Announcements

- HW 1 due tonight, 11PM
- HW 2 out: due Friday, July 8th at 11PM

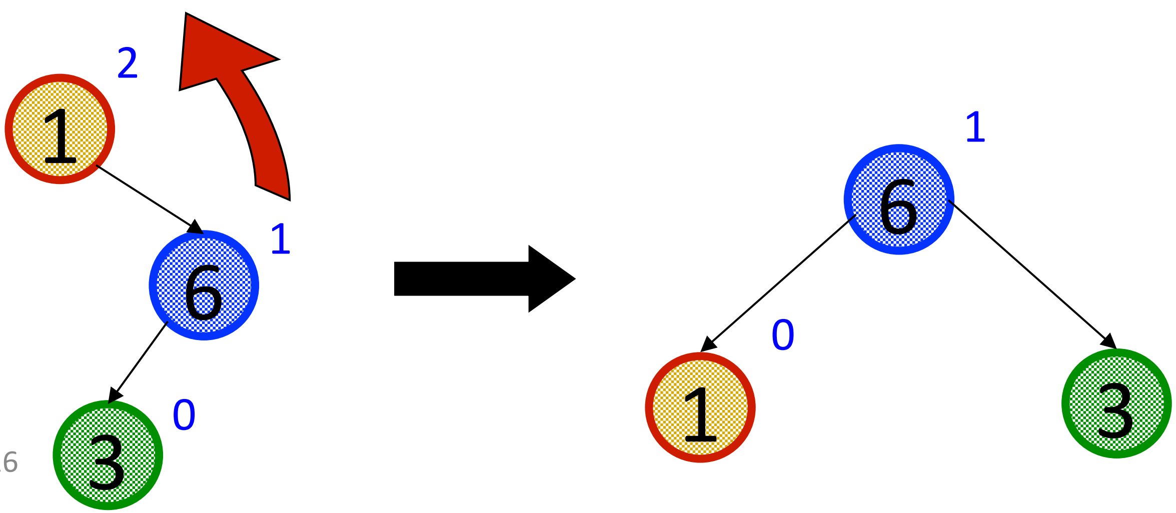
- Lilian and Dan holding office hours today

Two cases to go

Unfortunately, single rotations are not enough for insertions in the **left-right** subtree or the **right-left** subtree

Simple example: `insert`(1), `insert`(6), `insert`(3)

– First wrong idea: single rotation like we did for left-left

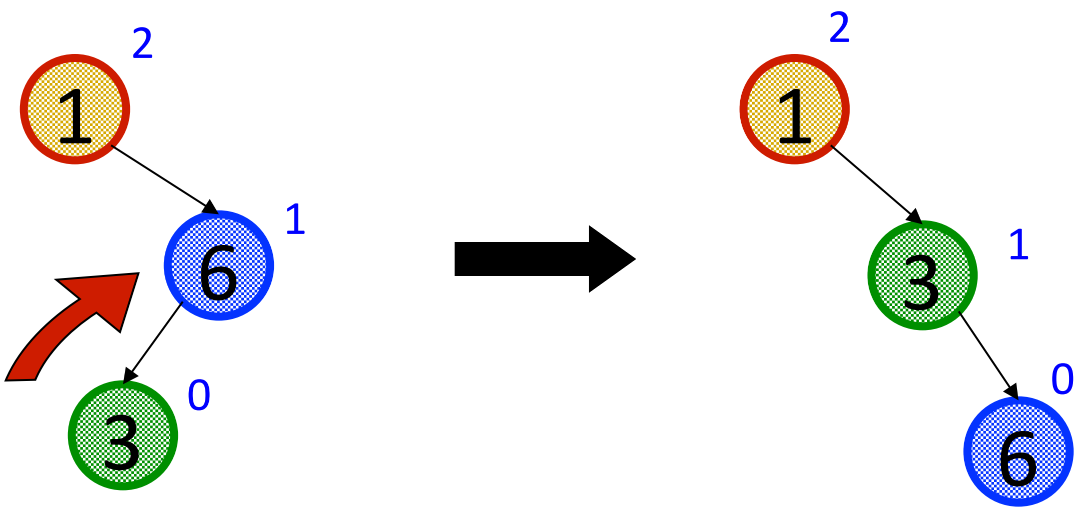


Two cases to go

Unfortunately, single rotations are not enough for insertions in the left-right subtree or the right-left subtree

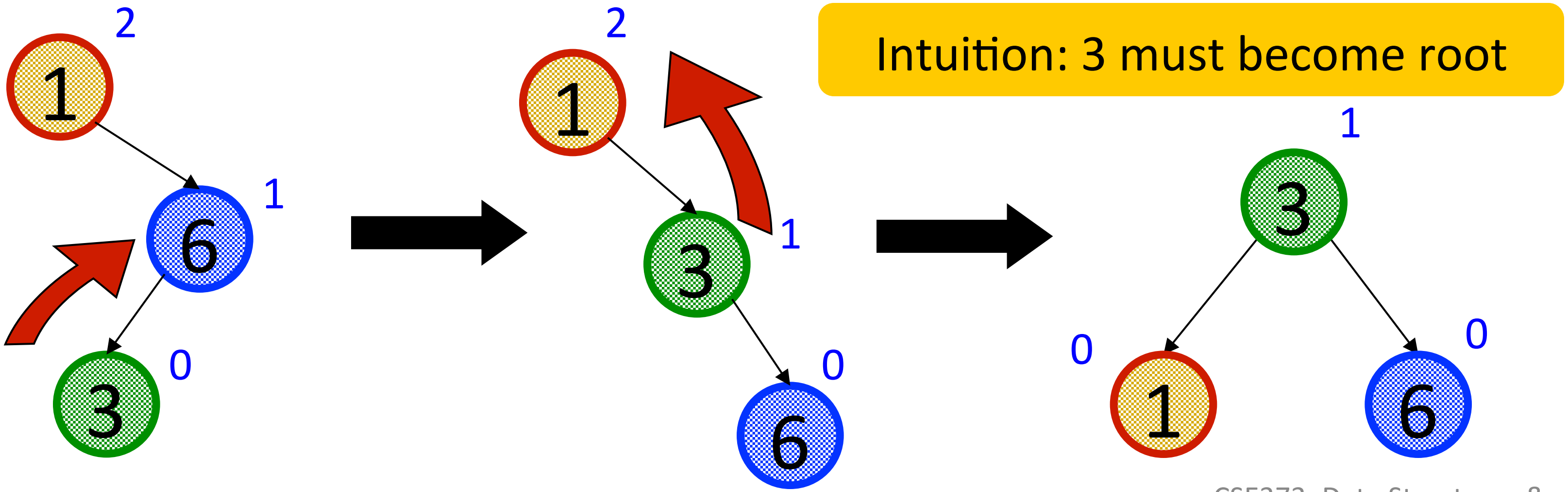
Simple example: **`insert`**(1), **`insert`**(6), **`insert`**(3)

– Second wrong idea: single rotation on the child of the unbalanced node

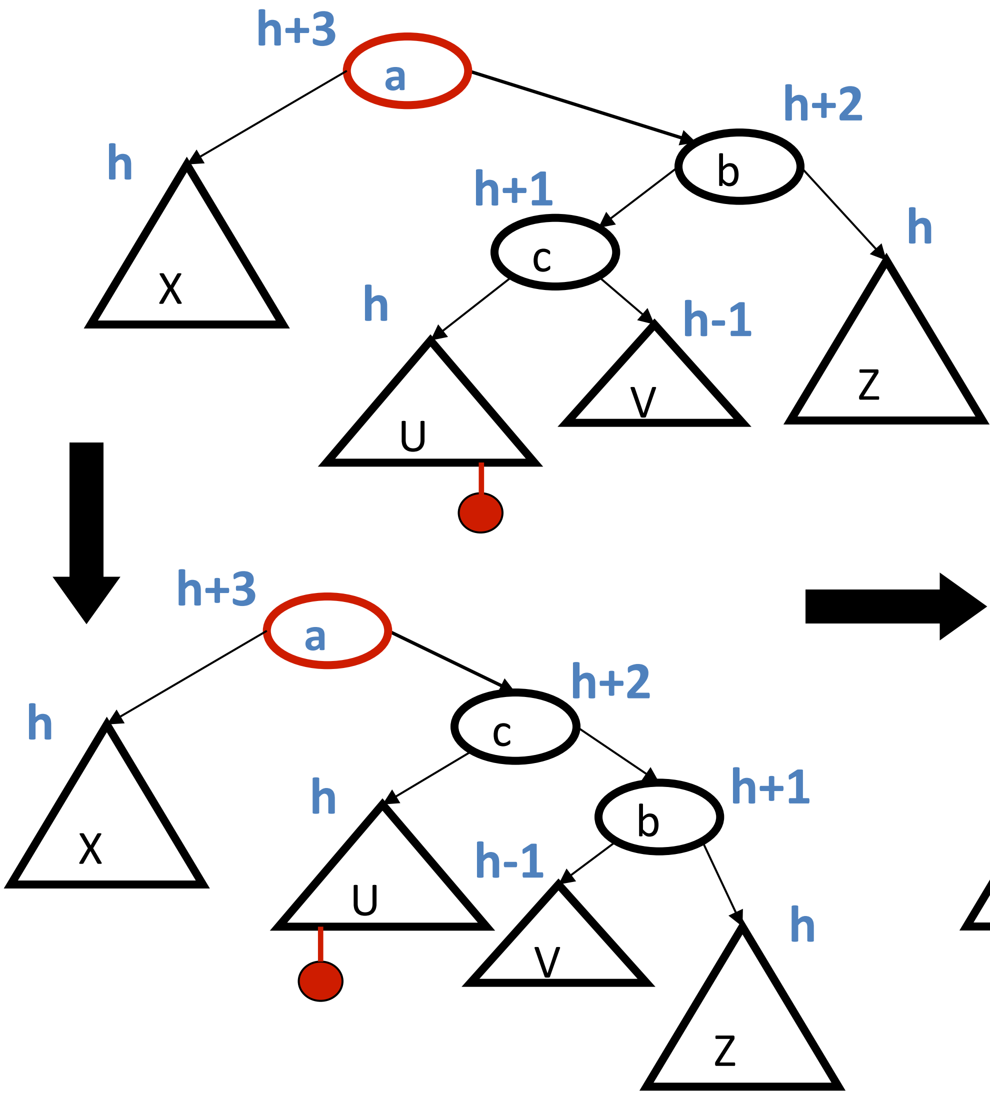


Sometimes two wrongs make a right

- First idea violated the BST property
- Second idea didn't fix balance
- But if we do both single rotations, starting with the second, it works! (And not just for this example.)
- Double rotation:
  1. Rotate problematic child and grandchild
  2. Then rotate between self and new child



The general right-left case

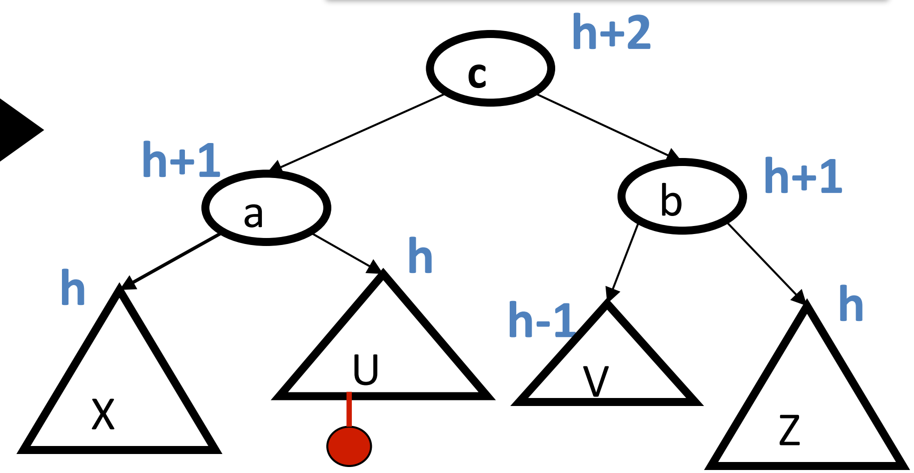


Rotation 1:

```
b.left = c.right
c.right = b
a.right = c
```

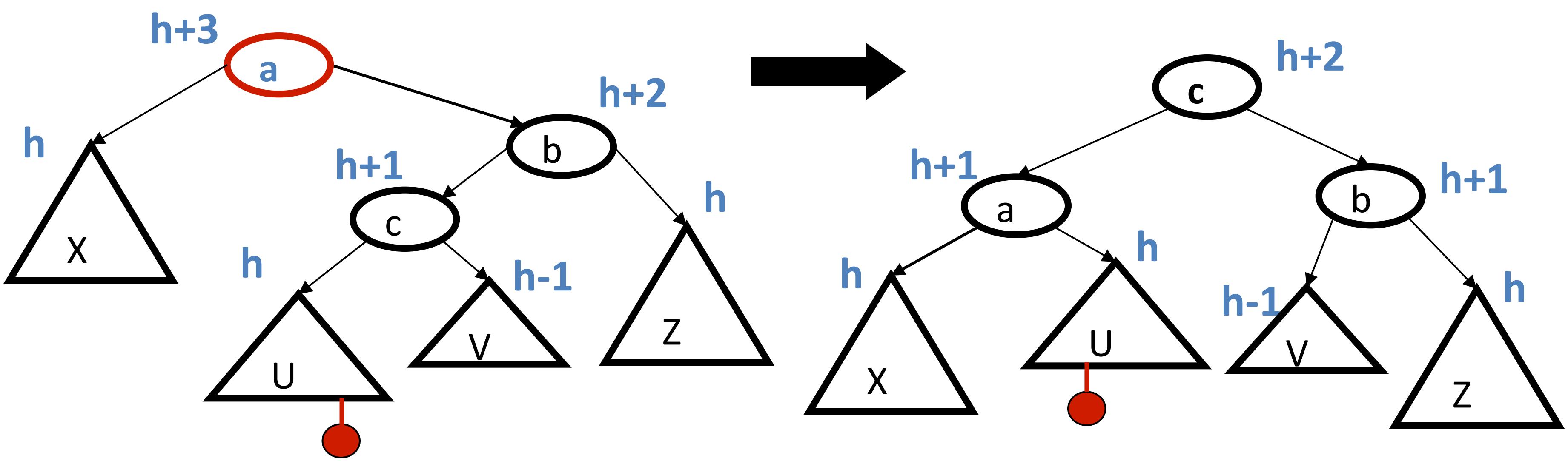
Rotation 2:

```
a.right = c.left
c.left = a
root = c
```



Comments

- Like in the left-left and right-right cases, the height of the subtree after rebalancing is the same as before the insert
  - So no ancestor in the tree will need rebalancing
- Does not have to be implemented as two rotations; can just do:



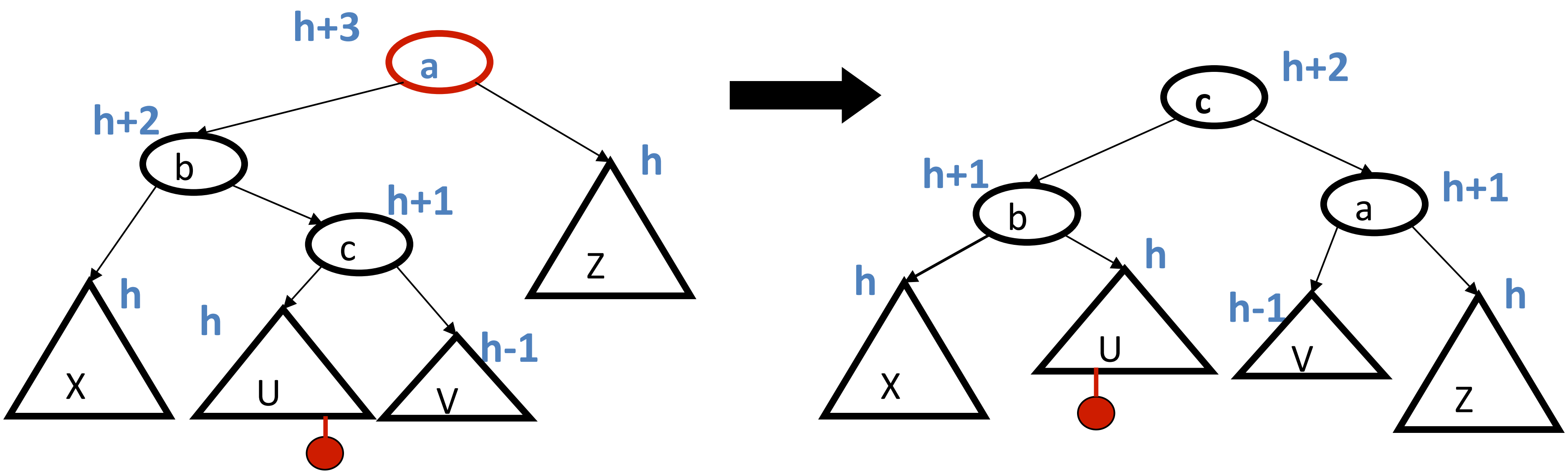
Easier to remember than you may think:

1) Move c to grandparent's position

2) Put a, b, X, U, V, and Z in the only legal positions for a BST

The last case: left-right

- Mirror image of right-left
  - Again, no new concepts, only new code to write



Insert, summarized

- Insert as in a BST
- Check back up path for imbalance, which will be 1 of 4 cases:
  - Node's left-left grandchild is too tall (**left-left single rotation**)
  - Node's left-right grandchild is too tall (**left-right double rotation**)
  - Node's right-left grandchild is too tall (**right-left double rotation)**
  - Node's right-right grandchild is too tall (**right-right double rotation**)
- Only one case occurs because tree was balanced before insert
- After the appropriate single or double rotation, the smallest-unbalanced subtree has the same height as before the insertion
  - So all ancestors are now balanced

Now efficiency

- Worst-case complexity of **`find`**: $O($**`log`** $n)$
  - Tree is balanced

- Worst-case complexity of **`insert`**: $O($**`log`** $n)$
  - Tree starts balanced
  - A rotation is $O(1)$ and there's an $O($**`log`** $n)$ path to root
  - (Same complexity even without one-rotation-is-enough fact)
  - Tree ends balanced

- Worst-case complexity of **`buildTree`**: $O(n$ **`log`** $n)$

Takes some more rotation action to handle **`delete`**…

Pros and Cons of AVL Trees

Arguments for AVL trees:

1. All operations logarithmic worst-case because trees are *always* balanced
2. Height balancing adds no more than a constant factor to the speed of `insert` and `delete`

Arguments against AVL trees:

1. Difficult to program & debug [but done once in a library!]
2. More space for height field
3. Asymptotically faster but rebalancing takes a little time
4. Most large searches are done in database-like systems on disk and use other structures (e.g., *B*-trees, a data structure in the text)
5. If *amortized* (later, I promise) logarithmic time is enough, use splay trees (also in text)

Dictionary Runtimes: More motivation

For a **dictionary** with *n* key, value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	O(1)	O(n)	O(n)
• Unsorted array	O(1)	O(n)	O(n)
• Sorted linked list	O(n)	O(n)	O(n)
• Sorted array	O(n)	O(`log` n)	O(n)
• *Balanced* tree	$O($`log` $n)$	$O($`log` $n)$	$O($`log` $n)$
• Magic array	$O(1)$	$O(1)$	$O(1)$

Sufficient "magic":

- Use key to compute array index for an item in $O(1)$ time [doable]
- Have a different index for every item [magic]

CSE373: Data Structures & Algorithms

Lecture 6: Hash Tables

Hunter Zahn

Summer 2016

Thanks to Kevin Quinn and Dan Grossman for slide materials

Motivating Hash Tables

For a **dictionary** with n key, value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$	$O(n)$	$O(n)$
• Unsorted array	$O(1)$	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$
• *Balanced* tree	$O(\log n)$	$O(\log n)$	$O(\log n)$
• Magic array	$O(1)$	$O(1)$	$O(1)$

Sufficient "magic":

- Use key to compute array index for an item in $O(1)$ time [doable]
- Have a different index for every item [magic]

Motivating Hash Tables

- Let's say you are tasked with counting the frequency of integers in a text file. You are guaranteed that only the integers 0 through 100 will occur:

For example: 5, 7, 8, 9, 9, 5, 0, 0, 1, 12

Result: 0 → 2 1 → 1 5 → 2 7 → 1 8 → 1 9 → 2

What structure is appropriate?

Tree?

List?

Array?

2	**1**				**2**		**1**	**1**	**2**
0	1	2	3	4	5	6	7	8	9

Motivating Hash Tables

Now what if we want to associate name to phone number?

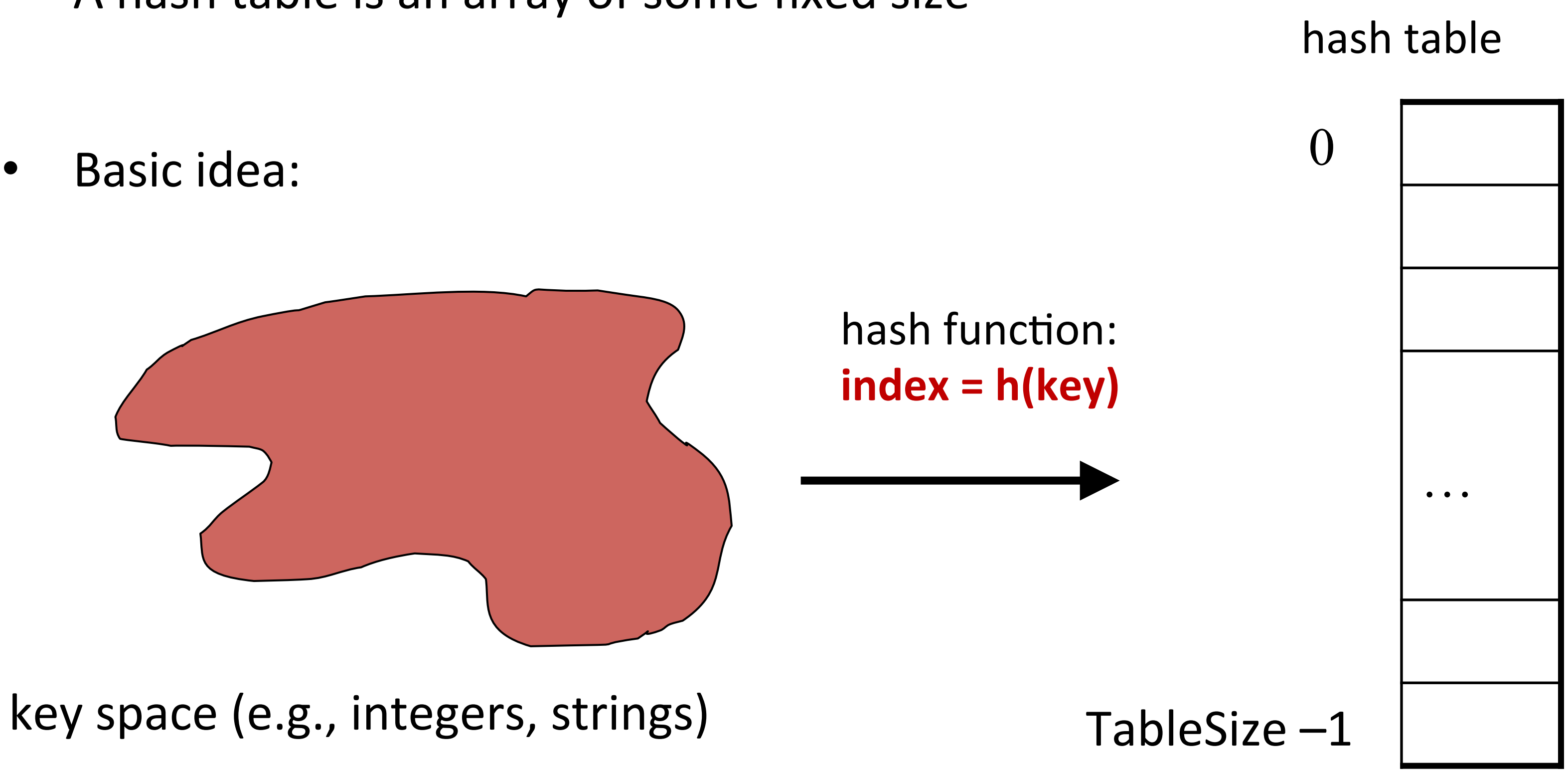
Suppose keys are first, last names

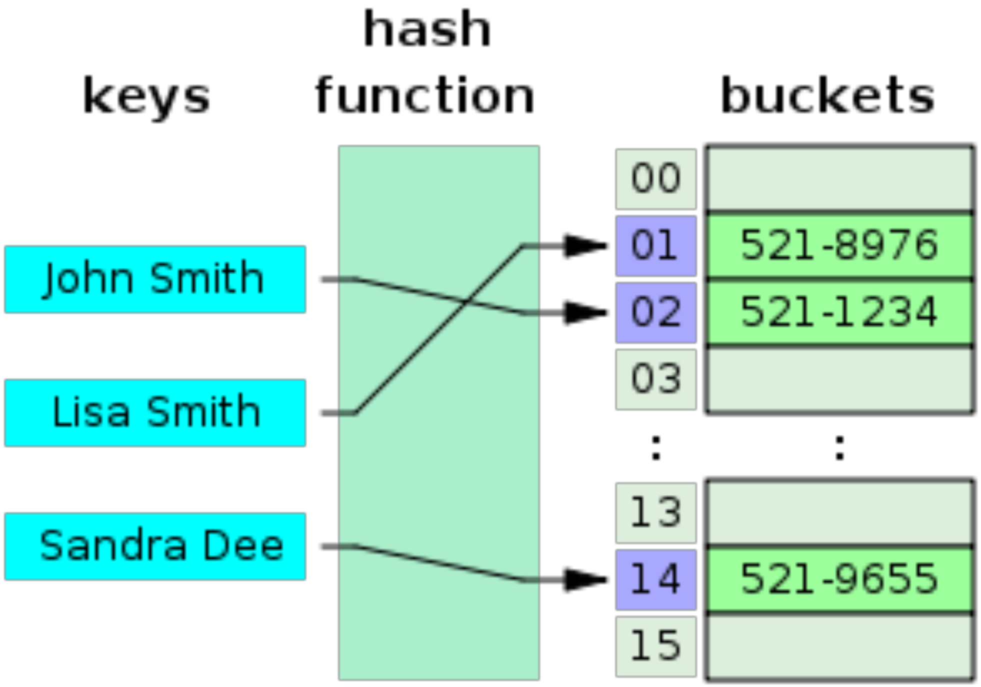
- how big is the key space?

Maybe we only care about students

Hash Tables

- Aim for constant-time (i.e., $O(1)$) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some often-reasonable assumptions
- A hash table is an array of some fixed size
- Basic idea:




hash
function
keys
buckets
John Smith
Lisa Smith
Sandra Dee
00
01
02
03
13
14
15
521-8976
521-1234
521-9655

Hash Tables vs. Balanced Trees

- In terms of a Dictionary ADT for just **`insert`**, **`find`**, **`delete`**, hash tables and balanced trees are just different data structures
  - Hash tables $O(1)$ on average (*assuming* we follow good practices)
  - Balanced trees $O(\log n)$ worst-case

- Constant-time is better, right?
  - Yes, but you need "hashing to behave" (must avoid collisions)
  - Yes, but **`findMin`**, **`findMax`**, **`predecessor`**, and **`successor`** go from $O(\log n)$ to $O(n)$, **`printSorted`** from $O(n)$ to $O(n \log n)$
    - Why your textbook considers this to be a different ADT

Hash Tables

- There are ***m*** possible keys (*m* typically large, even infinite)
- We expect our table to have only ***n*** items
- ***n*** is much less than ***m*** (often written ***n* << *m***)

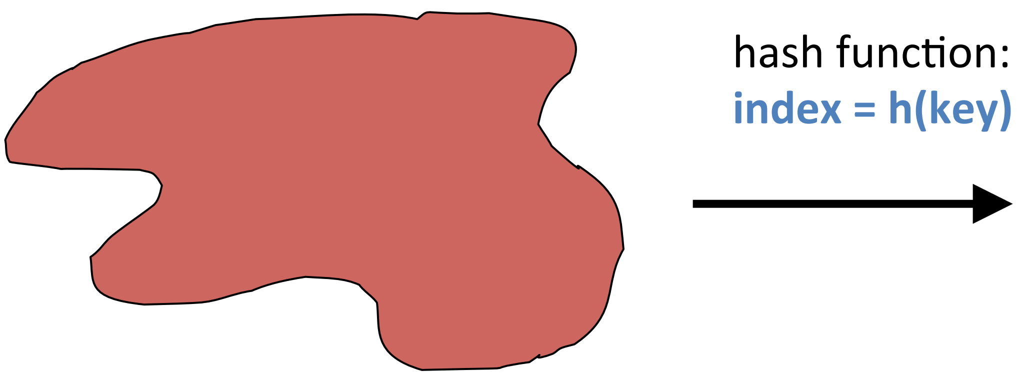
Many dictionaries have this property

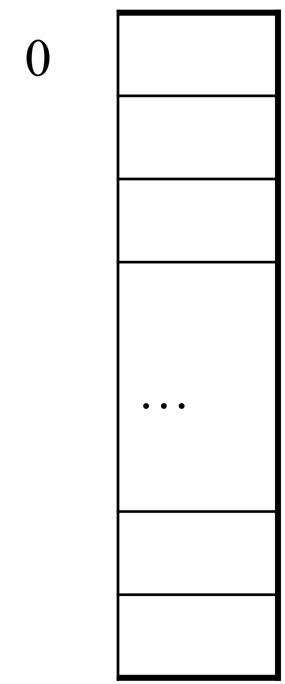
  - Compiler: All possible identifiers allowed by the language vs. those used in some file of one program
  - Database: All possible student names vs. students enrolled
  - AI: All possible chess-board configurations vs. those considered by the current player
  - …

Hash functions

An ideal hash function:

- Fast to compute
- "Rarely" hashes two "used" keys to the same index
  - Often impossible in theory but easy in practice
  - Will handle *collisions* later





Simple Integer Hash Functions

- key space K = integers
- TableSize = 7

- h(K) = K % 7

- **Insert**: 7, 18, 41

0	7
1	
2	
3	
4	18
5	
6	41

Simple Integer Hash Functions

- key space K = integers
- TableSize = 10
- h(K) = ??
- **Insert**: 7, 18, 41, 34
  – What happens when we insert 44?

0	
1	41
2	
3	
4	34
5	
6	
7	7
8	18
9	

Aside: Properties of Mod

To keep hashed values within the size of the table, we will generally do:

h(K) = function(K) % TableSize

(In the previous examples, function(K) = K.)

Useful properties of mod:

– (a + b) % c = [(a % c) + (b % c)] % c
– (a b) % c = [(a % c) (b % c)] % c
– a % c = b % c → (a – b) % c = 0

Designing Hash Functions

Often based on **modular hashing**:

$$h(K) = f(K) \% P$$

P is typically the TableSize

P is often chosen to be prime:

- Reduces likelihood of collisions due to patterns in data
- Is useful for guarantees on certain hashing strategies (as we'll see)

Equivalent objects MUST hash to the same location

Some String Hash Functions

key space = strings

$K = s_0\ s_1\ s_2 \ldots s_{m-1}$ (where s_i are chars: $s_i \in [0, 128]$)

1. $h(K) = s_0$ % TableSize   H("batman") = H("ballgame")

2. $h(K) = \left(\sum_{i=0}^{m-1} s_i \right)$ % TableSize   H("spot") = H("pots")

3. $h(K) = \left(\sum_{i=0}^{m-1} s_i \cdot 37^i \right)$ % TableSize

What to hash?

We will focus on the two most common things to hash: *ints* and *strings*

- For objects with several fields, usually best to have most of the "identifying fields" contribute to the hash to avoid collisions
- Example:

```
class Person {
   String first; String middle; String
last;
    Date birthdate;
 }
```

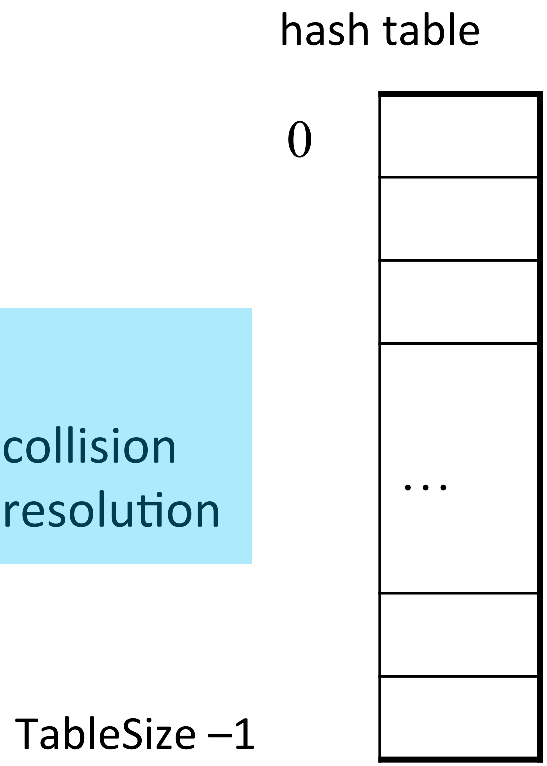
- An inherent trade-off: hashing-time vs. collision-avoidance
  - Bad idea(?): Use only first name
  - Good idea(?): Use only middle initial? Combination of fields?
  - Admittedly, what-to-hash-with is often unprincipled ☹

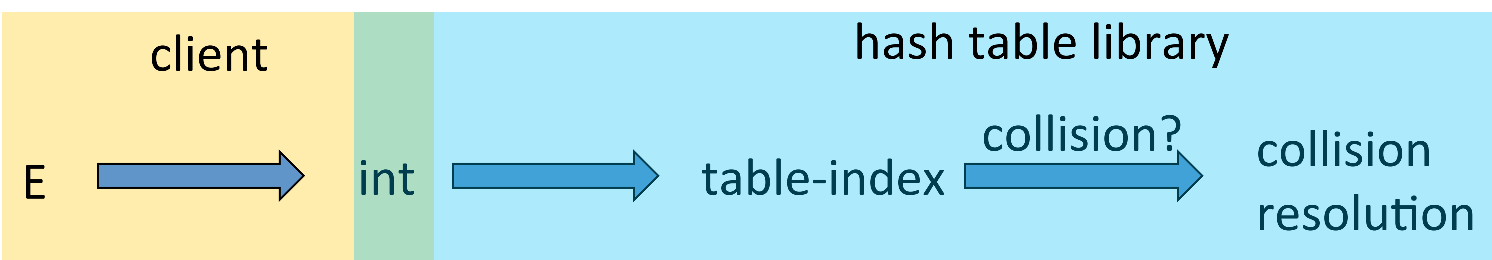
Deep Breath

- Recap

Hash Tables: Review

- Aim for constant-time (i.e., *O*(1)) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some reasonable assumptions
- A hash table is an array of some fixed size
  - But growable as we'll see





Collision resolution

Collision:

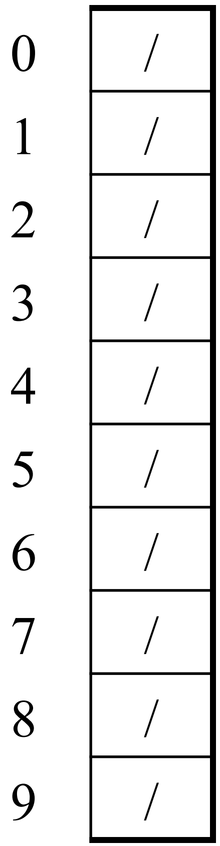
When two keys map to the same location in the hash table

We try to avoid it, but number-of-keys exceeds table size

So hash tables should support collision resolution

– Ideas?

Separate Chaining



Chaining:

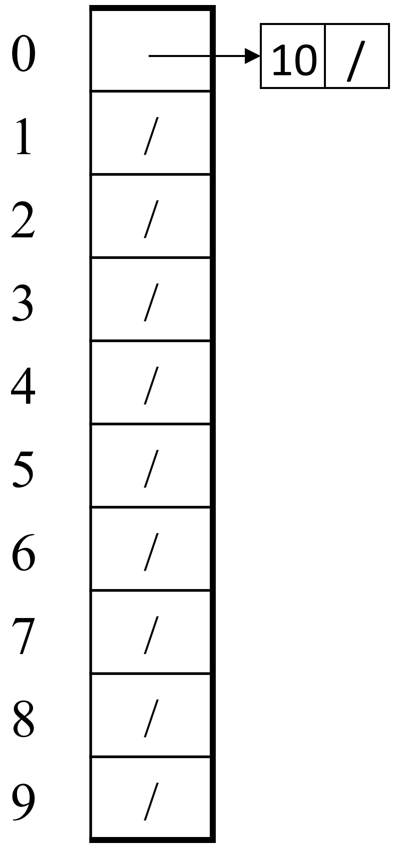
All keys that map to the same table location are kept in a list (a.k.a. a "chain" or "bucket")

As easy as it sounds

Example:

insert 10, 22, 107, 12, 42
with mod hashing
and **`TableSize`** = 10

Separate Chaining



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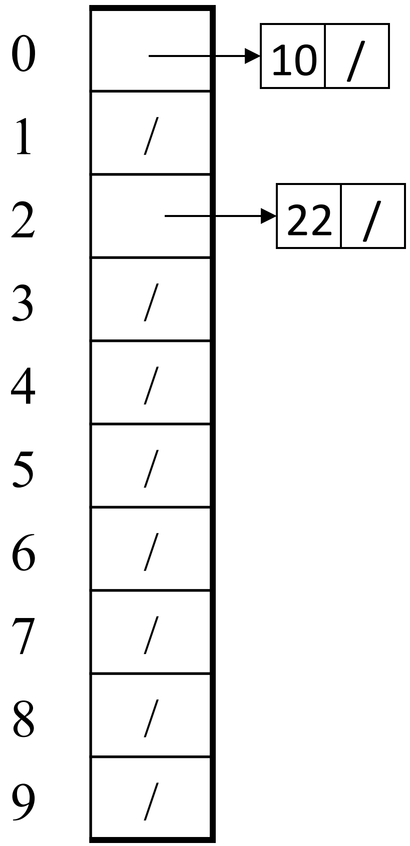

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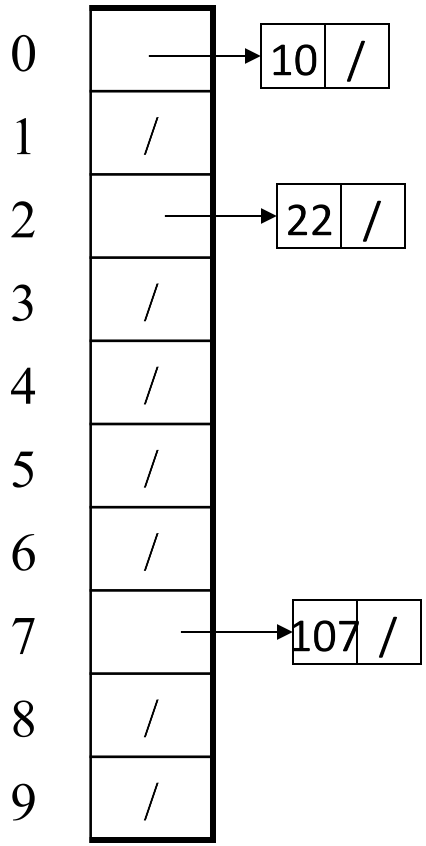

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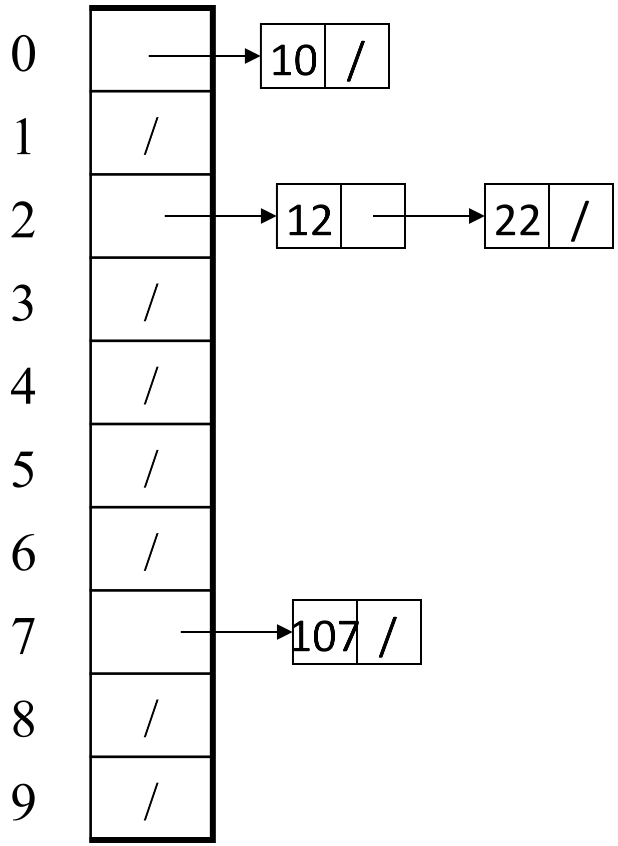

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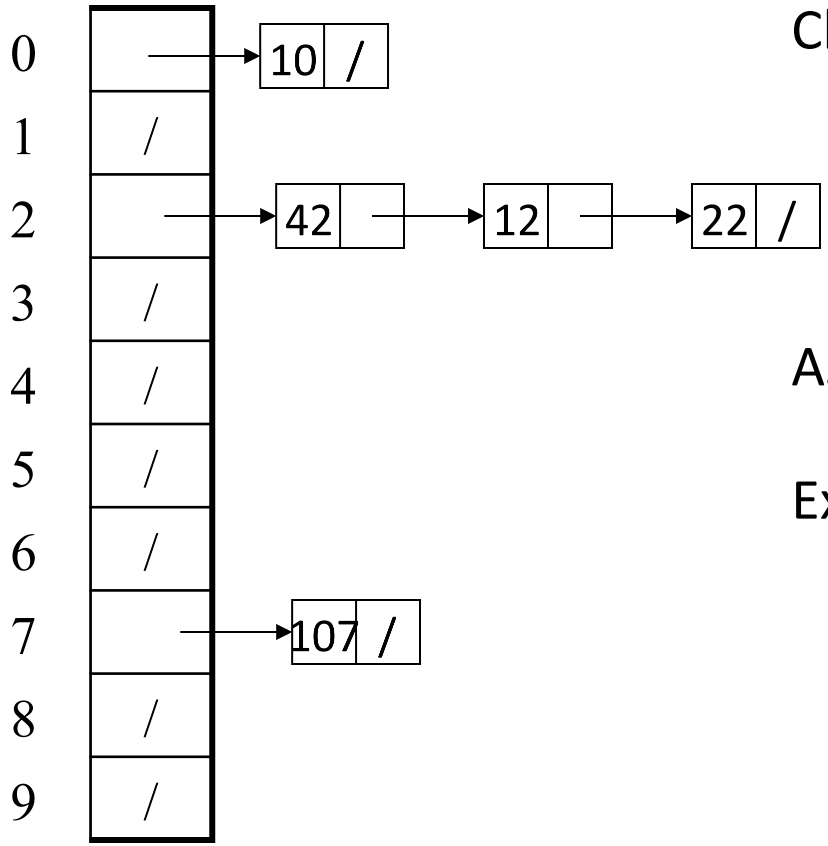

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As easy as it sounds

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More rigorous chaining analysis

Definition: The load factor, λ, of a hash table is

$$\lambda = \frac{\text{N}}{\text{TableSize}}$$ ← number of elements

Under chaining, the average number of elements per bucket is λ

So if some inserts are followed by *random* finds, then on average:

- Each "unsuccessful" `find` compares against λ items

So we like to keep λ fairly low (e.g., 1 or 1.5 or 2) for chaining