CSE 373 - Review Session 1

1 Induction

Proposition. *Let n be a positive integer, then $n^3 + 2n$ is divisible by 3.*

Proof. We will prove the proposition by induction on n.
Base Case. For $n = 1$ we have

$$n^3 + 2n = 1^3 + 2 \cdot 1 = 3$$

3 is clearly divisible by 3, therefore we have found that the proposition holds for the base case.
Induction Hypothesis. Assume that the proposition holds for some positive integer k, that is that $k^3 + 2k$ is divisible by 3.
Inductive step. We need to prove that the proposition holds for $k+1$, that is that $(k+1)^3 + 2(k+1)$ is divisible by 3.
Expanding and reordering we get:

$$\begin{aligned}
&(k+1)^3 + 2(k+1) = \\
&k^3 + 3k^2 + 3k + 1 + 2k + 2 = \\
&3k^2 + 3k + 3 + k^3 + 2k = \\
&3(k^2 + k + 1) + (k^3 + 2k)
\end{aligned}$$

The first term is clearly divisible by 3 and the second term is divisible by 3 according to the induction hypothesis. Therefore, it follows $(k+1)^3 + 2(k+1)$ is divisible by 3, that is we have proved the inductive step.

We have shown that the proposition holds for 1 and that given that it holds for k it follows that it holds for $k+1$, therefore it follows that the proposition holds for every positive integer. ■