CSE 373

DECEMBER 4TH – ALGORITHM DESIGN

ASSORTED MINUTIAE

- **P3P3 scripts running right now**
  - Pushing back resubmission to Friday
- **Next Monday office hours**
  - 12:00-2:00 – last minute exam questions
  - Topics list and old practice exams out after class
  - Practice exam (hopefully tomorrow), by Wednesday night

ASSORTED MINUTIAE

- **Course evaluations**
  - Very important to this class and this department
  - Above all, they're very important to me
  - Should only take ~5 minutes, and it's very valuable feedback
  - 17 of you so far… and I'm going to bug you until it's above 75%
  - Save yourself the 15 emails and just fill it out

ALGORITHM DESIGN

- Solving well known problems is great, but how can we use these lessons to approach new problems?

ALGORITHM DESIGN

- **Solving well known problems is great, but how can we use these lessons to approach new problems?**
  - Guess and Check (Brute Force)
  - Linear Solving
  - Divide and Conquer
  - Greedy-first
  - *Randomization and Approximation*
  - *Dynamic Programming*

BRUTE FORCE

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- **Classic naïve approach to algorithm design**
- **A Brute Force Algorithm revolves primarily around attempting all possible outcomes**
  - Bogo sort
  - Travelling salesman
  - Longest path

BRUTE FORCE

- **If the problem is very difficult, then brute force may not be the worst solution**
  - Cracking RSA
  - Low-reward problems
  - Small, non-time-constrained

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- **If the decider creates a set of correct answers, find one at a time**
  - Selection sort: find the lowest element at each run through
- **Sometimes, the best solution**
  - Find the smallest element of an unsorted array

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  - What piece of information brings you one step closer to the final answer?

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  - Exam problem – simple solution
  - Not always bad, O(n) problems lend themselves well to linear solving

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  - Why?

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  - If an algorithm is n2 work, and we divide into two halves, we've halved the work!
  - Recurrences are going to play a big role in this

GREEDY-FIRST

- **A Greedy-first algorithm is any algorithm that makes the move that seems best now**
  - These can be divide-and-conquer algorithms or linear algorithms
  - Dijkstra's and Ford-Fulkerson are both Greedy-first algorithms
  - Notice, however, Dijkstra's finds the correct answer easily, and Ford-Fulkerson requires some augmentation to guarantee correctness

ALGORITHM DESIGN

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  - EXP: Set of problems that can be solved in exponential time

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  - At each move, the computer needs to approximate the best move
  - Certainty always comes at a price

APPROXIMATION DESIGN

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  - Board quality – If you could easily rank which board layout in order of quality, chess is simply choosing the best board
  - It is very difficult, branching factor for chess is ~35
  - Look as many moves into the future as time allows to see which move yields the best outcome

APPROXIMATION DESIGN

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  - Does your client have a tolerance for error?
  - Can you map this problem to a similar problem?
  - "Greedy" algorithms are often approximators

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- **Two types of randomized algorithms**
  - Las Vegas – correct result in random time
  - Montecarlo – estimated result in deterministic time

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  - Terminate a random quicksort early!
  - If you haven’t gotten the problem in some constrained time, just return what you have.

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  - 90% of elements are smaller than the object to the right of it?
  - The longest sorted subsequence is 90% of the length?
- **Analysis for these problems can be very tricky, but it's an important approach**

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    - Still can be useful for some easier problems
    - Hugely dependent on how “good” the checker is

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  - P is our success probability
  - NP-complete means we can check a solution in $O(n^k)$ time, but we can find the exact solution in $O(k^n)$ time – very bad
  - Suppose we want to have a confidence equal to α, how do we get this?

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$k*\ln(1-p) = \ln \alpha$

$$k = \frac{(\ln \alpha)}{(\ln(1-p)}$$

$k = \log_{(1-p)} \alpha$

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- **Cool, I guess… but what does this mean?**
- **Suppose P = 0.5 (we only have a 50% chance of success on any given run) and α = 0.001, we only tolerate a 0.1% error**
- **How many runs do we need to get this level of confidence?**
  - Only 10! This is a constant multiple

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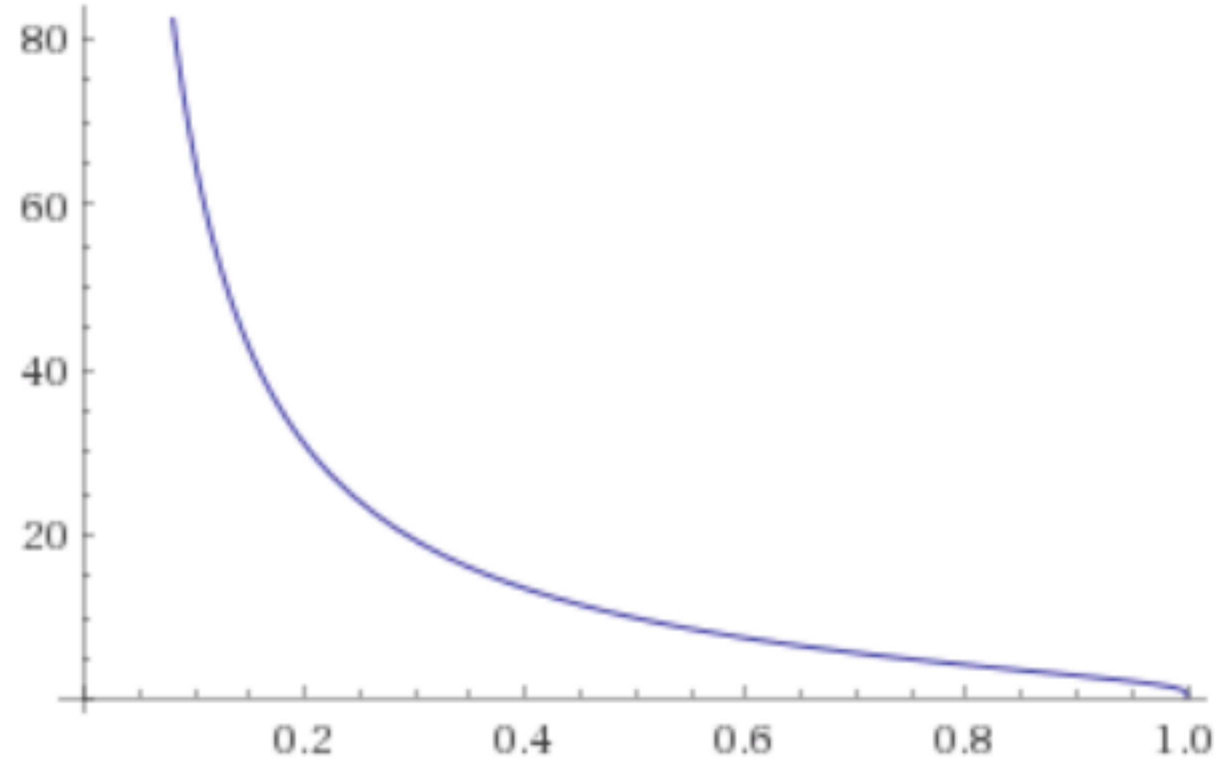

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- **Even if p is 0.1, only a 10% chance of success, we only need to run the algorithm 80 times to get a 0.001 confidence level**
- **What does this mean?**
  - Randomized algorithms don't have to be complicated, if you can create a *reasonable* guess and can verify it in a short amount of time, then you can get good performance just from running repeatedly.

RANDOMIZATION CONCLUSION

- **Good for estimating difficult problems in constrained time**

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- Relies on the quality of the guess
- Important approach to consider in modern computing

CONCLUSION

- **Be prepared for the algorithm design question on the final**
  - Understand how to go about getting the solution
  - Rigorous analysis, of both runtime and memory
  - Defend all design decisions
  - More points for explanation than for cleverness

CONCLUSION

- **Course evaluations**
  - https://uw.iasystem.org/survey/183488