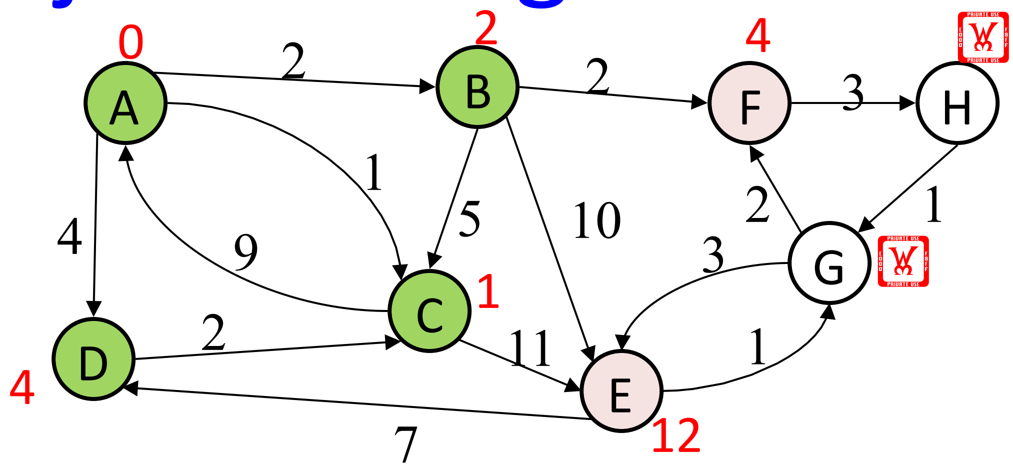
Dijkstra's algorithm

- The idea: reminiscent of BFS, but adapted to handle weights
  - Grow the set of nodes whose shortest distance has been computed
  - Nodes not in the set will have a "best distance so far"
  - A priority queue will turn out to be useful for efficiency

Dijkstra's Algorithm: Idea



- Initially, start node has cost 0 and all other nodes have cost ∞
- At each step:
  - Pick closest unknown vertex **v**
  - Add it to the "cloud" of known vertices
  - Update distances for nodes with edges from **v**
- That's it! (But we need to prove it produces correct answers)

The Algorithm

1. For each node **`v`**, set **`v.cost`** $= \infty$ and **`v.known = false`**
2. Set **`source.cost = 0`**
3. While there are unknown nodes in the graph
   a) Select the unknown node **`v`** with lowest cost
   b) Mark **`v`** as known
   c) For each edge **`(v,u)`** with weight **`w`**,

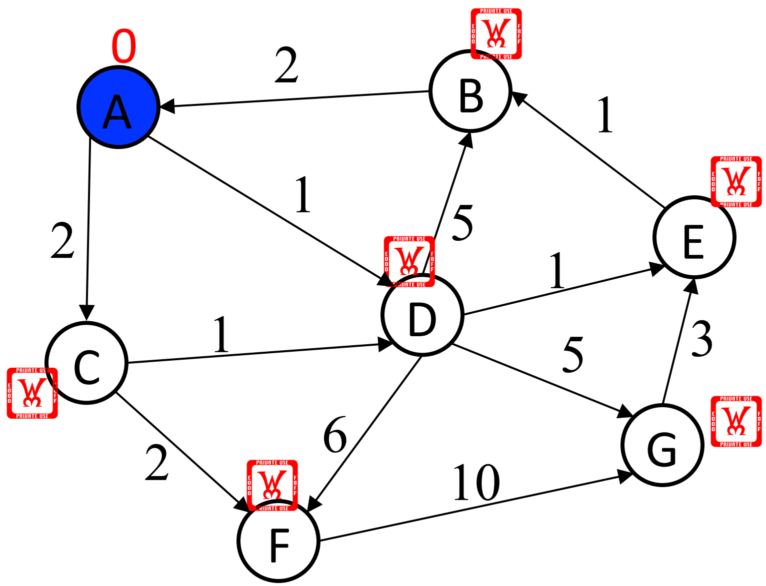
```
c1 = v.cost + w  // cost of best path through v to u
c2 = u.cost      // cost of best path to u previously known
if(c1 < c2){     // if the path through v is better
  u.cost = c1
  u.path = v     // for computing actual paths
}
```

Important features

- When a vertex is marked known, the cost of the shortest path to that node is known
  - The path is also known by following back-pointers

- While a vertex is still not known, another shorter path to it *might* still be found

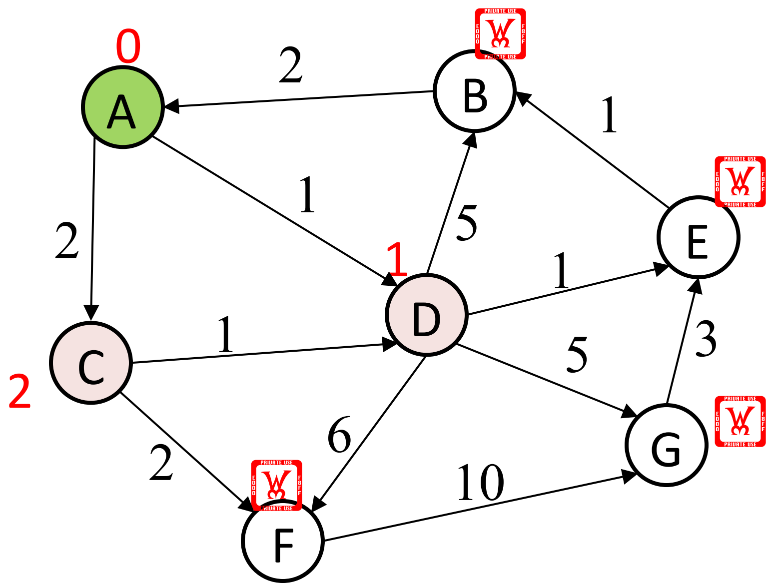
Example #2



Order Added to Known Set:

vertex	known?	cost	path
A		0	
B		??	
C		??	
D		??	
E		??	
F		??	
G		??	

Example #2

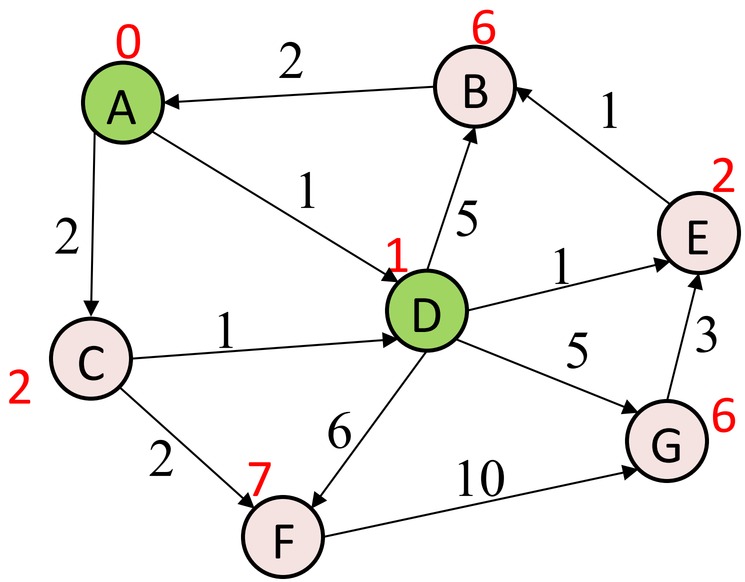


Order Added to Known Set:

A

vertex	known?	cost	path
A	Y	0	
B		??	
C		≤ 2	A
D		≤ 1	A
E		??	
F		??	
G		??	

Example #2

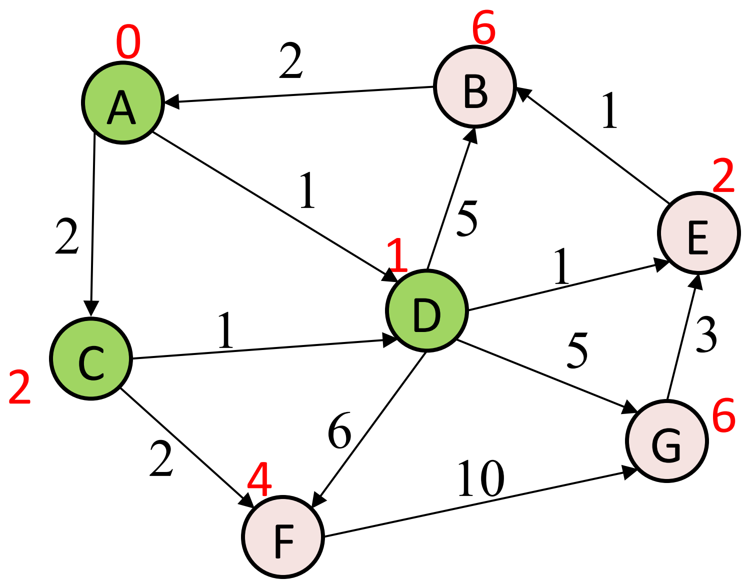


vertex	known?	cost	path
A	Y	0	
B		≤ 6	D
C		≤ 2	A
D	Y	1	A
E		≤ 2	D
F		≤ 7	D
G		≤ 6	D

Order Added to Known Set:

A, D

Example #2

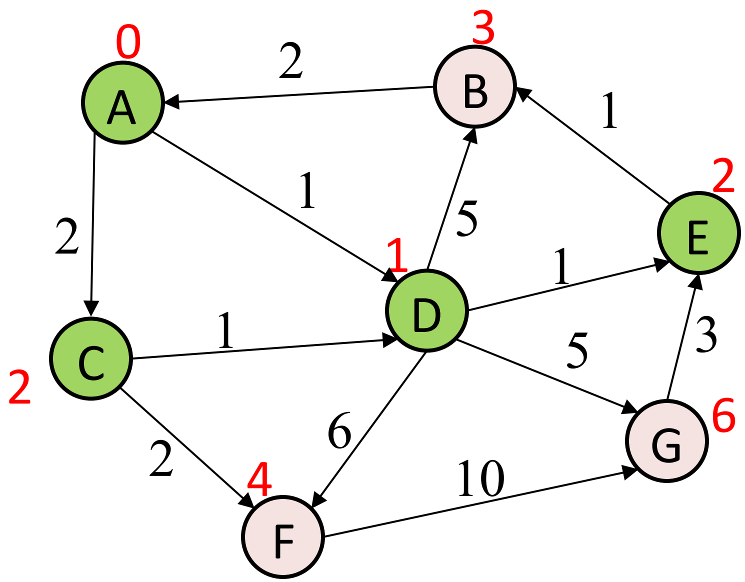


Order Added to Known Set:

A, D, C

vertex	known?	cost	path
A	Y	0	
B		≤ 6	D
C	Y	2	A
D	Y	1	A
E		≤ 2	D
F		≤ 4	C
G		≤ 6	D

Example #2

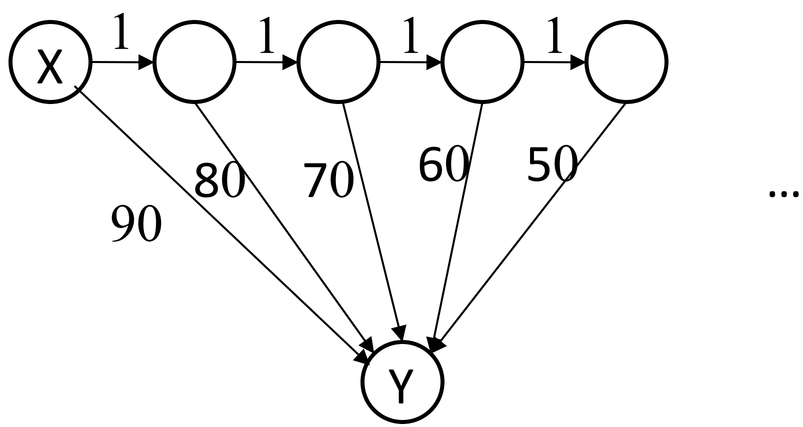


vertex	known?	cost	path
A	Y	0	
B		≤ 3	E
C	Y	2	A
D	Y	1	A
E	Y	2	D
F		≤ 4	C
G		≤ 6	D

Order Added to Known Set:

A, D, C, E

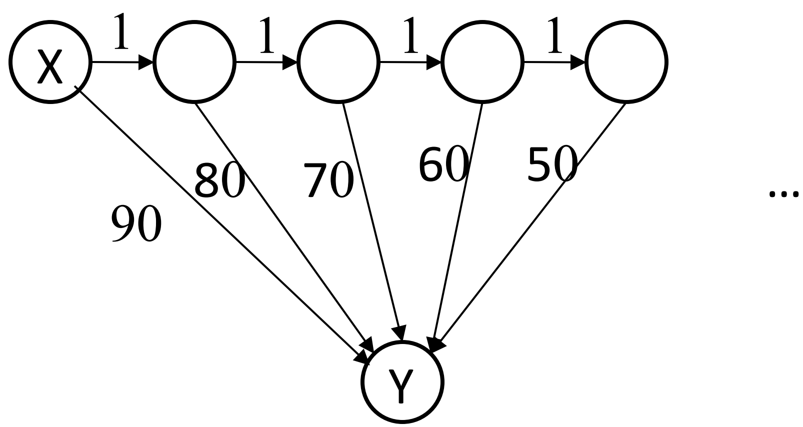
Example #3



How will the best-cost-so-far for Y proceed?

Is this expensive?

Example #3



How will the best-cost-so-far for Y proceed? 90, 81, 72, 63, 54, …

Is this expensive? No, each *edge* is processed only once

A Greedy Algorithm

- Dijkstra's algorithm
  - For single-source shortest paths in a weighted graph (directed or undirected) with no negative-weight edges

- An example of a *greedy algorithm*:
  - At each step, irrevocably does what seems best at that step
    - A locally optimal step, not necessarily globally optimal
  - Once a vertex is known, it is not revisited
    - Turns out to be globally optimal

Where are We?

- Had a problem: Compute shortest paths in a weighted graph with no negative weights
- Learned an algorithm: Dijkstra's algorithm
- What should we do after learning an algorithm?
  - Prove it is correct
    - Not obvious!
    - We will sketch the key ideas
  - Analyze its efficiency
    - Will do better by using a data structure we learned earlier!

Correctness: Intuition

Rough intuition:

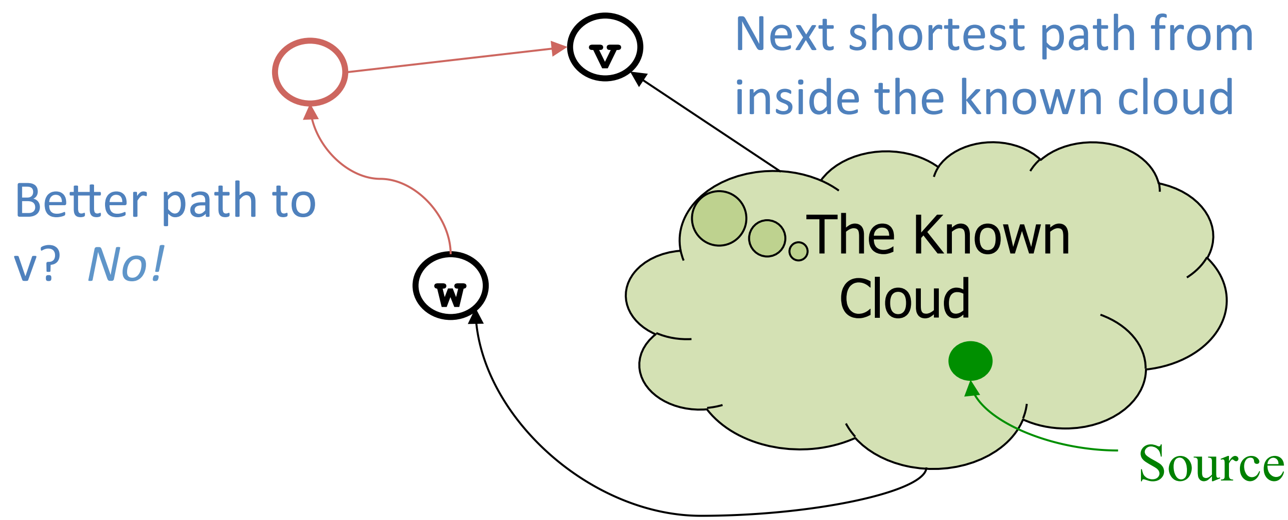
All the “known” vertices have the correct shortest path

- True initially: shortest path to start node has cost 0
- If it stays true every time we mark a node “known”, then by induction this holds and eventually everything is “known”

Key fact we need: When we mark a vertex “known” we won’t discover a shorter path later!

- This holds only because Dijkstra’s algorithm picks the node with the next shortest path-so-far
- The proof is by contradiction…

Correctness: The Cloud (Rough Sketch)



Suppose v is the next node to be marked known ("added to the cloud")

- The best-known path to v must have only nodes "in the cloud"
  - Else we would have picked a node closer to the cloud than v
- Suppose the actual shortest path to v is different
  - It won't use only cloud nodes, or we would know about it
  - So it must use non-cloud nodes. Let w be the *first* non-cloud node on this path. The part of the path up to w is already known and must be shorter than the best-known path to v. So v would not have been picked. Contradiction.

Naïve asymptotic running time

- So far: $O(|V|^2)$
- We had a similar "problem" with topological sort being $O(|V|^2)$ due to each iteration looking for the node to process next
  - We solved it with a queue of zero-degree nodes
  - But here we need the lowest-cost node and costs can change as we process edges
- Solution?

Improving asymptotic running time

- So far: $O(|V|^2)$
- We had a similar "problem" with topological sort being $O(|V|^2)$ due to each iteration looking for the node to process next
  - We solved it with a queue of zero-degree nodes
  - But here we need the lowest-cost node and costs can change as we process edges
- Solution?
  - A priority queue holding all unknown nodes, sorted by cost
  - But must support **`decreaseKey`** operation
    - Must maintain a reference from each node to its current position in the priority queue
    - Conceptually simple, but can be a pain to code up

Efficiency, second approach

Use pseudocode to determine asymptotic run-time

```
dijkstra(Graph G, Node start) {
  for each node: x.cost=infinity, x.known=false
  start.cost = 0
  build-heap with all nodes
  while(heap is not empty) {
    b = deleteMin()
    b.known = true
    for each edge (b,a) in G
     if(!a.known)
       if(b.cost + weight((b,a)) < a.cost){
         decreaseKey(a,"new cost – old cost")
         a.path = b
       }
}
```

Efficiency, second approach

Use pseudocode to determine asymptotic run-time

```
dijkstra(Graph G, Node start) {
  for each node: x.cost=infinity, x.known=false
  start.cost = 0
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     if(!a.known)
       if(b.cost + weight((b,a)) < a.cost){
         decreaseKey(a,"new cost – old cost")
         a.path = b
       }
}
```

O(|V|)

O(|V|log|V|)

O(|E|log|V|)

O(|V|log|V|+|E|log|V|)

CSE373: Data Structures & Algorithms

Software-Design Interlude – Preserving Abstractions

Hunter Zahn

Summer 2016

Motivation

- Essential: knowing available data structures and their trade-offs
  - You're taking a whole course on it! ☺
- However, you will rarely if ever re-implement these "in real life"
  - Provided by libraries
- But the key idea of an *abstraction* arises *all the time* "in real life"
  - Clients do not know how it is implemented
  - Clients do not need to know
  - Clients cannot "break the abstraction" *no matter what they do*

Interface vs. implementation

- Provide a reusable interface without revealing implementation
- More difficult than it sounds due to aliasing and field-assignment
  - Some common pitfalls
- So study it in terms of ADTs vs. data structures
  - Will use priority queues as example in lecture, but any ADT would do
  - Key aspect of grading your homework on graphs

Recall the abstraction

Clients:

"not trusted by ADT implementer"

- Can perform any sequence of ADT operations
- Can do anything type-checker allows on any accessible objects

```
new PQ(…)

insert(…)

deleteMin(…)

isEmpty()
```

Data structure:

- Should document how operations can be used and what is checked (raising appropriate exceptions)
  - E.g., parameter for method x not `null`
- If used correctly, correct priority queue for any client
- Client "cannot see" the implementation
  - E.g., binary min heap

Our example

- A priority queue with to-do items, so earlier dates "come first"
  - Simpler example than using Java generics
- Exact method names and behavior not essential to example

```
public class Date {
   … // some private fields (year, month, day)
   public int getYear() {…}
   public void setYear(int y) {…}
   … // more methods
}
public class ToDoItem {
   … // some private fields (date,
description)
   public void setDate(Date d) {…}
   public void setDescription(String d) {…}
   … // more methods
}
// continued next slide…
```

Our example

- A priority queue with to-do items, so earlier dates "come first"
  - Simpler example than using Java generics
- Exact method names and behavior not essential to example

```
public class Date { … }
public class ToDoItem { … }
public class ToDoPQ {
   … // some private fields (array, size, …)
   public ToDoPQ() {…}
   void insert(ToDoItem t) {…}
   ToDoItem deleteMin() {…}
   boolean isEmpty() {…}
}
```

An obvious mistake

- Why we trained you to "mindlessly" make fields **`private`**:

```
public class ToDoPQ {
   … // other fields
   public ToDoItem[] heap;
   public ToDoPQ() {…}
   void insert(ToDoItem t) {…}
   …
}
// client:
pq = new ToDoPQ();
pq.heap = null;
pq.insert(…); // likely exception
```

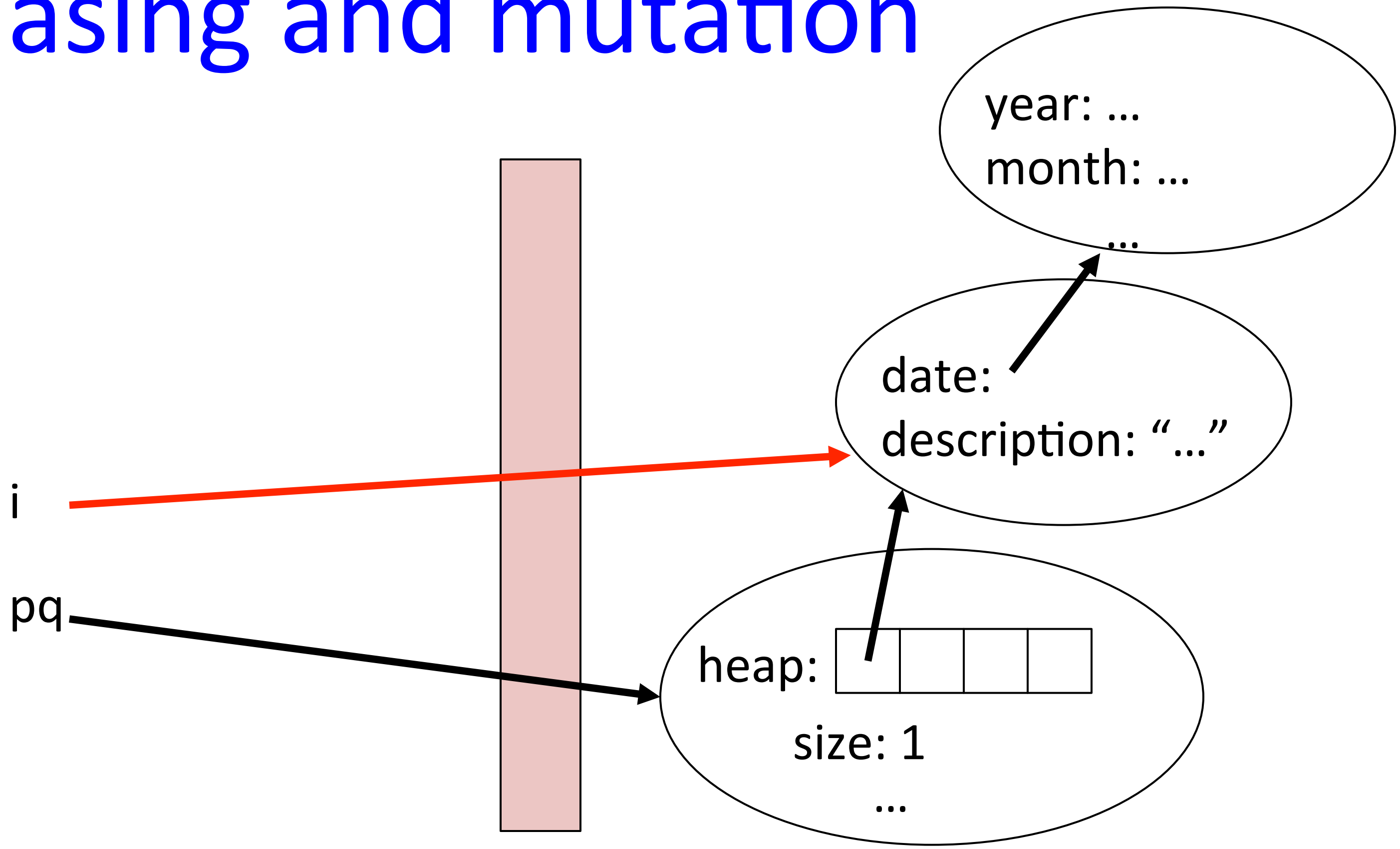
- Today's lecture: `private` does not solve all your problems!
  - Upcoming pitfalls can occur even with all `private` fields

Less obvious mistakes

```
public class ToDoPQ {
   … // all private fields
   public ToDoPQ() {…}
   void insert(ToDoItem i) {…}
   …
}

// client:
ToDoPQ   pq = new ToDoPQ();
ToDoItem i  = new ToDoItem(…);
pq.insert(i);
i.setDescription("some different thing");
pq.insert(i); // same object after update
x = deleteMin(); // x's description???
y = deleteMin(); // y's description???
```

Aliasing and mutation

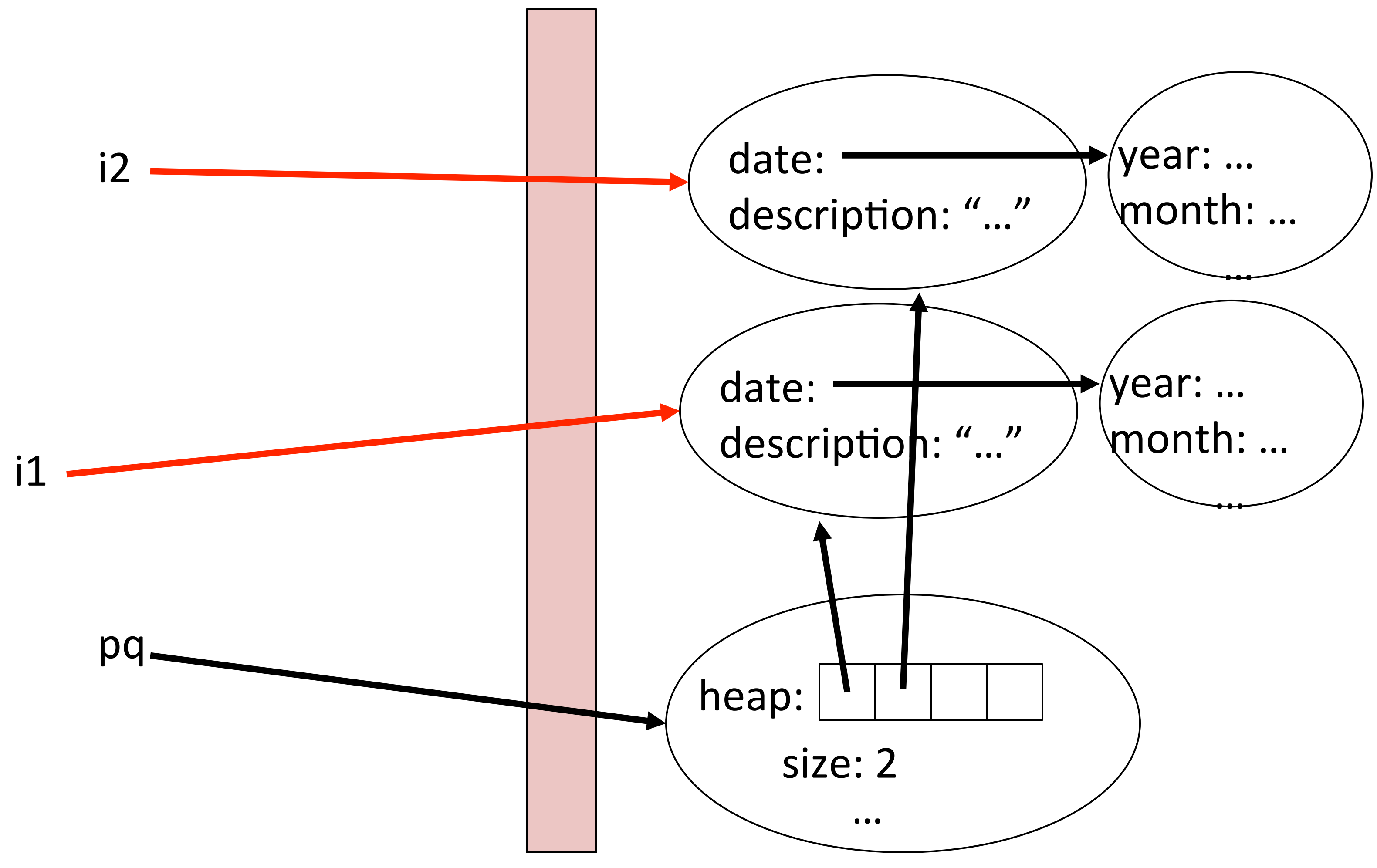


- Client was able to update something inside the abstraction because client had an alias to it!
  - It is too hard to reason about and document what should happen, so better software designs avoid the issue!

More bad clients

```
ToDoPQ   pq = new ToDoPQ();
ToDoItem i1 = new ToDoItem(…); // year 2013
ToDoItem i2 = new ToDoItem(…); // year 2014
pq.insert(i1);
pq.insert(i2);
i1.setDate(…); // year 2015
x = deleteMin(); // “wrong” (???) item?
       // What date does returned item have???
```

More bad clients



More bad clients

```
pq = new ToDoPQ();
ToDoItem i1 = new ToDoItem(…);
pq.insert(i1);
i1.setDate(null);
ToDoItem i2 = new ToDoItem(…);
pq.insert(i2); // NullPointerException???
```

Get exception inside data-structure code even if `insert` did a careful check that the date in the `ToDoItem` is not `null`

- Bad client later invalidates the check

The general fix

- Avoid aliases into the internal data (the "red arrows") by copying objects as needed
  - Do not use the same objects inside and outside the abstraction because two sides do not know all mutation (field-setting) that might occur
  - "Copy-in-copy-out"
- A first attempt:

```
public class ToDoPQ {
   …
   void insert(ToDoItem i) {
     ToDoItem internal_i =
       new ToDoItem(i.date,i.description);
     … // use only the internal object
   }
}
```

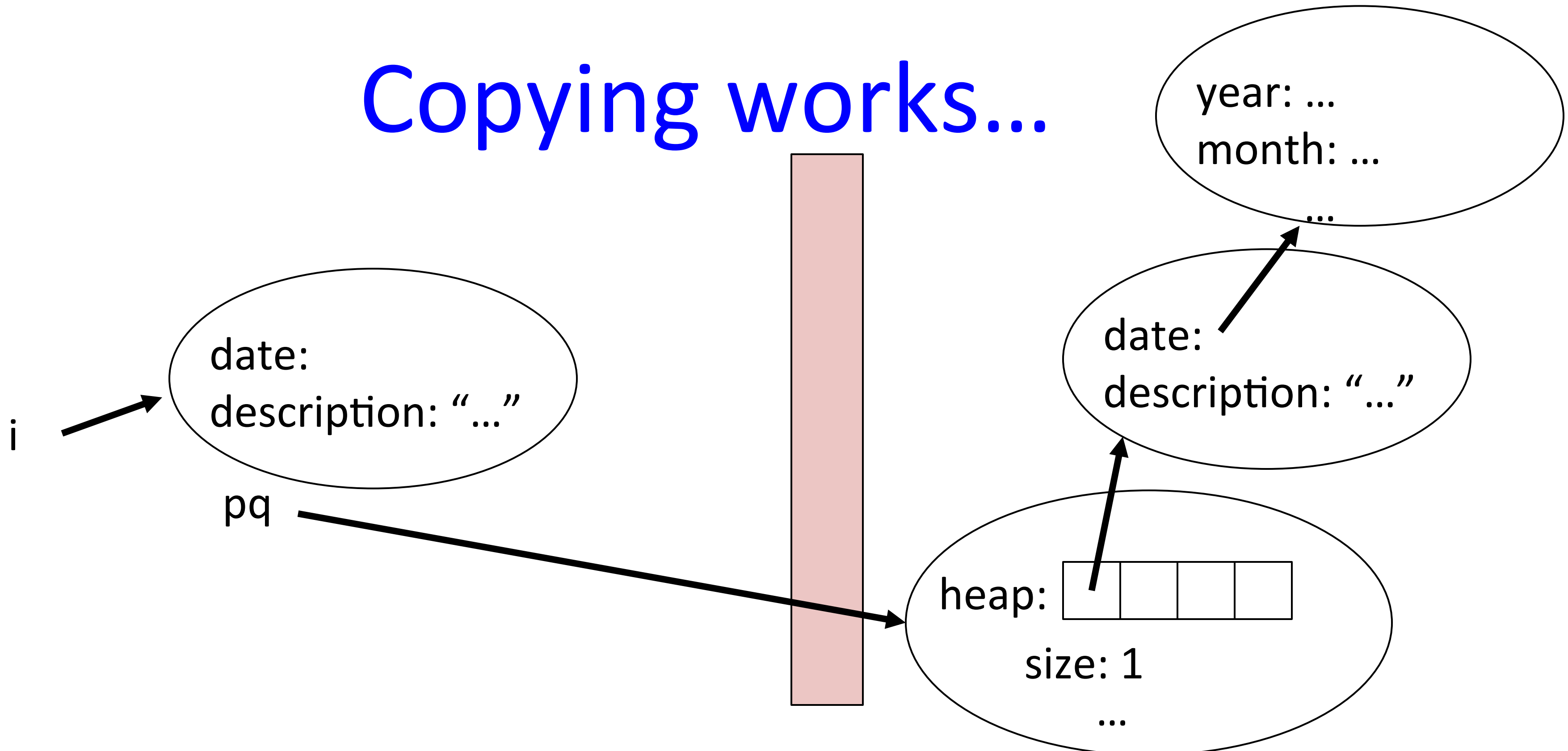
Must copy the object

```
public class ToDoPQ {
   …
   void insert(ToDoItem i) {
     ToDoItem internal_i =
       new ToDoItem(i.date,i.description);
     … // use only the internal object
   }
}
```

- Notice this version accomplishes nothing
  - Still the alias to the object we got from the client:

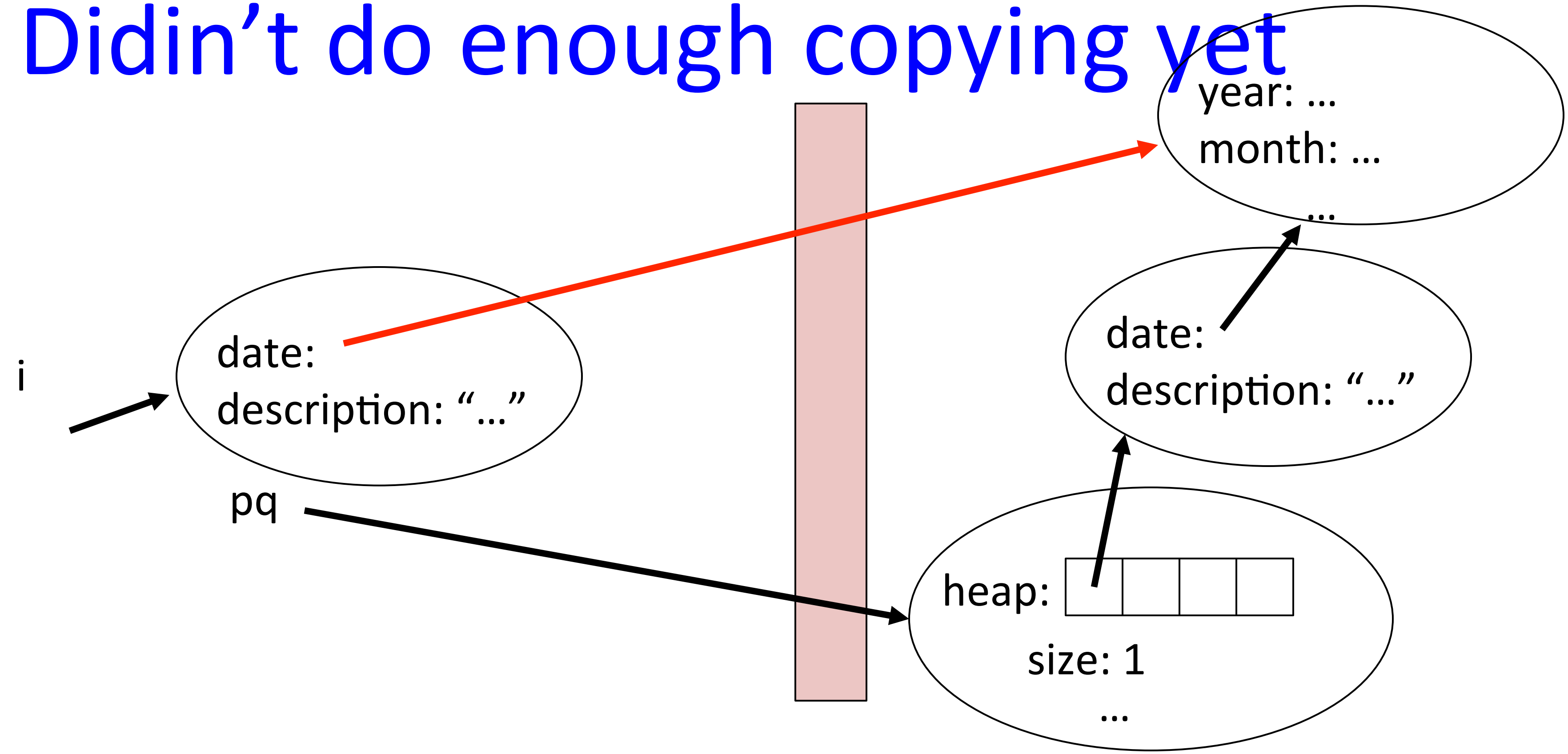
```
public class ToDoPQ {
   …
   void insert(ToDoItem i) {
     ToDoItem internal_i = i;
     … // internal_i refers to same object
   }
}
```

Copying works…




```
ToDoItem i = new ToDoItem(…);
pq = new ToDoPQ();
pq.insert(i);
i.setDescription("some different thing");
pq.insert(i);
x = deleteMin();
y = deleteMin();
```

Didin’t do enough copying yet




```
Date d = new Date(…)
ToDoItem i = new ToDoItem(d,“buy beer”);
pq = new ToDoPQ();
pq.insert(i);
d.setYear(2015);
…
```

Deep copying

- For copying to work fully, usually need to also make copies of all objects referred to (and that they refer to and so on…)
  – All the way down to **`int`**, **`double`**, **`String`**, …
  – Called *deep copying* (versus our first attempt *shallow-copy*)
- Rule of thumb: Deep copy of things passed into abstraction

```
public class ToDoPQ {
   …
   void insert(ToDoItem i) {
     ToDoItem internal_i =
       new ToDoItem(new Date(…),
                    i.description);
     … // use only the internal object
   }
}
```

Constructors take input too

- General rule: Do not "trust" data passed to constructors
  - Check properties and make deep copies
- Example: Floyd's algorithm for **`buildHeap`** should:
  - Check the array (e.g., for **`null`** values in fields of objects or array positions)
  - Make a deep copy: new array, new objects

```
public class ToDoPQ {
  // a second constructor that uses
  // Floyd's algorithm, but good design
  // deep-copies the array (and its contents)
  void PriorityQueue(ToDoItem[] items) {
     …
  }
}
```

That was copy-in, now copy-out…

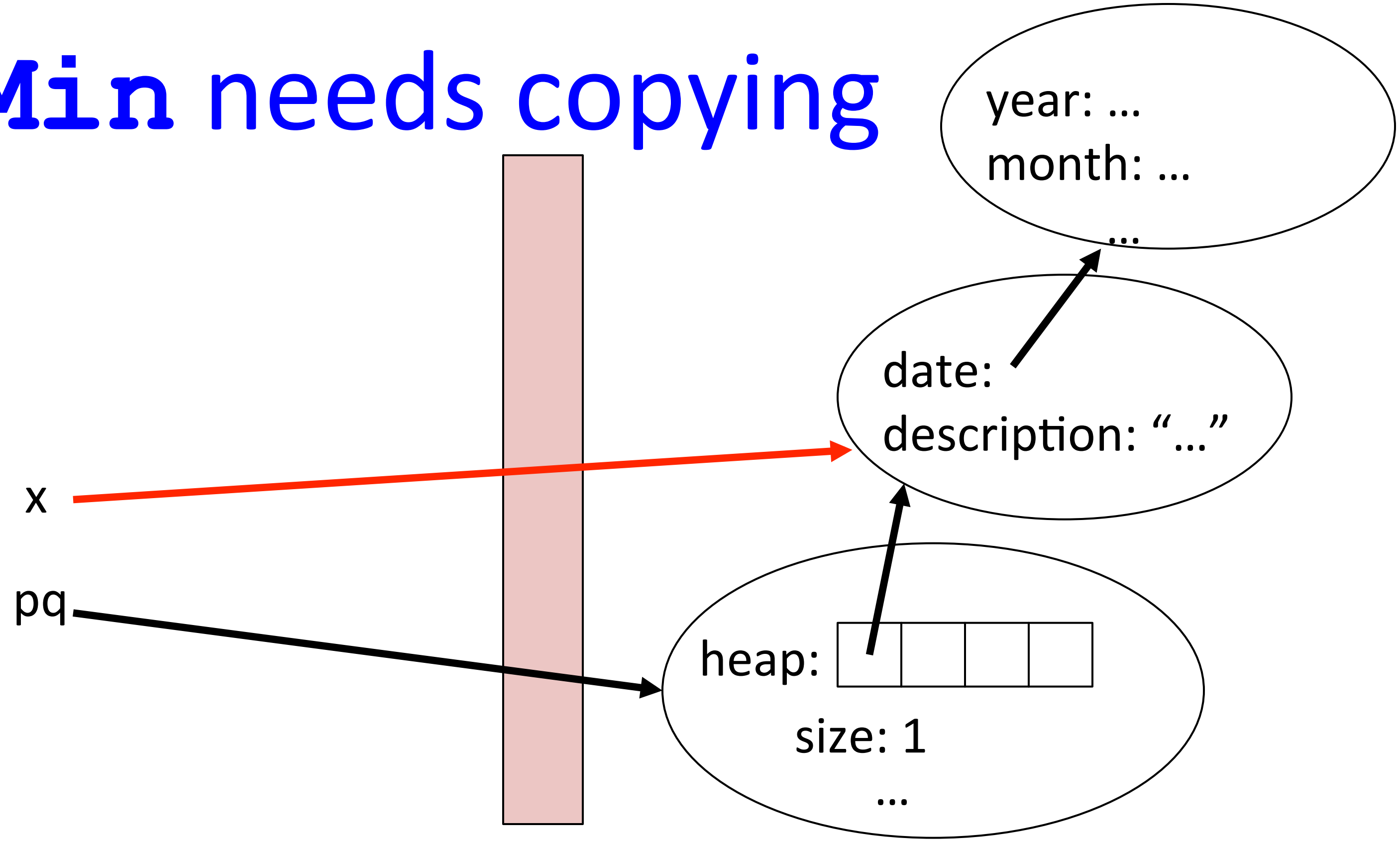
- So we have seen:
  - Need to deep-copy data passed into abstractions to avoid pain and suffering
- Next:
  - Need to deep-copy data passed out of abstractions to avoid pain and suffering (unless data is "new" or no longer used in abstraction)
- Then:
  - If objects are immutable (no way to update fields or things they refer to), then copying unnecessary

`deleteMin` is fine

```
public class ToDoPQ {
    …
    ToDoItem deleteMin() {
      ToDoItem ans = heap[0];
      … // algorithm involving percolateDown
      return ans;
}
```

- Does not create a “red arrow” because object returned is no longer part of the data structure
- Returns an alias to object that was in the heap, but now it is not, so conceptual “ownership” “transfers” to the client

getMin needs copying




```
ToDoItem i = new ToDoItem(…);
pq = new ToDoPQ();
x = pq.getMin();
x.setDate(…);
```

```
public class ToDoPQ {
  ToDoItem getMin() {
    int ans = heap[0];
    return ans;
  }
}
```

- Uh-oh, creates a "red arrow"

The fix

- Just like we deep-copy objects from clients before adding to our data structure, we should deep-copy parts of our data structure and return the copies to clients

- Copy-in *and* copy-out

```
public class ToDoPQ {
  ToDoItem getMin() {
    int ans = heap[0];
    return new ToDoItem(new Date(…),
                        ans.description);
  }
}
```

Less copying

- (Deep) copying is one solution to our aliasing problems

- Another solution is *immutability*
  - Make it so nobody can ever change an object or any other objects it can refer to (deeply)
  - Allows "red arrows", but immutability makes them harmless

- In Java, a **`final`** field cannot be updated after an object is constructed, so helps ensure immutability
  - But **`final`** is a "shallow" idea and we need "deep" immutability

This works

```
public class Date {
    private final int year;
    private final String month;
    private final String day;
}
public class ToDoItem {
    private final Date date;
    private final String description;
}
public class ToDoPQ {
    void insert(ToDoItem i){/*no copy-in needed!*/}
    ToDoItem getMin(){/*no copy-out needed!*/}
    …
}
```

Notes:

- `String` objects are immutable in Java
- (Using `String` for `month` and `day` is not great style though)

This does not work

```
public class Date {
   private final int year;
   private String month; // not final
   private final String day;
   …
}
public class ToDoItem {
   private final Date date;
   private final String description;
}
public class ToDoPQ {
   void insert(ToDoItem i){/*no copy-in*/}
   ToDoItem getMin(){/*no copy-out*/}
   …
}
```

Client could mutate a `Date`'s `month` that is in our data structure

- So must do entire deep copy of `ToDoItem`

`final` is shallow

```
public class ToDoItem {
   private final Date date;
   private final String description;
}
```

- Here, **`final`** means no code can update the **`date`** or **`description`** fields after the object is constructed
- So they will always refer to the same **`Date`** and **`String`** objects
- But what if those objects have *their* contents change
  - Cannot happen with **`String`** objects
  - For **`Date`** objects, depends how we define **`Date`**
- So **`final`** is a "shallow" notion, but we can use it "all the way down" to get deep immutability

This works

- When deep-copying, can “stop” when you get to immutable data
  - Copying immutable data is wasted work, so poor style

```
public class Date { // immutable
   private final int year;
   private final String month;
   private final String day;
   …
}
public class ToDoItem {
   private Date date;
   private String description;
}
public class ToDoPQ {
   ToDoItem getMin(){
    int ans = heap[0];
    return new ToDoItem(ans.date, // okay!
                        ans.description);
   }
}
```

What about this?

```
public class Date { // immutable
    …
}
public class ToDoItem { // immutable (unlike last slide)
    …
}
public class ToDoPQ {
  // a second constructor that uses
  // Floyd’s algorithm
  void PriorityQueue(ToDoItem[] items) {
      // what copying should we do?
      …
    }
}
```

What about this?

```
public class Date { // immutable
    …
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public class ToDoPQ {
  // a second constructor that uses
  // Floyd's algorithm
  void PriorityQueue(ToDoItem[] items) {
     // what copying should we do?
     …
   }
}
```

Copy the array, but do not copy the `ToDoItem` or `Date` objects

Homework 4

- You are implementing a graph abstraction
- As provided, **`Vertex`** and **`Edge`** are immutable
  - But **`Collection<Vertex>`** and **`Collection<Edge>`** are not
- You might choose to add fields to **`Vertex`** or **`Edge`** that make them not immutable
  - Leads to more copy-in-copy-out, but that's fine!
- *Or* you might leave them immutable and keep things like "best-path-cost-so-far" in another dictionary (e.g., a **`HashMap`**)

There is more than one good design, but preserve your abstraction

- *Great practice with a key concept in software design*