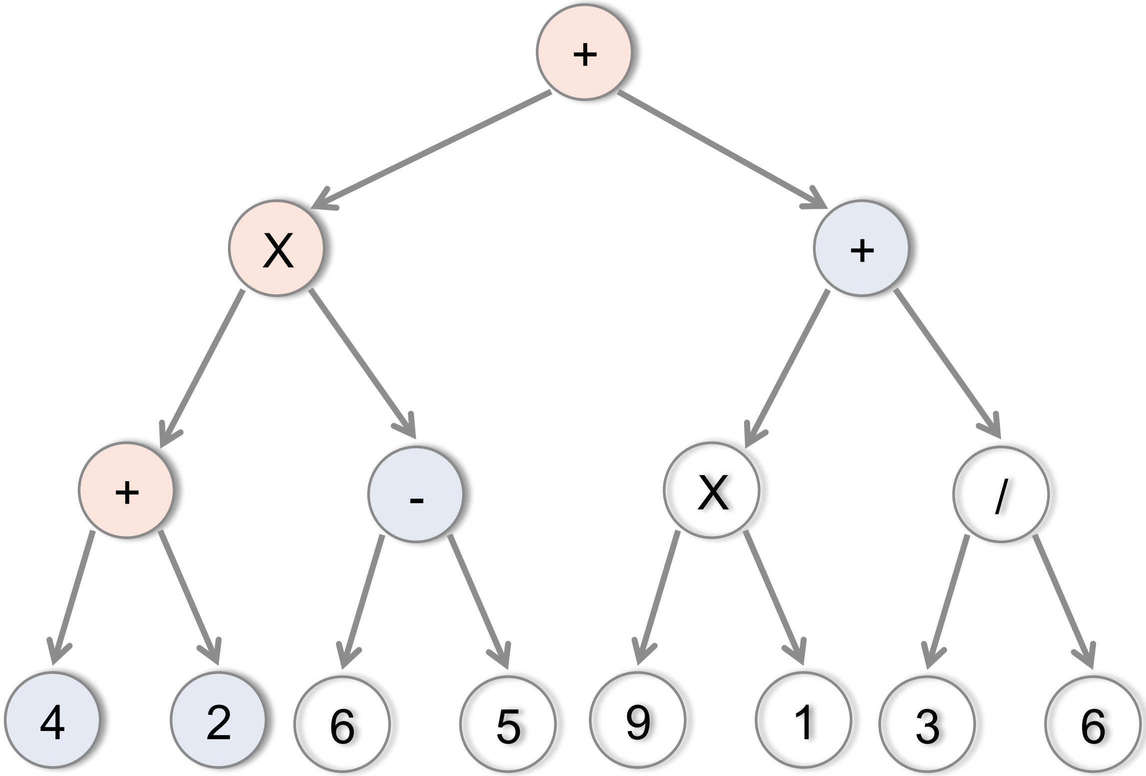
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	+ | - | +
Output:	+X

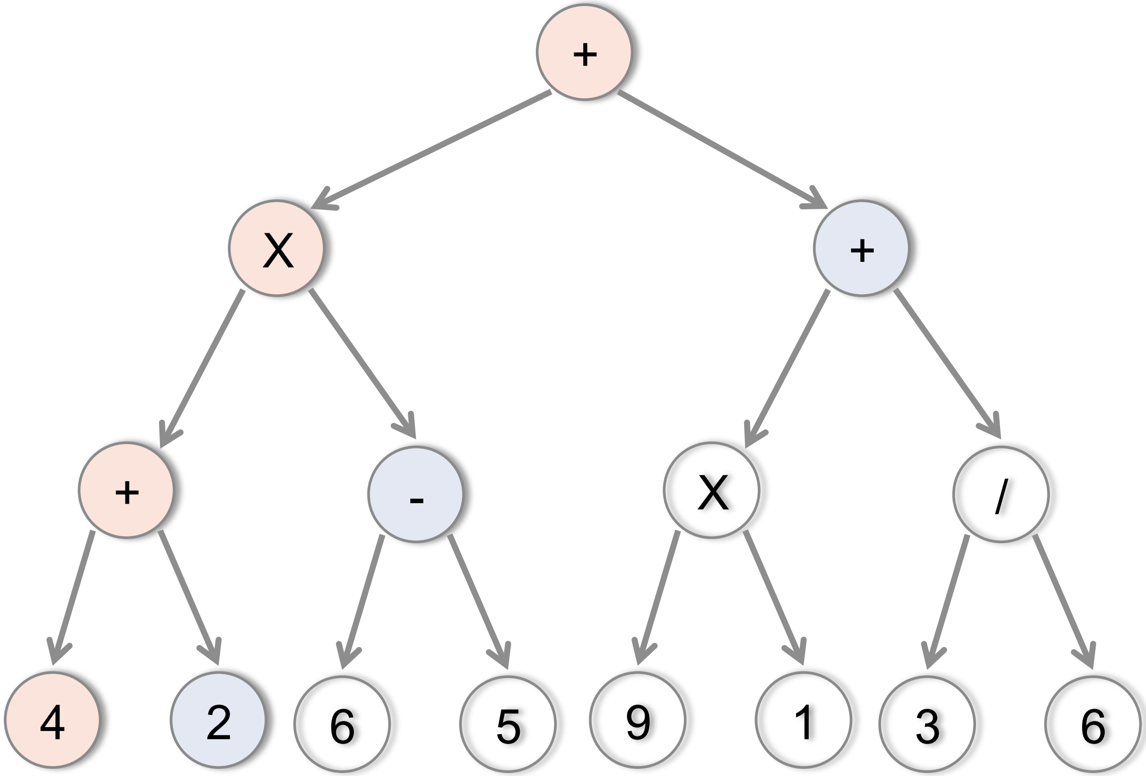
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	4 | 2 | - | +
Output:	+X+

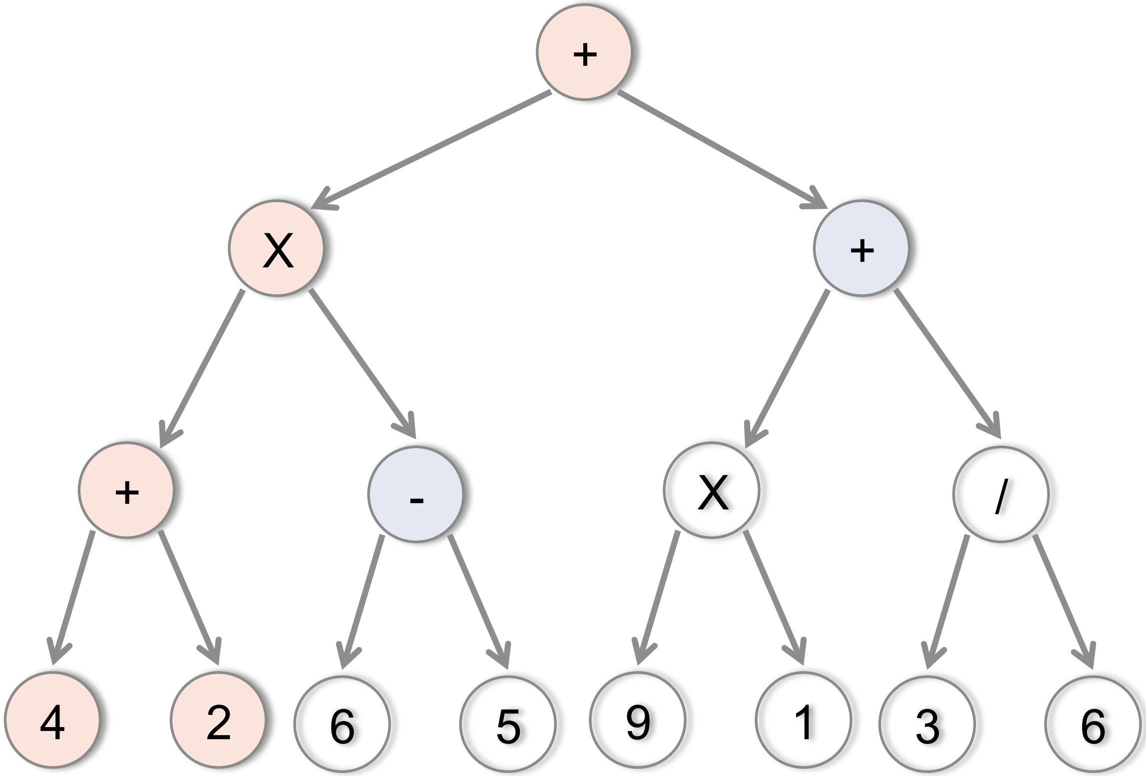
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	2 | - | +
Output:	+X+4

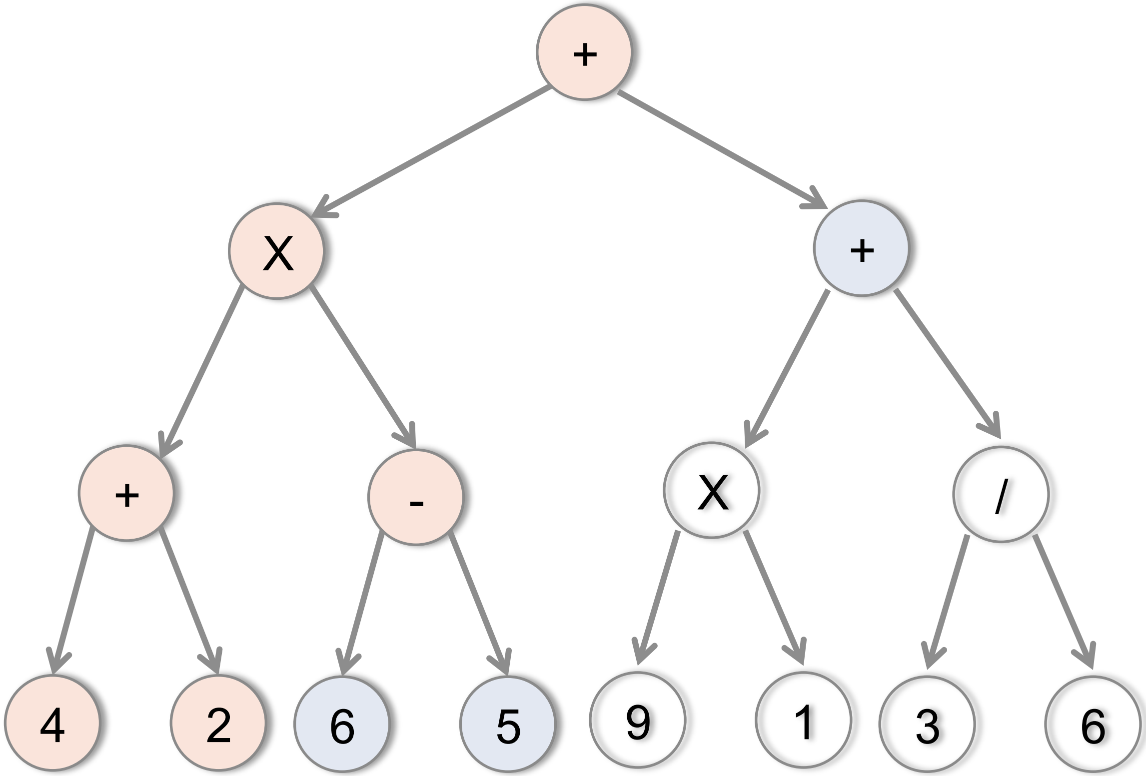
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	- | +
Output:	+X+42

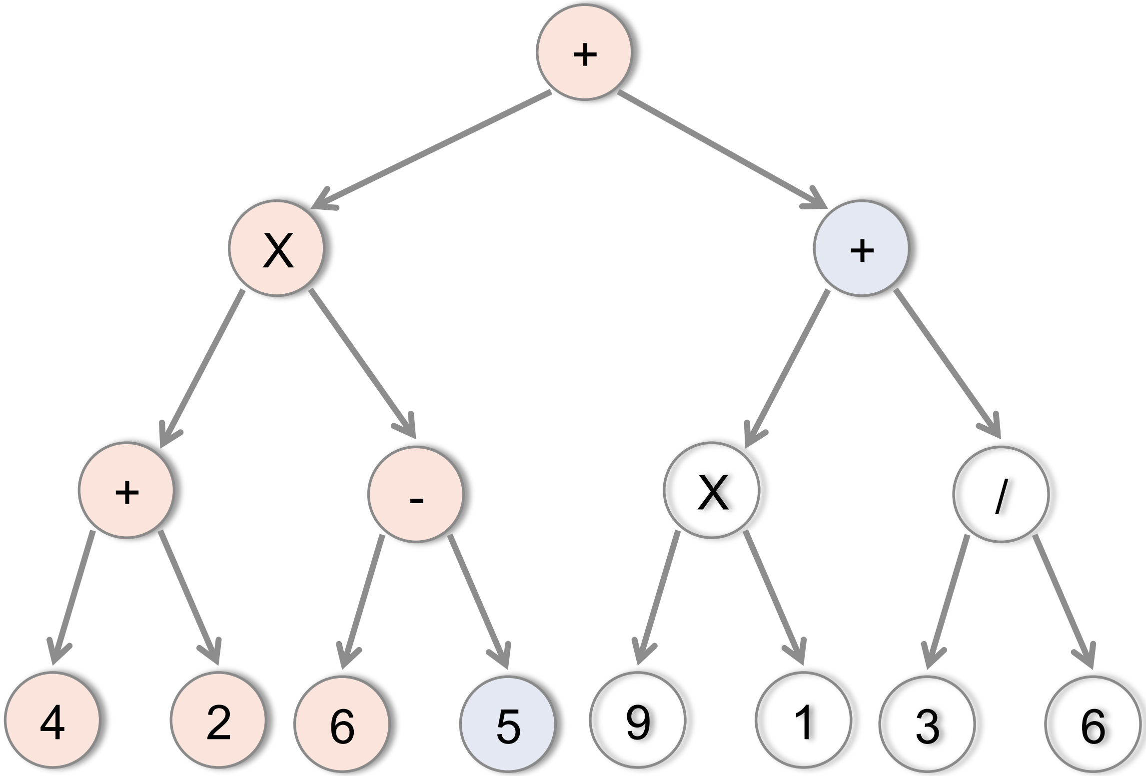
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	6 | 5 | +
Output:	+X+42-

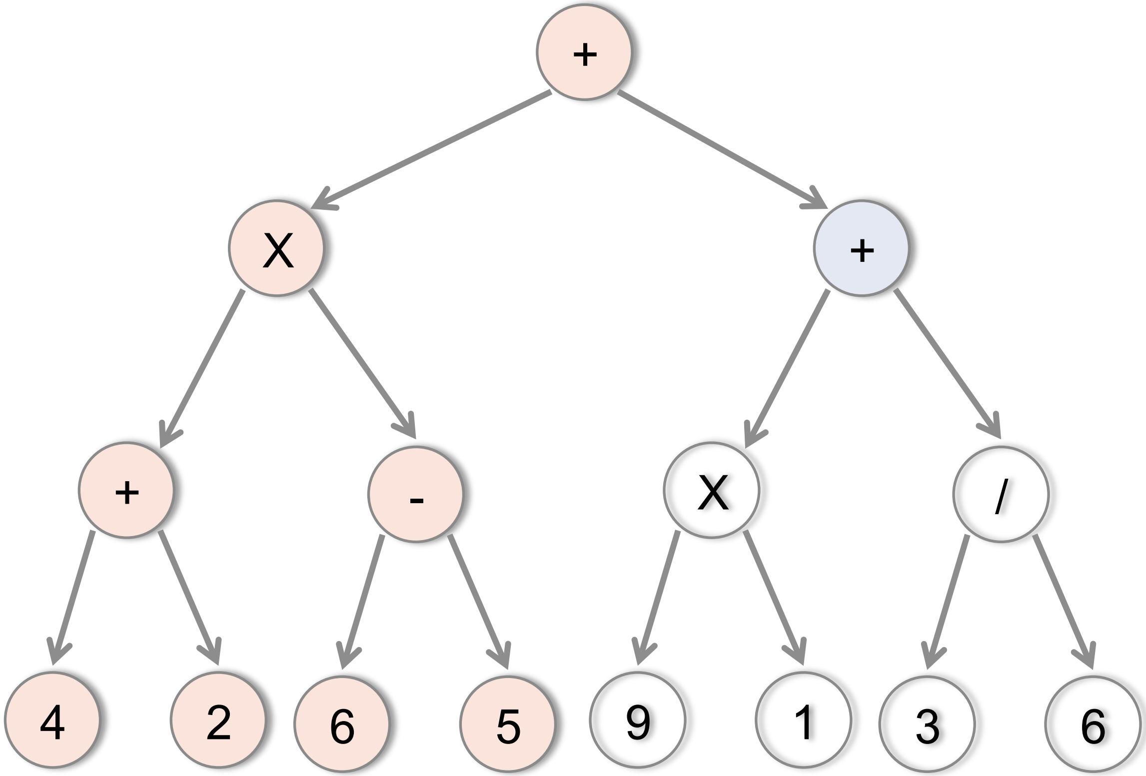
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	5 | +
Output:	+X+42-6

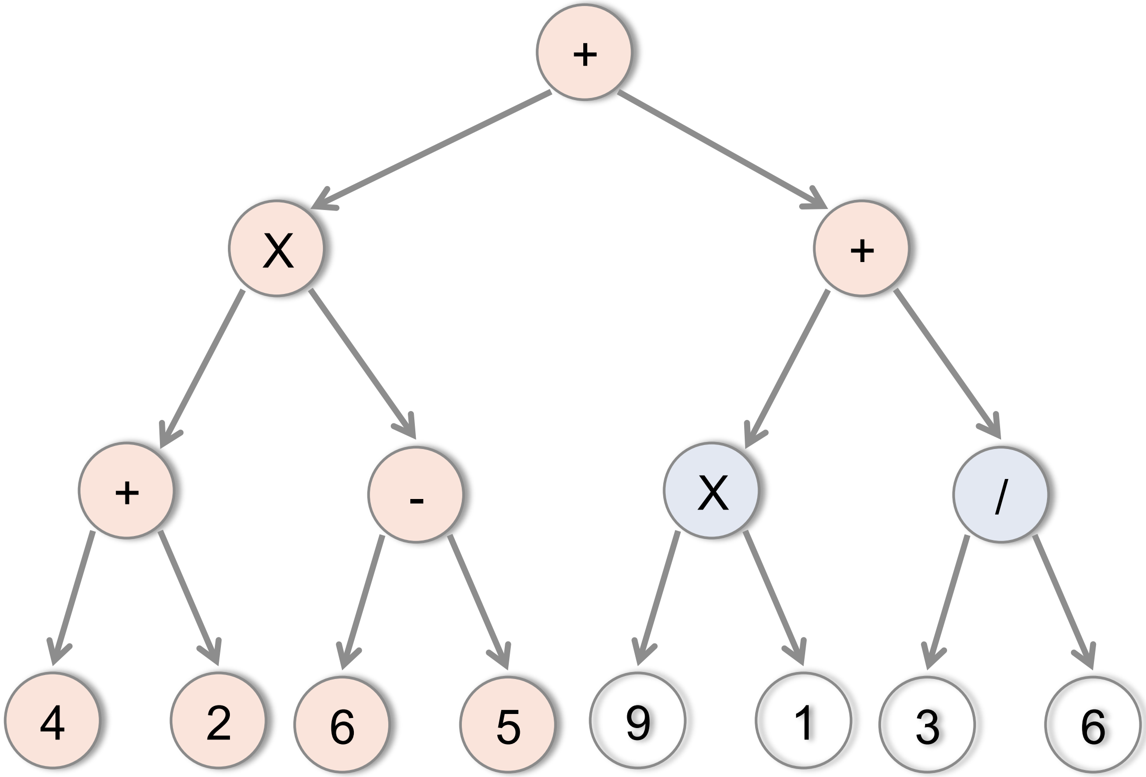
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	+
Output:	+X+42-65

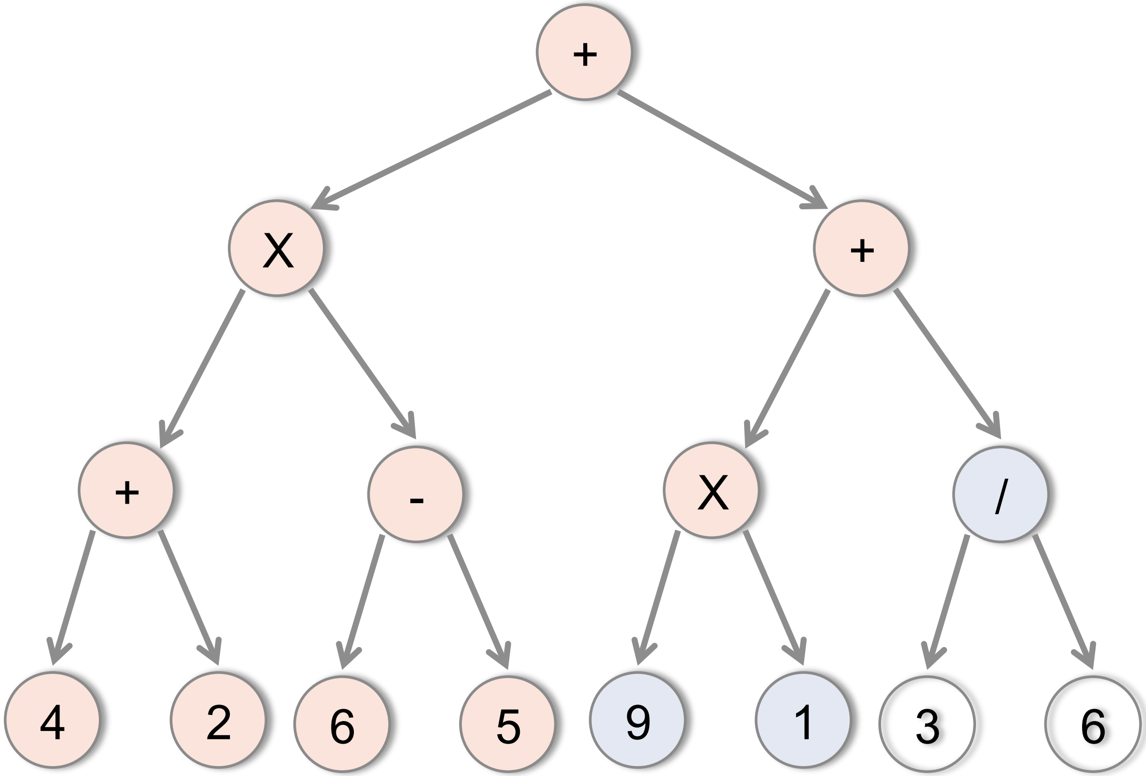
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	X | /
Output:	+X+42-65+

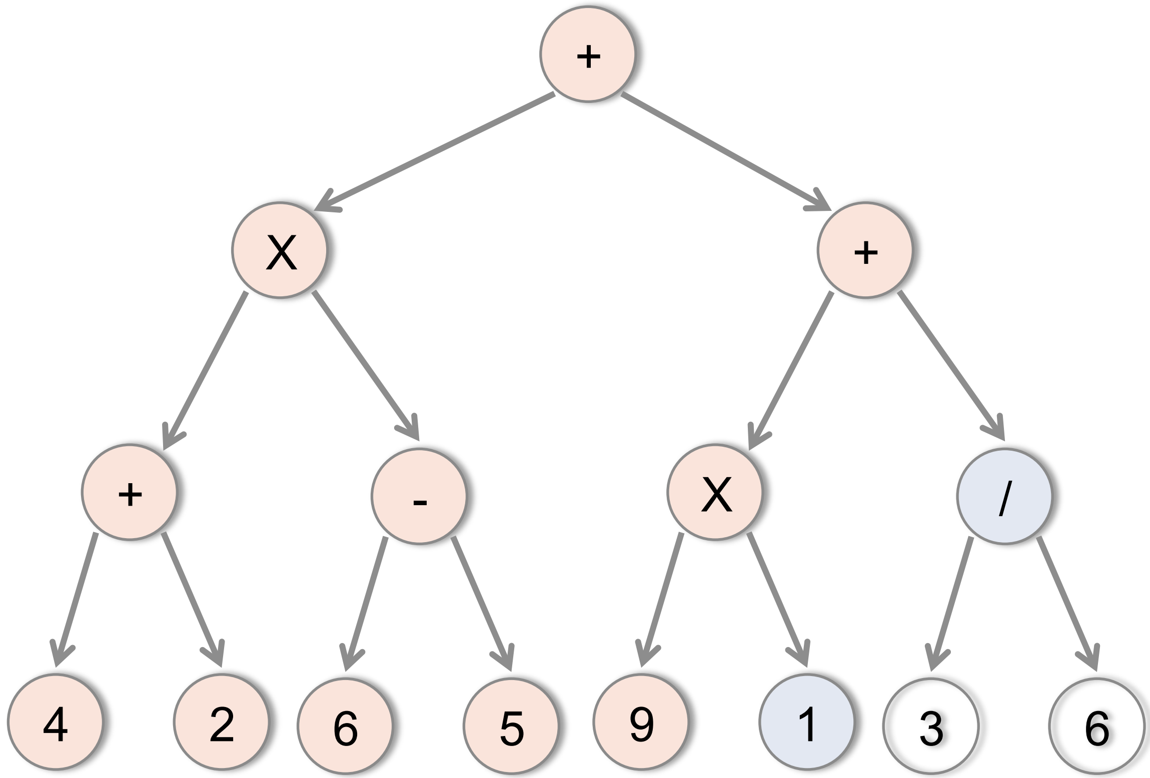
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	9 | 1 | /
Output:	+X+42-65+X

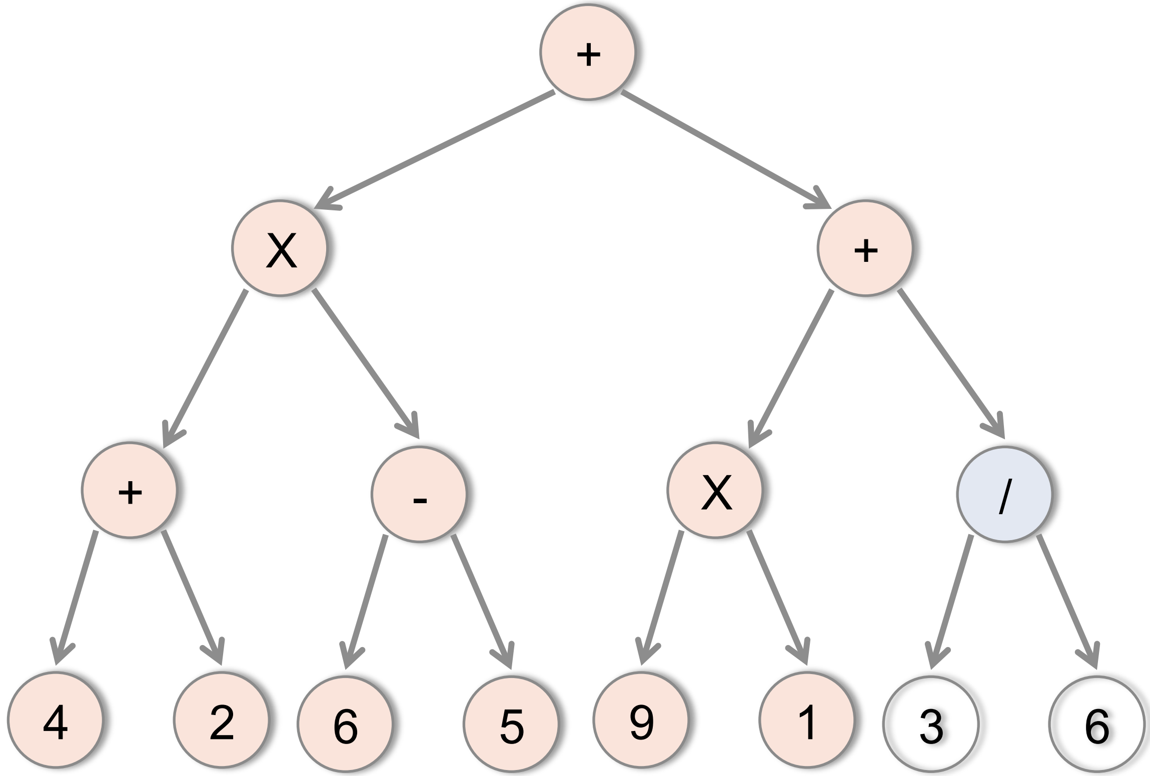
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	1 | /
Output:	+X+42-65+X9

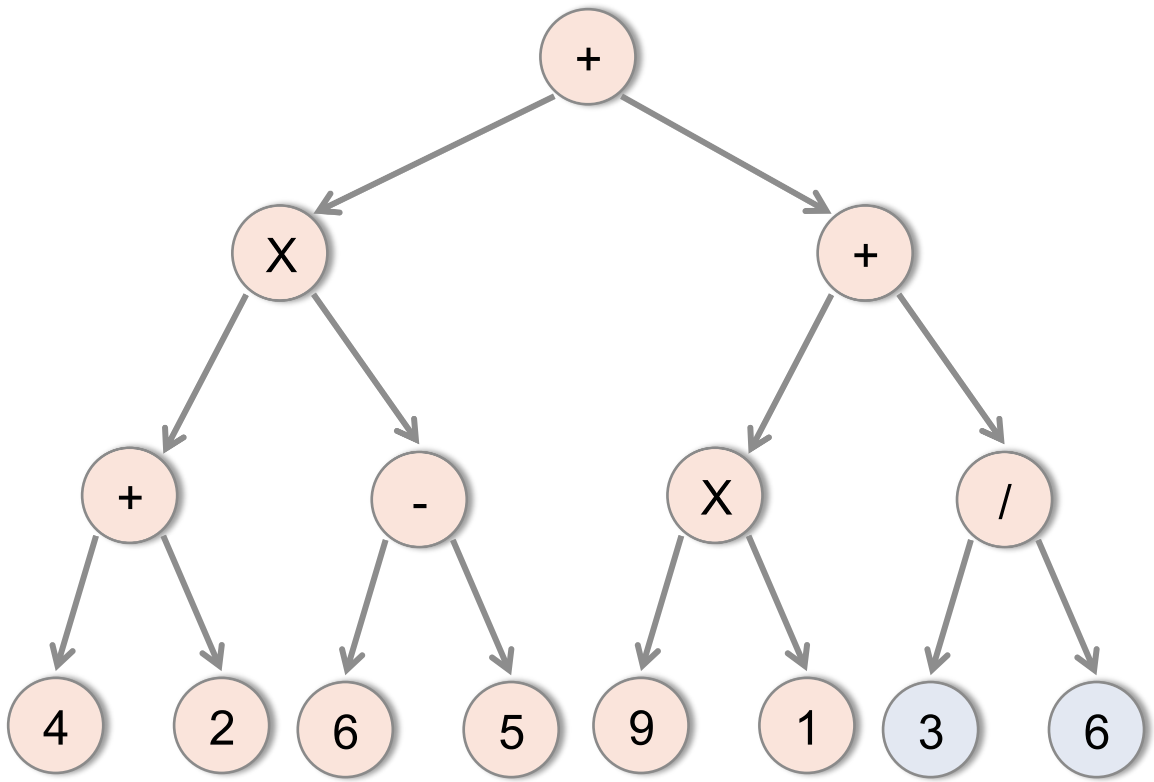
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	/
Output:	+X+42-65+X91

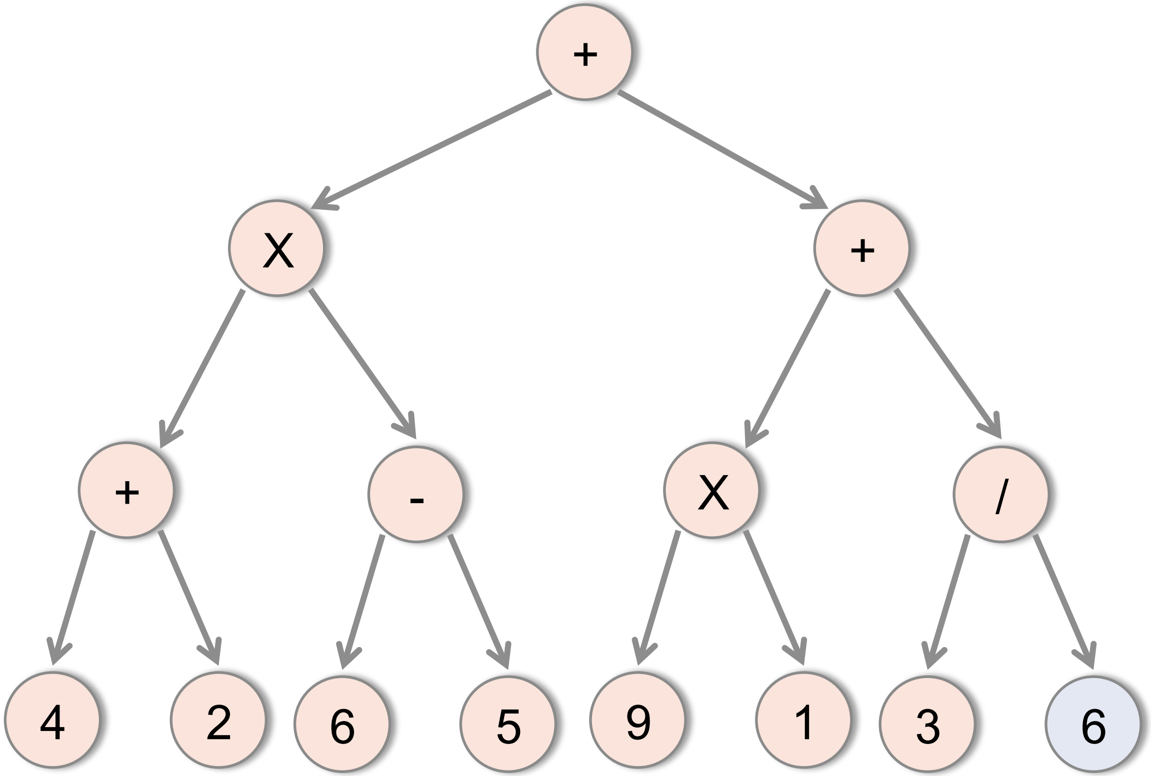
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	3 | 6
Output:	+X+42-65+X91/

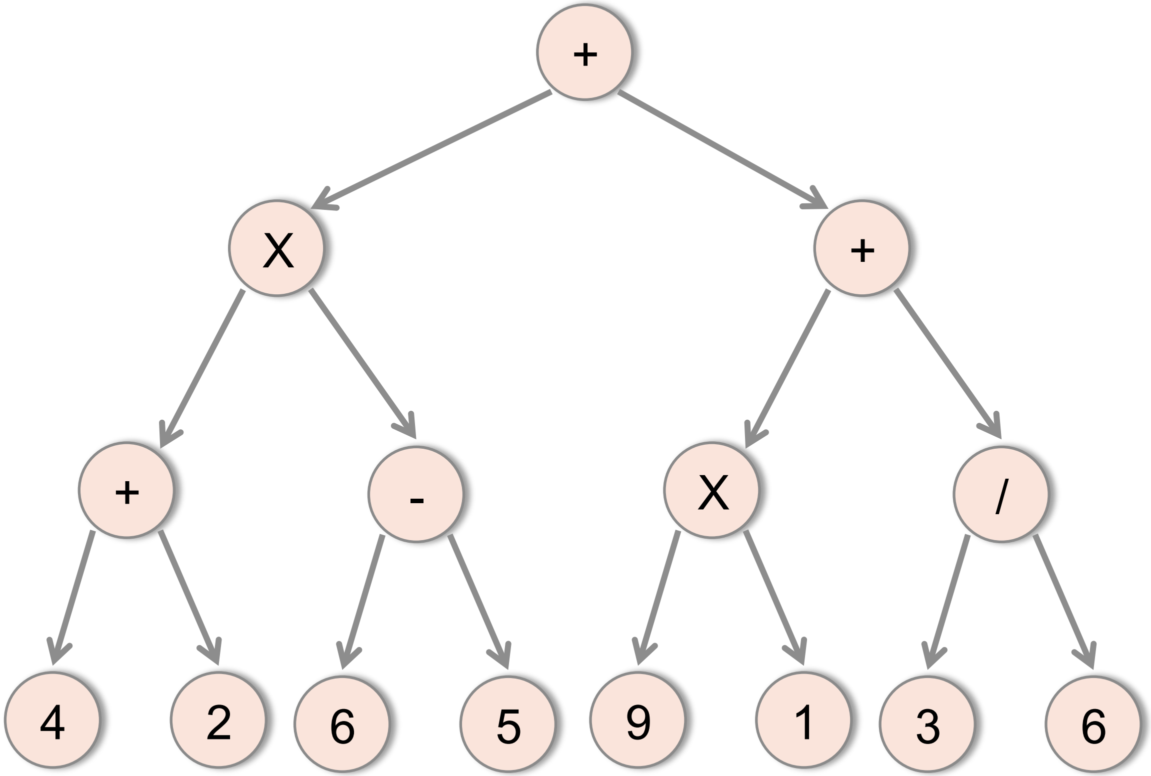
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	6
Output:	+X+42-65+X91/3

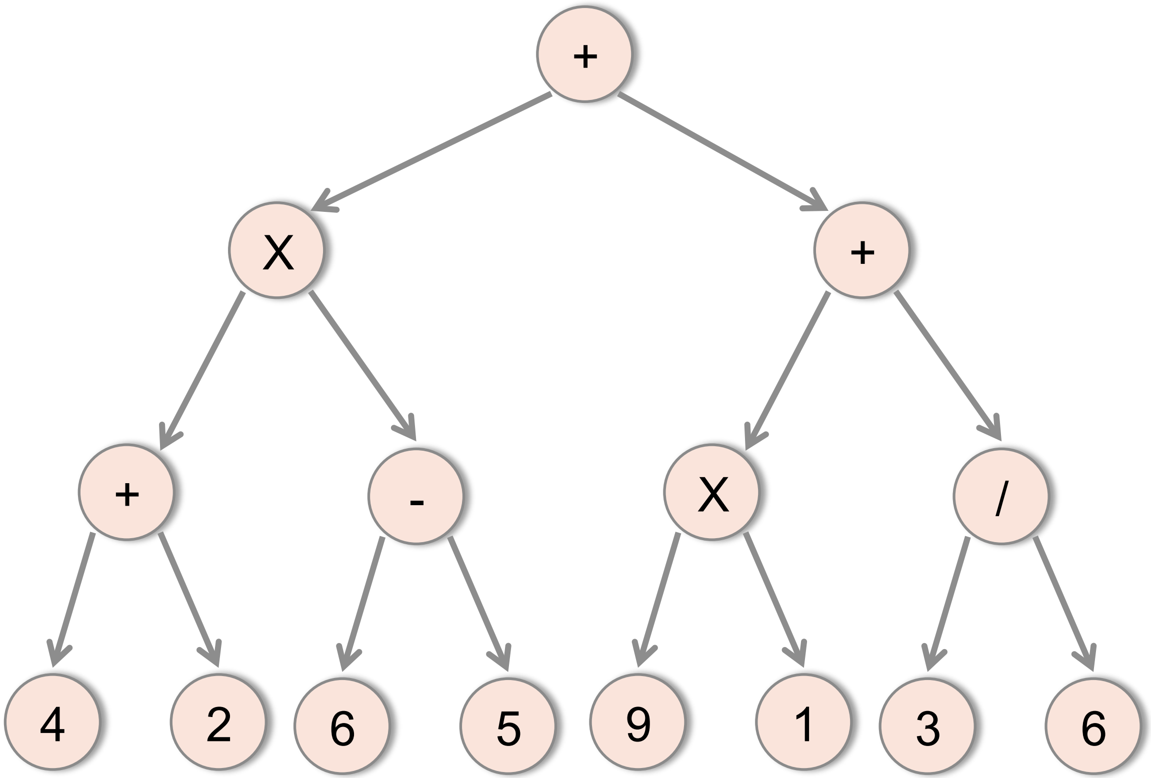
PREORDER TRAVERSAL



Process the node and then add children (right then left)

Stack:	
Output:	+X+42-65+X91/36

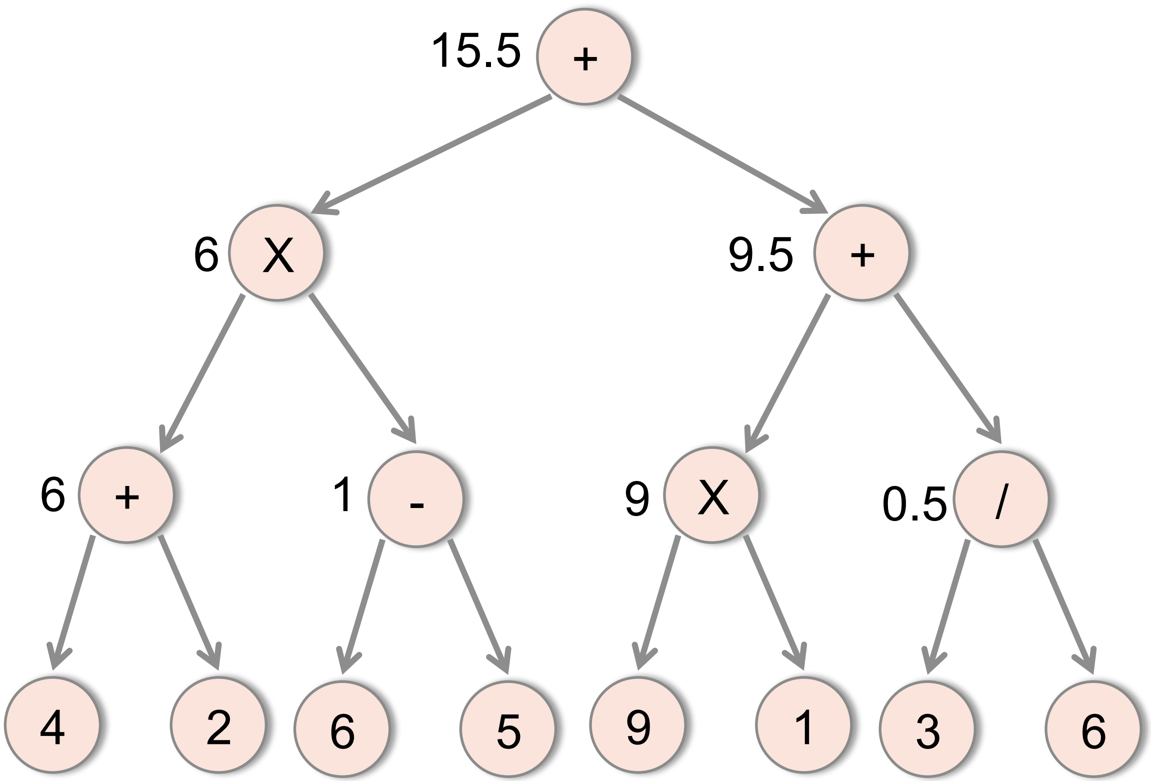
PREORDER TRAVERSAL



What does this evaluate to?

Stack:	
Output:	+X+42-65+X91/36

PREORDER TRAVERSAL



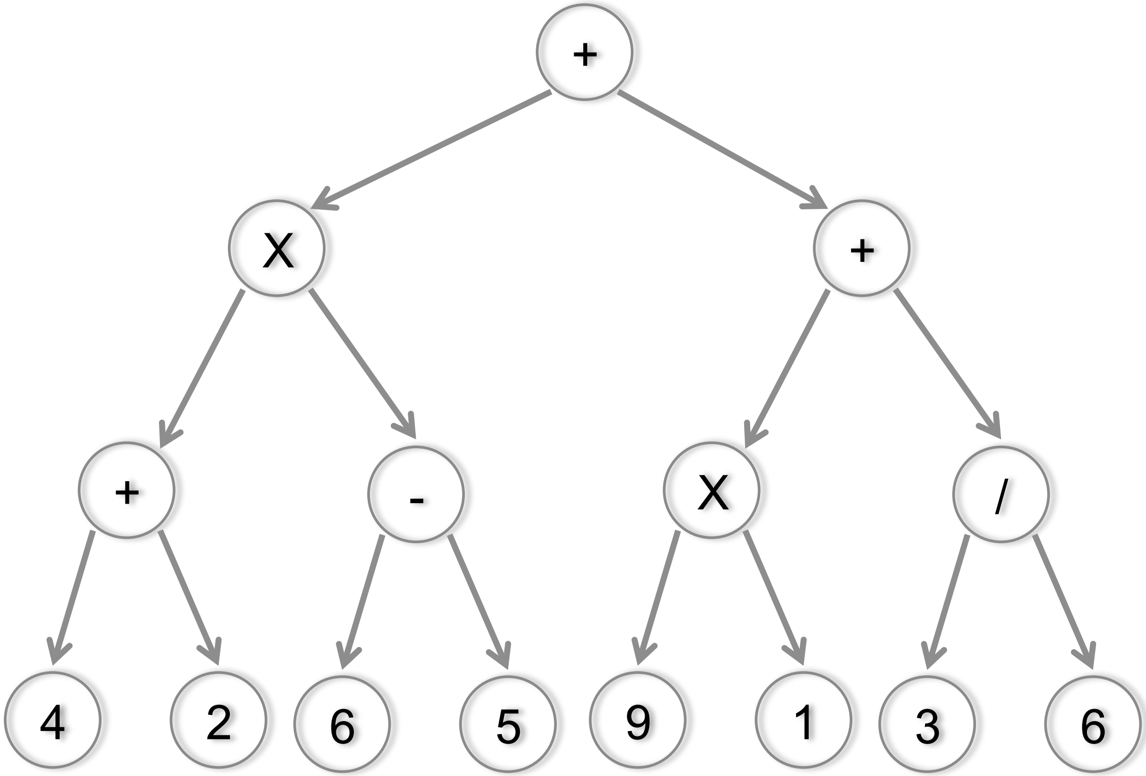
What does this evaluate to?

Stack:	
Output:	+X+42-65+X91/36

PREORDER TRAVERSAL

- **Knowing the rule of preorder, is that string ambiguous?**
  - +X+42-65+X91/36
- **Given that preorder traversal is DFS with ordering:**
  - Process, Left, Right
- **What string results from postorder?**
  - Left Right Process?

POSTORDER TRAVERSAL



POSTORDER TRAVERSAL

- **Pre-order**
  - +X+42-65+X91/36
- **Post-order**
  - 42+65-X91X36/++

POSTORDER TRAVERSAL

- **Pre-order** (Polish Notation)
  - +X+42-65+X91/36
- **Post-order** (Reverse Polish Notation)
  - 42+65-X91X36/++

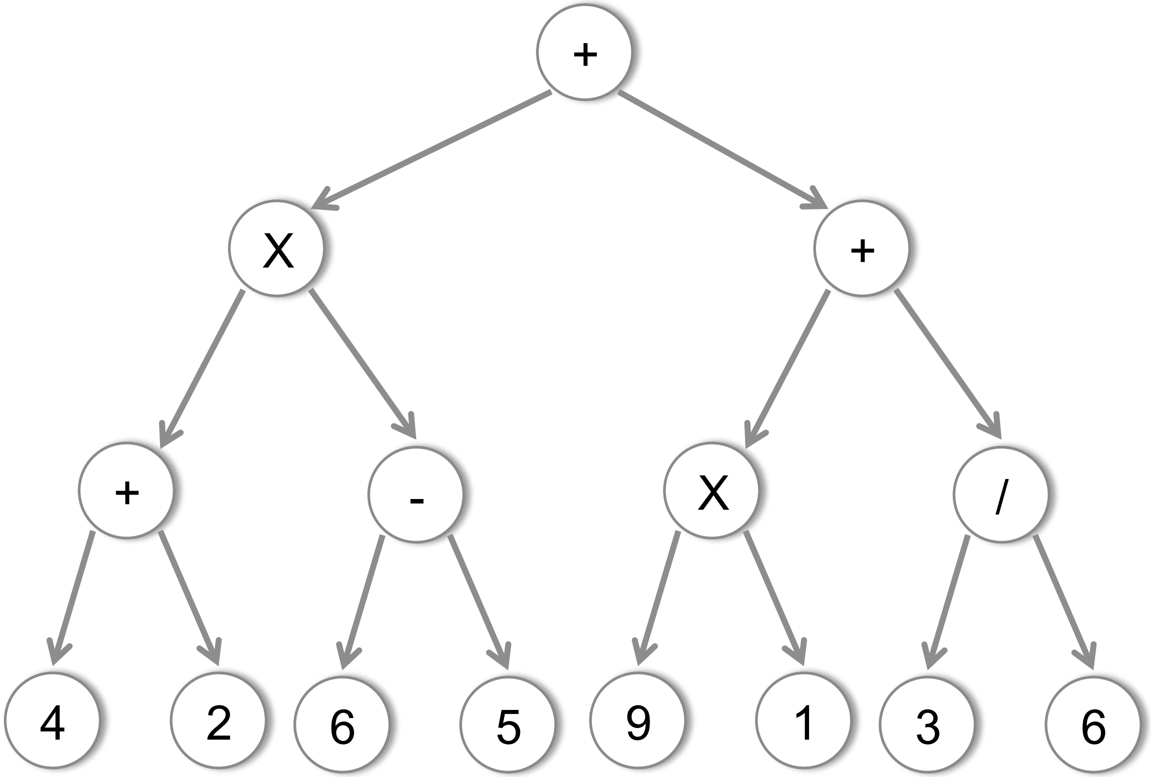
POSTORDER TRAVERSAL

- **Pre-order** (Polish Notation)
  - +X+42-65+X91/36
- **Post-order** (Reverse Polish Notation)
  - 42+65-X91X36/++
- **These are unambiguous strings**

POSTORDER TRAVERSAL

- **Pre-order** (Polish Notation)
  - +X+42-65+X91/36
- **Post-order** (Reverse Polish Notation)
  - 42+65-X91X36/++
- **These are unambiguous strings**
- **What about the final ordering?**
  - Left, Process, Right?

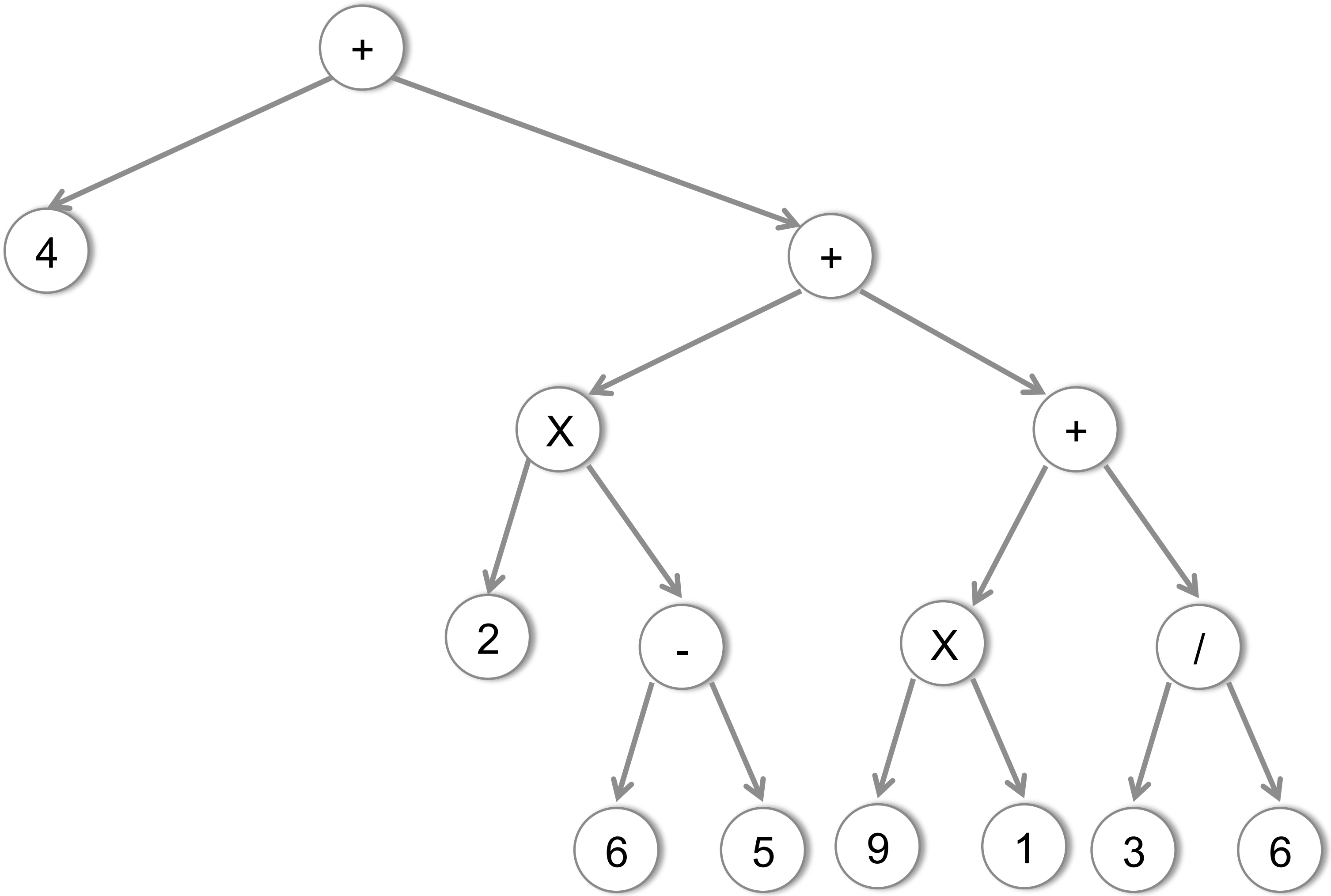
IN-ORDER TRAVERSAL



IN-ORDER TRAVERSAL

- **In-order**
  - 4+2X6-5+9X1+3/6
- **What is the problem here?**

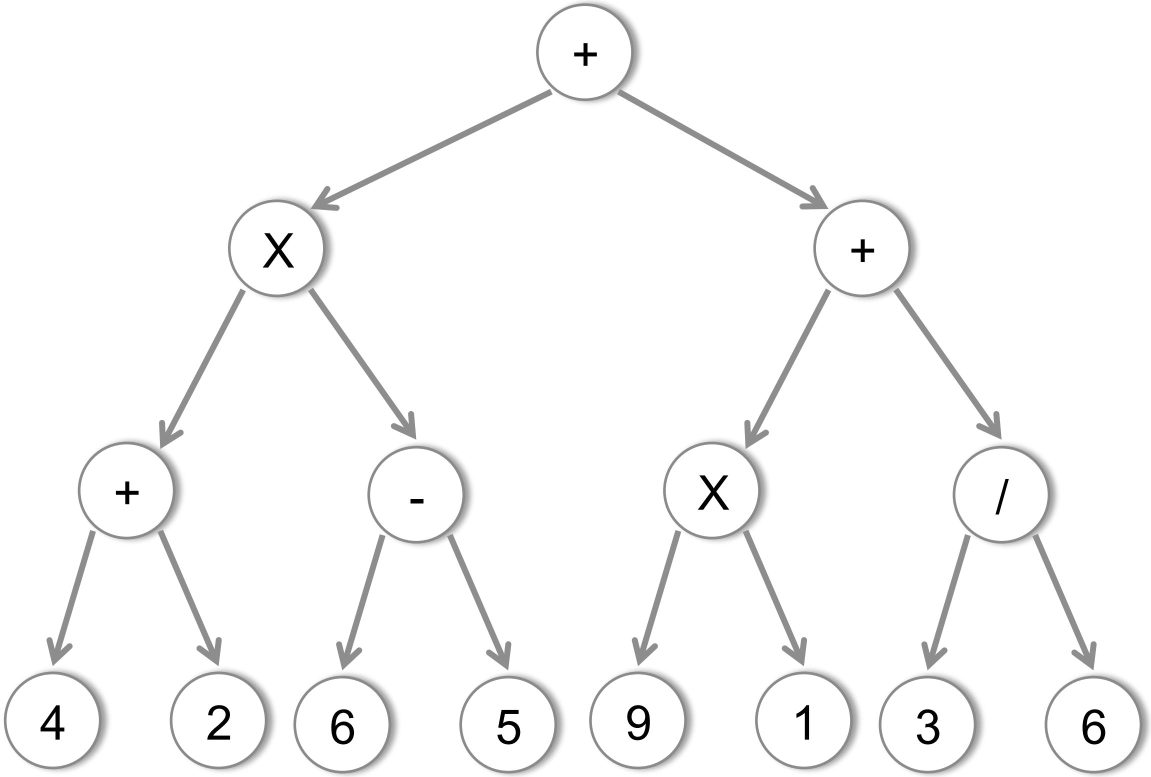
IN-ORDER TRAVERSAL



TRAVERSALS

- **In-order**
  - 4+2X6-5+9X1+3/6
- **What is the problem here?**
  - There are multiple trees!
- **In order returns the left-to-right sorted order**
  - In-order traversal of a BST is sorted result

IN-ORDER TRAVERSAL



4+2X6-5+9X1+3/6

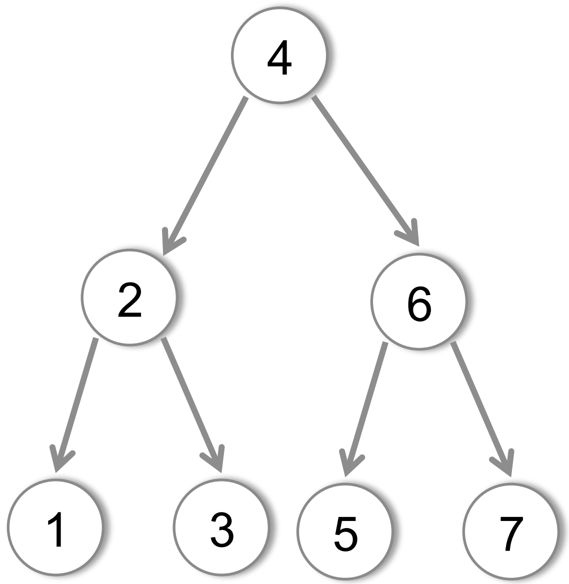
TRAVERSALS

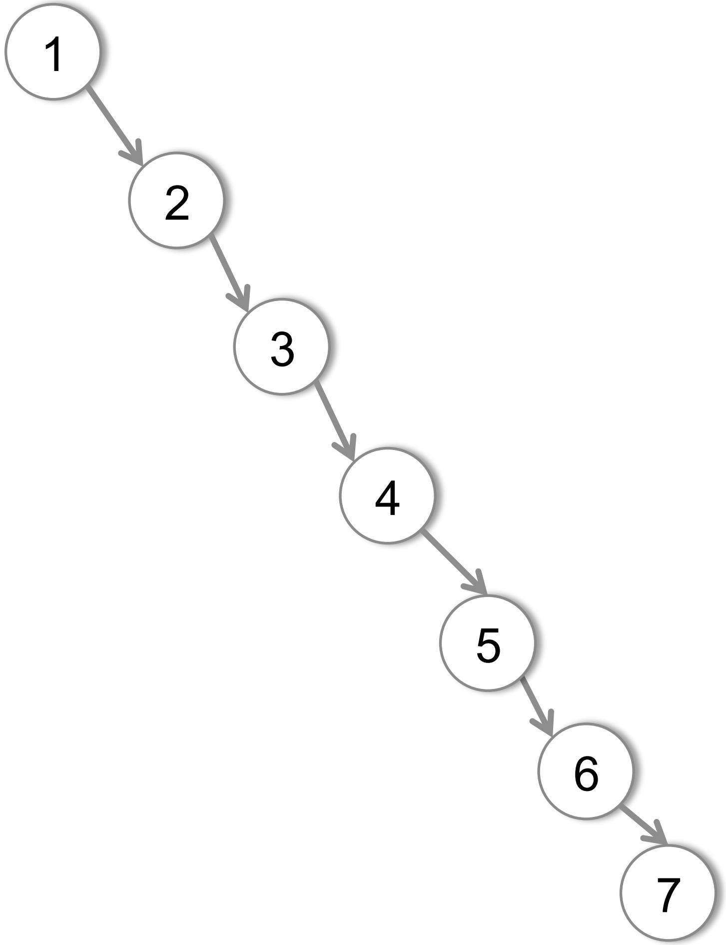
- **Pre-order and post-order are unambiguous, why?**
  - They can only represent one tree because we can distinguish parents from leaves
  - Parents are operators and leaves are numbers
  - If they are all numbers, the multiple trees represent the multiple ways of storing the data

BALANCE AND HEIGHT

- **If the same data can be represented multiple ways, what is best?**

BALANCE AND HEIGHT





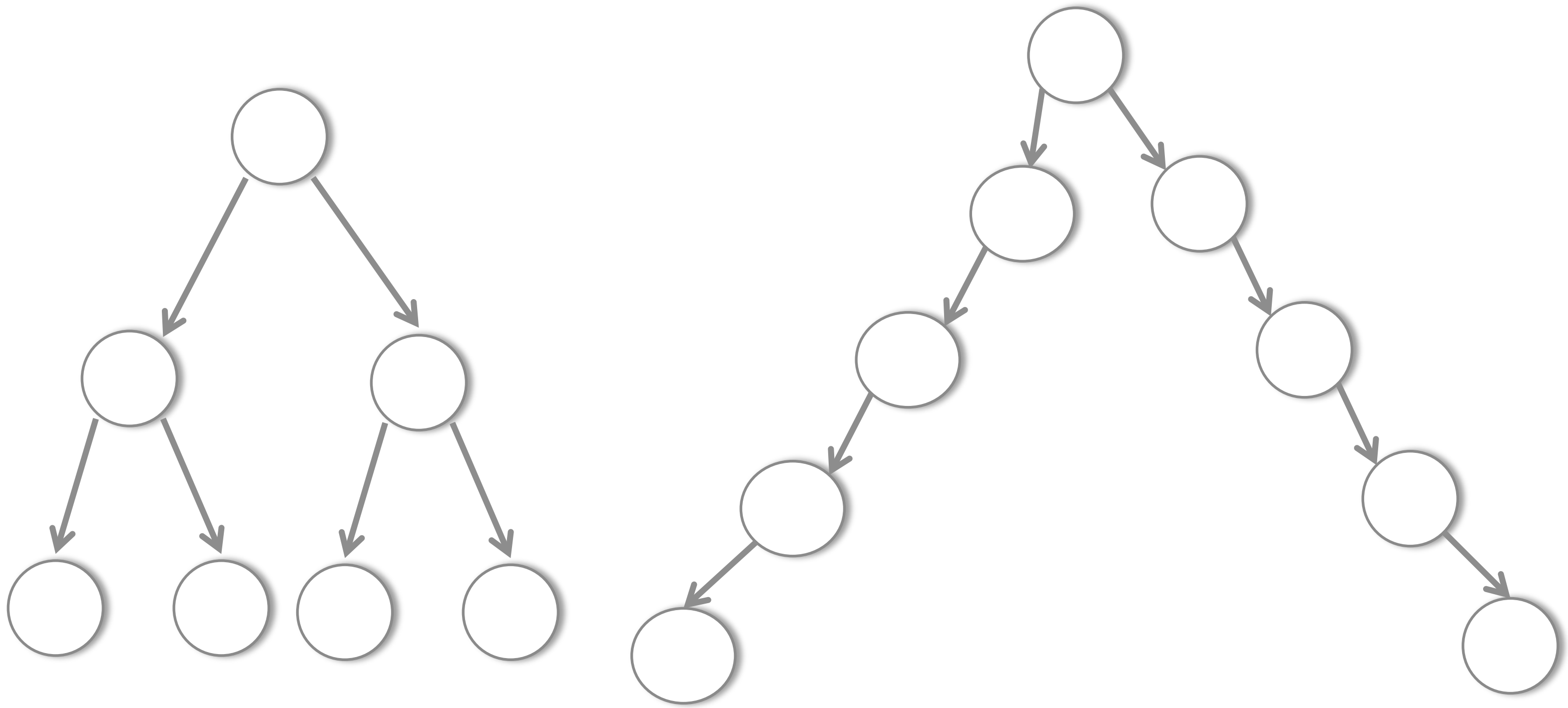
BALANCE AND HEIGHT

- **Height is key for how fast functions on our tree are!**
  - If we can structure the same data two different ways, we want to choose the better one.
  - Balanced is better for BSTs
  - Can we enforce balance?

BALANCE AND HEIGHT

- **Balance**
  - How can we define balance?
  - Abs(height(left) – height(right))
  - If the heights of the left and right trees are balanced, the tree is balanced.
  - Anything wrong with this?

BALANCE AND HEIGHT

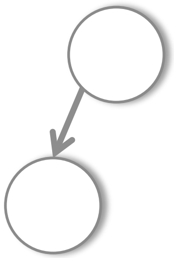


BALANCE AND HEIGHT

- **Not enough for the root to be balanced!**
- **All nodes must be balanced!**
- **Ideally, our "balance" property will say:**
  - For all nodes in the tree, height(left) = height(right)
  - What is the problem with this?
  - **Not always enforceable!**

BALANCE AND HEIGHT

- **Consider adding an element to a tree.**
  - When the tree is empty, it is balanced
- **We add one element**
  - Height(left) = height(right) = 0
- **Add another element**
  - Oh no! There is no way to enforce balance!



BALANCE AND HEIGHT

- **New property**
  - If Abs(height(left) – height(right)) is balance
  - We can only enforce if this is <=1
  - That is, the height left and right subtrees can differ by at most one
  - Still must preserve this for every node!
- **This is the AVL property**
- **AVL Trees are Binary Search Trees that have the AVL property**
  - They have worst case O(log n) find!

NEXT CLASS

- **AVL Trees**
  - Prove that they have O(log n) height
  - Come up with implementations for insert and delete
  - Want to get O(1) time for these, ideally