Prove $1+2+4+8+\dots+2^n = 2^{n+1}-1 \quad \forall n \geq 1$

Let $P(n)$ be "$\sum_{i=0}^{n} 2^i = 2^{n+1}-1$"

<u>Base case (n=0):</u>

$$2^0 = 1 = 2-1 = 2^{0+1}-1 \quad \checkmark$$

<u>Inductive Hypothesis (I.H.)</u>

Assume $P(K)$ is <u>true</u> for some arbitrary $K \in \mathbb{N}$

remember, $P(K)$ is

"$\sum_{i=0}^{k} 2^i = 2^{k+1}-1$"

Inductive Step

note: our goal is to show that $P(K+1)$ is true

$P(K+1)$ is "$\sum_{i=0}^{K+1} 2^i = 2^{(K+1)+1} - 1$"

if we ~~that~~ show this, we're done.

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1) $\sum_{i=0}^{K+1} 2^i = \sum_{i=0}^{K} 2^i + 2^{K+1}$ split summation

2) $= 2^{K+1} - 1 + 2^{K+1}$ by I.H. ★

3) $= (2^{K+1} + 2^{K+1}) - 1$ Assoc. +

4) $= 2(2^{K+1}) - 1$ factor

5) $= 2^{K+2} - 1 = 2^{(K+1)+1} - 1$ simplify.

we've shown that $P(K) \rightarrow P(K+1)$.
given this and our base case, we've shown that $P(n)$ is true for all natural number ★

```
X:=∅
for i=1 to N do
    for j=1 to i do
        x:= x+3
return x
```

Let $P(n)$ be "after the outer for-loop executes n times, $x = 3n(n+1)/2$

↪ we want to show this is true $\forall n \geq 0$

Base case ($n=0$):

outer loop is not entered... $x=0$ ✓

Inductive Hypothesis (I.H.)

Assume $P(K)$ for some arbitraly $K \geq \emptyset$

$P(K)$ = "after the outer loop executes K times,

$x = 3K(K+1)/2$

Inductive Step

our goal is to prove $P(K+1)$, or that after $K+1$ iterations of the outer loop, $x = 3(K+1)((K+1)+1)/2$

We know that after K iterations, $x = 3K(K+1)/2$ (from I.H.)

the next iteration adds $\underbrace{3(K+1)}$ to x. see note at end *

so after $K+1$ iterations,

$x = 3K(K+1)/2 + 3(K+1)$

1) $= \dfrac{3K(K+1) + 6(K+1)}{2}$ common denom.

2) $= \dfrac{(K+1)(3K+6)}{2}$ factor

3) $= \dfrac{3(K+1)(K+2)}{2}$ factor

$= \dfrac{3(K+1)((K+1)+1)}{2}$ ... this is $P(K+1)$

Conclusion

we've shown $P(0)$ and that $P(k) \rightarrow P(k+1)$, so by induction, ~~P(k)~~ $P(n)$ is true $\forall\, n \geq 0$

note: once the inner for loop is done executing:

```
for (j=1 to i):
    x: x+3
```

x will have been incremented by $3i$.