CSE 373

OCTOBER 25TH – B-TREES

ASSORTED MINUTIAE

- **Project 2 is due tonight**
  - Make canvas group submissions
  - Load factor: total number of elements / current table size
  - Can select any load factor (but since we don't measure memory consumption, lower may be better)

TODAY'S LECTURE

- **Review of relevant info from Monday**
- **New, memory-conscious data structure**
  - B-trees

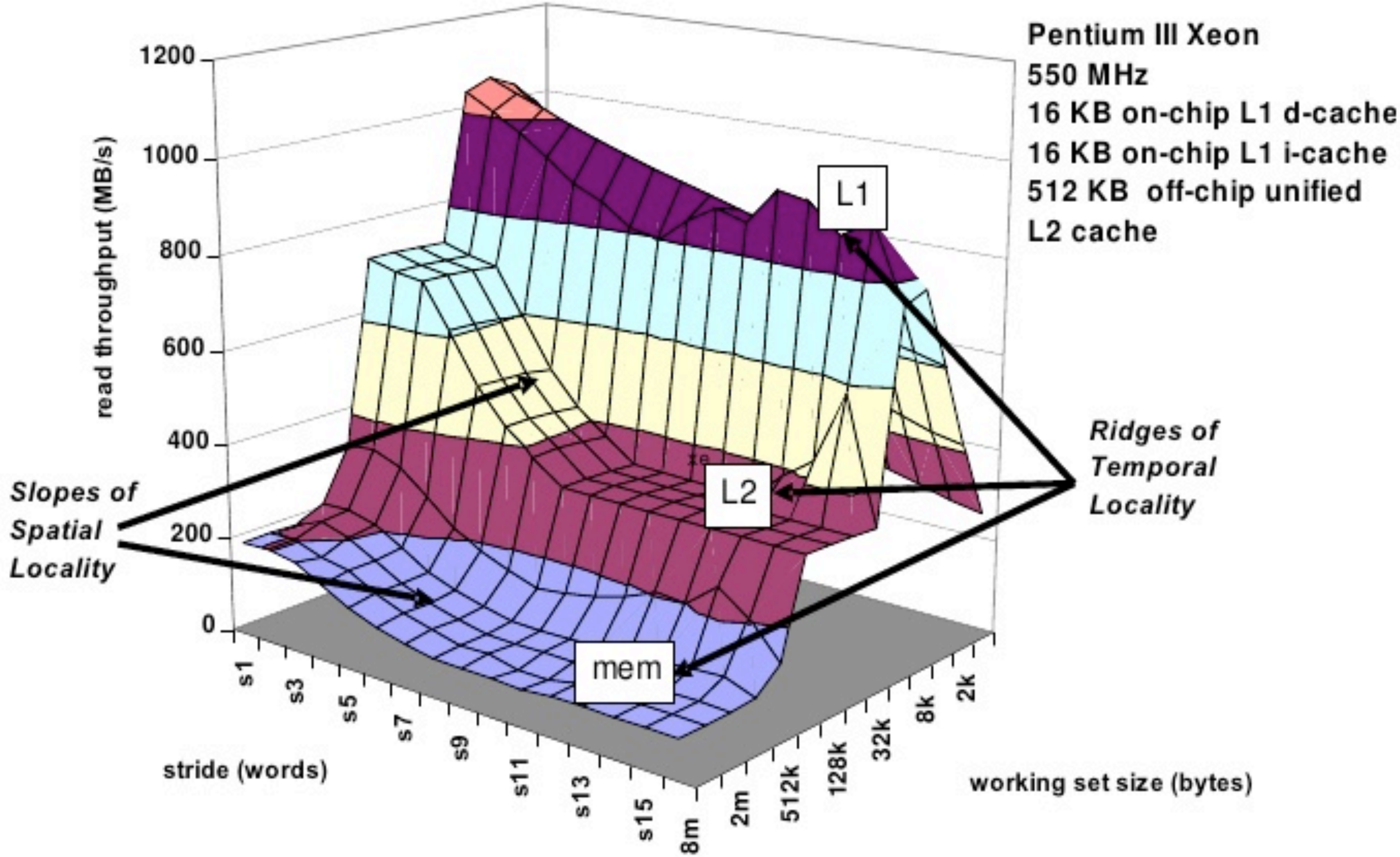
HARDWARE CONSTRAINTS

- **So far, we've taken for granted that memory access in the computer is constant and easily accessible**
  - This isn't always true!
  - At any given time, some memory might be cheaper and easier to access than others
  - Memory can't always be accessed easily
  - Sometimes the OS lies, and says an object is "in memory" when it's actually on the disk

HARDWARE CONSTRAINTS

- **Back on 32-bit machines, each program had access to 4GB of memory**
  - This isn't feasible to provide!
  - Sometimes there isn't enough available, and so memory that hasn't been used in a while gets pushed to the disk
- **Memory that is frequently accessed goes to the cache, which is even faster than RAM**

The Memory Mountain



LOCALITY AND PAGES

- **Secondly, the OS uses temporal locality,**
  - Memory recently accessed is likely to be accessed again
  - Bring recently used data into faster memory
- **Types of memory (by speed)**
  - Register
  - L1,L2,L3
  - Memory
  - Disk
  - The interwebs (the cloud)

LOCALITY AND PAGES

- **The OS is always processing this information and deciding which is the best**
  - This is why arrays are faster in practice, they are always next to each other in memory
  - Each new node in a tree may not even be in the same page in memory!!
- **Important to consider when designing and explaining design problems.**

COST OF MEMORY ACCESSES

- **Registers (128B): Instantaneous access**
- **L2 Cache (128KB): 0.5 nanoseconds**
- **L3 Cache (2MB): 7 nanoseconds**
- **Main Memory (32 GB): 100 nanoseconds**
- ***Disk (TBs): 8,000,000 nanoseconds***

LARGE AVL

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LARGE AVL

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  - Height is about 50
  - How many disk accesses will a find take?
  - Between 0 and 50!
  - This is the difference between nanoseconds and almost half a second!
  - If lots data is stored on the disk, O(log n) finds don’t happen in practice

PROBLEMS

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- **Why is AVL so bad on disk?**
  - Each piece of data is its own node
  - Each call of `new` may not place objects next to each other
  - Has large height, for the number of elements?

SOLUTIONS

- **What changes might we want to make to an AVL to make it better for disk?**
  - Still want to keep log n height
  - Allocate more objects closer together
  - Have a higher branching factor so that data you want is at a lower depth
  - Take advantage of page sizes

B-TREE

- Noded data structure

B-TREE

- **Noded data structure**
  - As an aside, what we will discuss in this course is called a B+ tree, which has slight differences if you go and look for resources online

B-TREE

- **Noded data structure**
  - Two types of nodes:
    - internal “signpost” nodes
    - leaf “data” nodes
  - Each node has a capacity
    - M for “signpost” nodes
    - L for “leaf/data” nodes

B-TREE

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  - Other than the root, internal nodes have between M/2 and M children and leaves have between L/2 and L data
  - Elements in the leaves are stored in sorted order
  - The number of subtrees for a signpost is one more than the number of elements in the signpost
  - The signpost has the smallest piece of data to the right of it – *all data is in a leaf*

B-TREE

- Example

B-TREE

- Find

B-TREE

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    - $O(Log_2 M)$

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    - $O(\text{Log}_2 L)$
  - Total find = $O(\text{Log}_2 L + \text{Log}_2 M * \text{Log}_M N)$

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  - Recursively overflow as necessary
  - If the root overflows, make a new root

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  - *Splitting is actually fairly uncommon*
  - *Care most about # of disc accesses*
    - *$\mathrm{Log}_M n$*

B-TREE

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  - Recursively underflow up to root if necessary

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  - Total time: $O(L + M \log_m n)$

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- **Practice tool here:**
  - https://www.cs.usfca.edu/~galles/visualization/BPlusTree.html

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  - Many keys stored in each signpost
  - Each can be brought up in one disk access
    - Binary search is fast because it’s all in memory
  - Internal nodes have only the keys (values waste space)
  - What values of M and L do we want?
    - Want each node to be one page

B-TREE

- Choosing M and L

B-TREE

- **Choosing M and L**
  - Let a page be *p* bytes
  - Keys are *k* bytes
  - Pointers are *t* bytes
  - Values are *v* bytes
- **p = M*p + M-1*k; M = p+k / t+k**
- **L = (p-t) / (k+v)**