

let $g(n) = 1000n$ and $f(n) = n^2$

Show that $g(n)$ is in $O(f(n))$.

recall the definition:

$g(n)$ is in $O(f(n))$ if there exist constants c and n_0 s.t. $g(n) \leq c f(n) \quad \forall n \geq n_0$

note that the function names are arbitrary,
you might also want to prove $f(n)$ is in $O(g(n))$
or $A(n)$ is in $B(n)$.

need c, n_0 s.t. $g(n) \leq c \cdot f(n)$
 $\forall n \geq n_0$

$$1000n \leq c \cdot n^2$$

$$c = 1$$

$$1000n \leq n^2$$

this holds for all $n \geq 1000$, so $n_0 = 1000$

step 1: pick c or n_0

step 2: solve for other
(if possible)

likewise, we could have chosen

$$L=2 \text{ and } n_0 = 500$$

or even

$$L=2 \text{ and } n_0 = 1,000,000.$$

there are many valid L and n_0 pairs.