CSE 373: Data Structures and Algorithms

Dictionaries and Binary Search Trees

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Winter 2016

This lecture material represents the work of multiple instructors at the University of Washington. Thank you to all who have contributed!

Today’s Outline

Today’s Topics
- Finish Asymptotic Analysis
- Dictionary ADT (a.k.a. Map): associate keys with values
  - Extremely common
- Binary Trees

Summary of Asymptotic Analysis

Analysis can be about:
- The problem or the algorithm (usually algorithm)
- Time or space (usually time)
  - Or power or dollars or …
- Best-, worst-, or average-case (usually worst)
- Upper-, lower-, or tight-bound (usually upper)
- The most common thing we will do is give an \(O\) upper bound to the worst-case running time of an algorithm.

Big-Oh Caveats

- Asymptotic complexity focuses on behavior for large \(n\) and is independent of any computer / coding trick
- But you can “abuse” it to be misled about trade-offs
- Example: \(n^{1/10}\) vs. \(\log n\)
  - Asymptotically \(n^{1/10}\) grows more quickly
  - But the “cross-over” point is around \(5 \times 10^{17}\)
  - So if you have input size less than \(2^{58}\), prefer \(n^{1/10}\)
- For small \(n\), an algorithm with worse asymptotic complexity might be faster
  - If you care about performance for small \(n\) then the constant factors can matter

Addendum: Timing vs. Big-Oh Summary

- Big-oh is an essential part of computer science’s mathematical foundation
  - Examine the algorithm itself, not the implementation
  - Reason about (even prove) performance as a function of \(n\)
- Timing also has its place
  - Compare implementations
  - Focus on data sets you care about (versus worst case)
  - Determine what the constant factors “really are”

Let’s take a breath

- So far we’ve covered
  - Some simple ADTs: stacks, queues, lists
  - Some math (proof by induction)
  - How to analyze algorithms
  - Asymptotic notation (Big-Oh)
- Coming up…
  - Many more ADTs
    - Starting with dictionaries
The Dictionary (a.k.a. Map) ADT

- **Data:**
  - set of (key, value) pairs
  - keys must be comparable

- **Operations:**
  - `insert(key, value)`
  - `find(key)`
  - `delete(key)`
  - ...

Will tend to emphasize the keys; don’t forget about the stored values

<table>
<thead>
<tr>
<th>David Swanson</th>
<th>OH: Wed 3:30-4:20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicholas Shahan</td>
<td>OH: Wed 11:30-12:20</td>
</tr>
<tr>
<td>Megan Hopp</td>
<td>OH: Mon 10-10:50</td>
</tr>
</tbody>
</table>

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently

- Lots of programs do that!

- **Search:** inverted indexes, phone directories, ...
- **Networks:** router tables
- **Operating systems:** page tables
- **Compilers:** symbol tables
- **Databases:** dictionaries with other nice properties
- **Biology:** genome maps
- ...

Possibly the most widely used ADT

Simple implementations

For dictionary with n key/value pairs

- Unsorted linked list: `insert O(1)`
- Unsorted array: `O(n)`
- Sorted linked list: `O(n)`
- Sorted array: `O(log n)`

`*` Unless we need to check for duplicates

We’ll see a Binary Search Tree (BST) probably does better but not in the worst case (unless we keep it balanced)

<table>
<thead>
<tr>
<th>10</th>
<th>12</th>
<th>24</th>
<th>30</th>
<th>41</th>
<th>42</th>
<th>44</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
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</tr>
</tbody>
</table>

Lazy Deletion

A general technique for making `delete` as fast as `find`:

- Instead of actually removing the item just mark it deleted
- Plusses:
  - Simpler
  - Can do removals later in batches
  - If re-added soon thereafter, just unmark the deletion
- Minuses:
  - Extra space for the “is-it-deleted” flag
  - Data structure full of deleted nodes wastes space
  - May complicate other operations

Better dictionary data structures

There are many good data structures for (large) dictionaries

1. Binary trees
   - Binary search trees with guaranteed balancing
2. AVL trees
3. Hash tables
   - Not tree-like at all

Skipping: Other trees (e.g., B-trees, red-black, splay)

Tree terms (review?)

<table>
<thead>
<tr>
<th>Tree T</th>
<th>Root (tree)</th>
<th>Depth (node)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>F</td>
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<td>G</td>
<td>H</td>
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<tr>
<td>M</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leaf (tree)</th>
<th>Height (tree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Children (node)</th>
<th>Degree (node)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A,C</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parent (node)</th>
<th>Branching factor (tree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A,C</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Siblings (node)</th>
<th>Ancestors (node)</th>
<th>Descendants (node)</th>
<th>Subtree (node)</th>
</tr>
</thead>
</table>
More tree terms

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of “next” as the one child)
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree
- A tree can be balanced or not
  - A balanced tree with \( n \) nodes has a height of \( O(\log n) \)
  - Different tree data structures have different “balance conditions” to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- \( n \)-ary tree: Each node has at most \( n \) children (branching factor \( n \))
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right

What is the height of a perfect binary tree with \( n \) nodes?
A complete binary tree?

Binary Trees

- Binary tree: Each node has at most 2 children (branching factor 2)
  - Binary tree is
    - A root (with data)
    - A left subtree (may be empty)
    - A right subtree (may be empty)
- Representation:

```
          A
         / \
        B   C
       /   /
      D   E   F
```
- For a dictionary, data will include a key and a value

Binary Tree Representation

```
A
/\   \
B   C
/
D
```

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height \( h \):
- max # of leaves: \( 2^h \)
- max # of nodes: \( 2^{(h+1)} - 1 \)
- min # of leaves: \( 1 \)
- min # of nodes: \( h + 1 \)

For \( n \) nodes, we cannot do better than \( O(\log n) \) height and we want to avoid \( O(n) \) height

Calculating height

What is the height of a tree with root \( \text{root} \)?

```java
int treeHeight(Node root) {
    ???
}
```
Calculating height

What is the height of a tree with root \( root \)?

```java
int treeHeight(Node root) {
    if (root == null)
        return -1;
    return 1 + max(treeHeight(root.left),
                    treeHeight(root.right));
}
```

Running time for tree with \( n \) nodes: \( O(n) \) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion’s call stack

Tree Traversals

A traversal is an order for visiting all the nodes of a tree

- **Pre-order**: root, left subtree, right subtree
  \[ + \cdot 2 \cdot 4 \cdot 5 \]
- **In-order**: left subtree, root, right subtree
  \[ 2 \cdot 4 + 5 \]
- **Post-order**: left subtree, right subtree, root
  \[ 2 \cdot 4 \cdot 5 + \]

(An expression tree)

More on traversals

```java
void inOrderTraversal(Node t) {
    if (t != null) {
        inOrderTraversal(t.left);
        process(t.element);
        inOrderTraversal(t.right);
    }
}
```

\( A \) = current node \( B \) = processing (on the call stack)
\( C \) = completed node \( ✓ \) = element has been processed
```java
void inOrderTraversal(Node t) {
    if (t != null) {
        inOrderTraversal(t.left);
        process(t.element);
        inOrderTraversal(t.right);
    }
}
```