CSE373: Data Structures & Algorithms

Lecture 6: Binary Search Trees

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Spring 2016

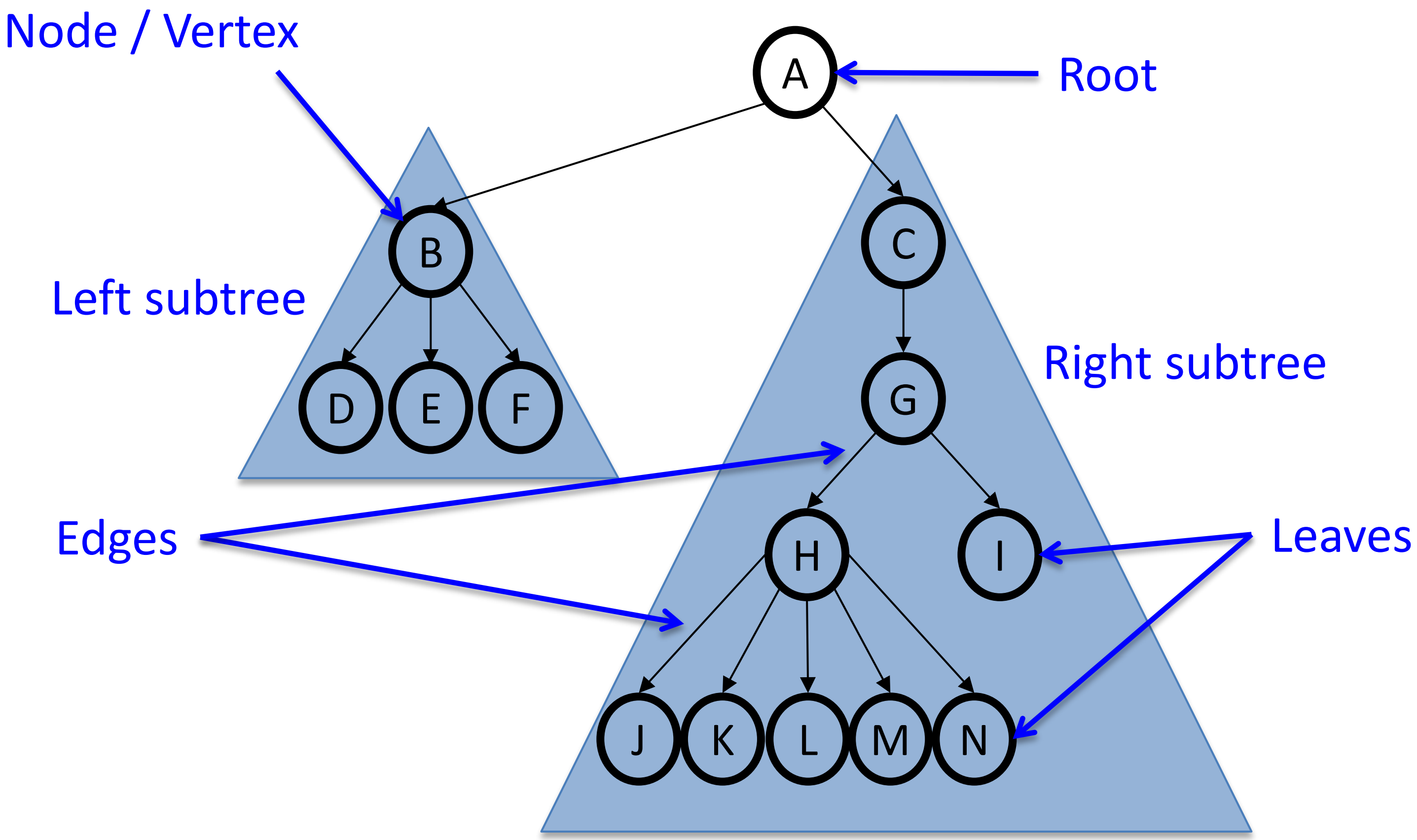
Announcements

- HW2 due start of class Wednesday April 13 on paper.

Previously

- Dictionary ADT
  - stores (key, value) pairs
  - **`find`**, **`insert`**, **`delete`**
- Trees
  - Terminology
  - Binary Trees

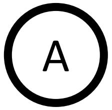
Reminder: Tree terminology



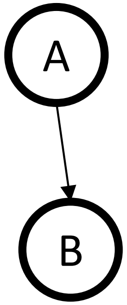
Example Tree Calculations

Recall: Height of a tree is the maximum number of edges from the root to a leaf.

What is the height of this tree?

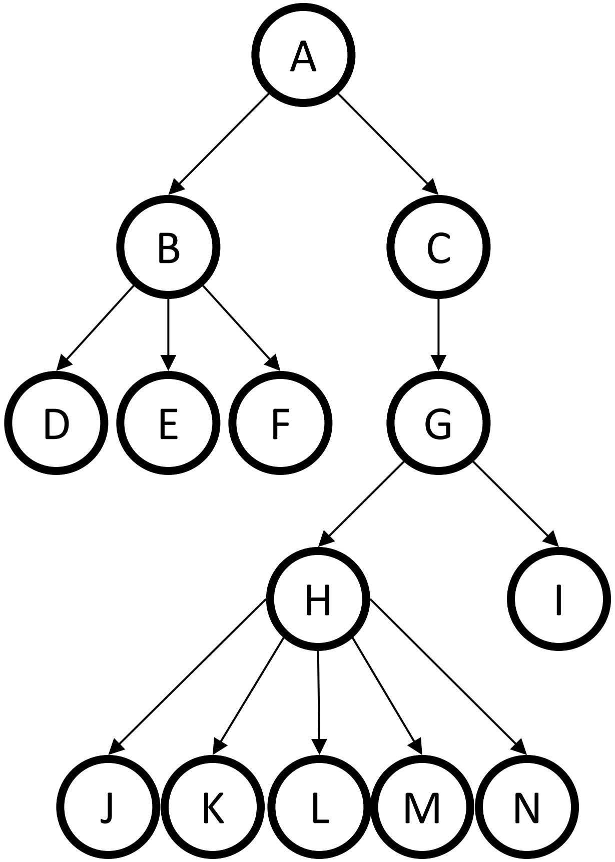


Height = 0



Height = 1

Height = 4



What is the depth of node G?

Depth = 2

What is the depth of node L?

Depth = 4

Binary Trees

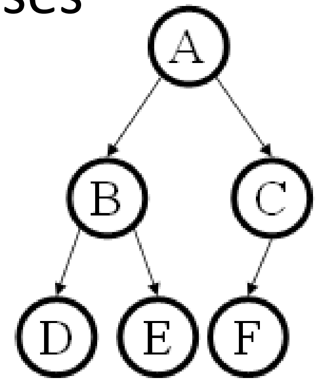
- Binary tree: Each node has at most 2 children (branching factor 2)

- Binary tree is
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*

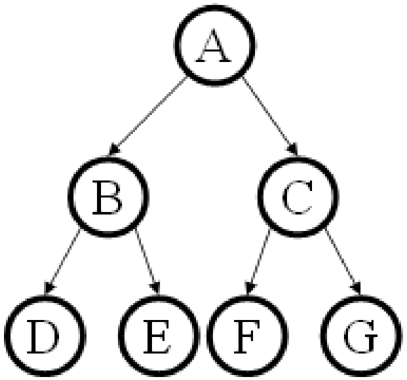
What is full?

Every node has 0 or 2 children.

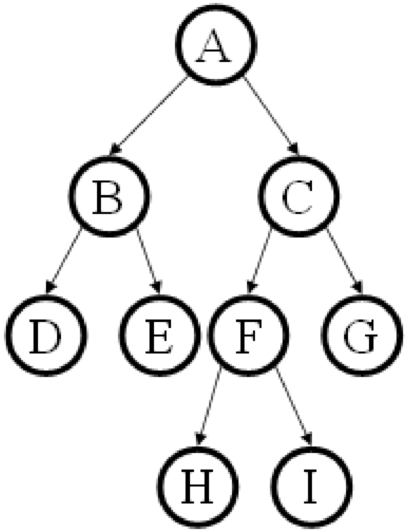
- Special Cases



Complete Tree



Perfect Tree



Full Tree

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree

  + * 2 4 5

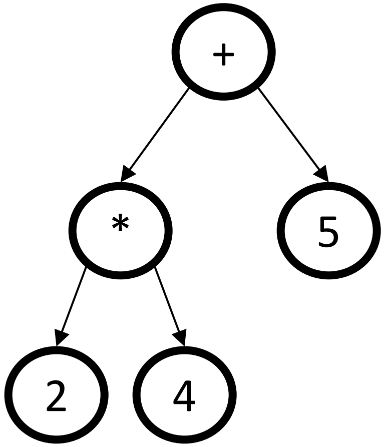
- *In-order*: left subtree, root, right subtree

  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

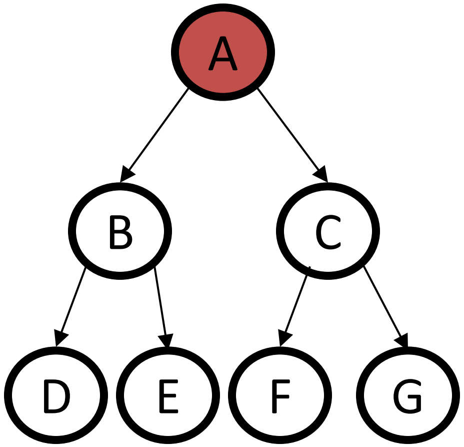
  2 4 * 5 +

(an expression tree)



More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



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A = processing (on the call stack)

A = completed node

✓= element has been processed

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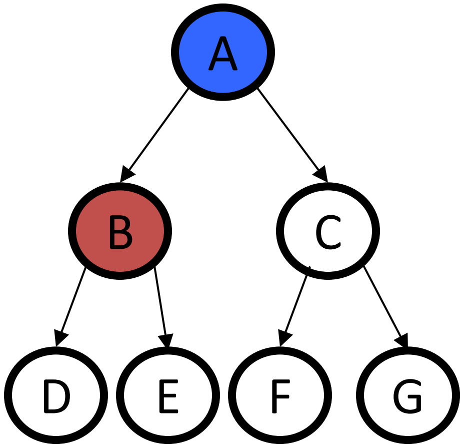

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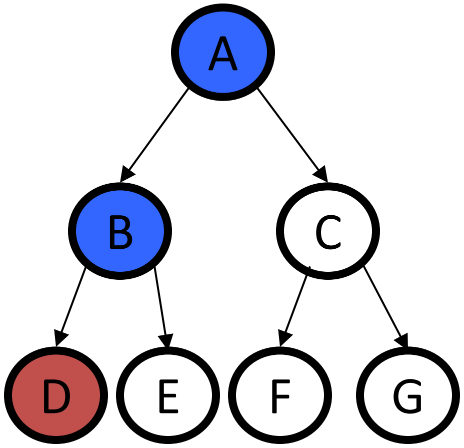

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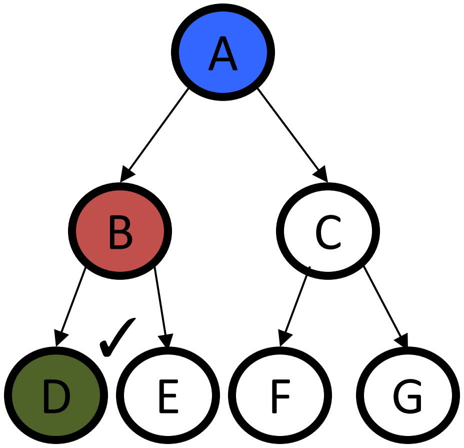

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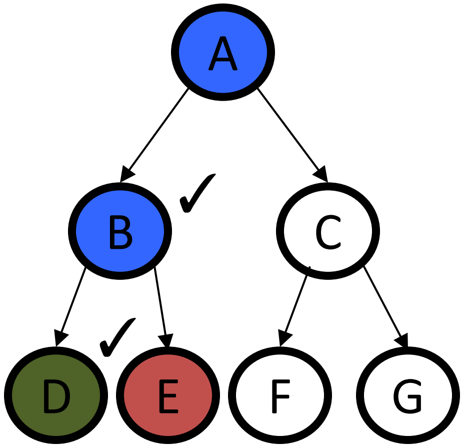

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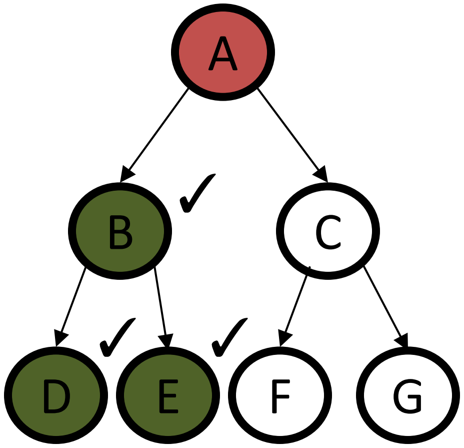

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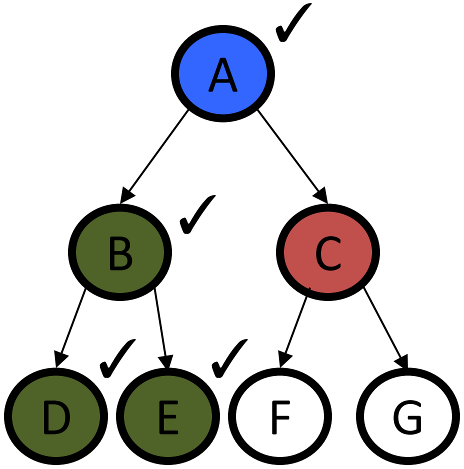

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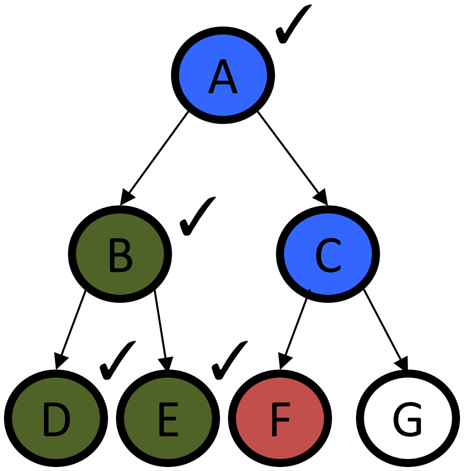

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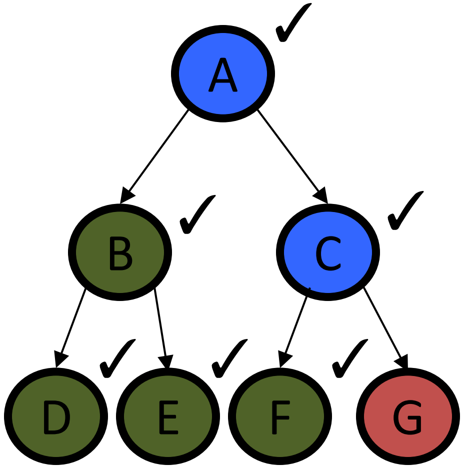

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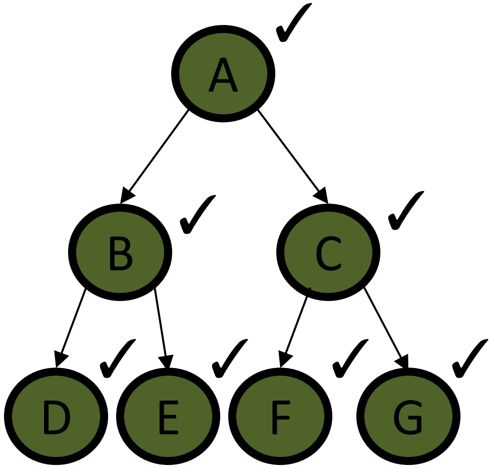

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D B E A F C G

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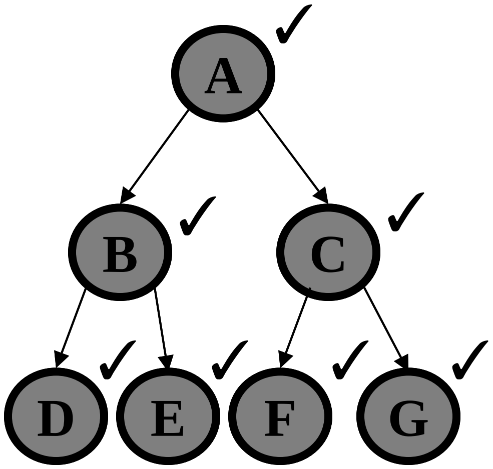


Sometimes order doesn’t matter

- Example: sum all elements

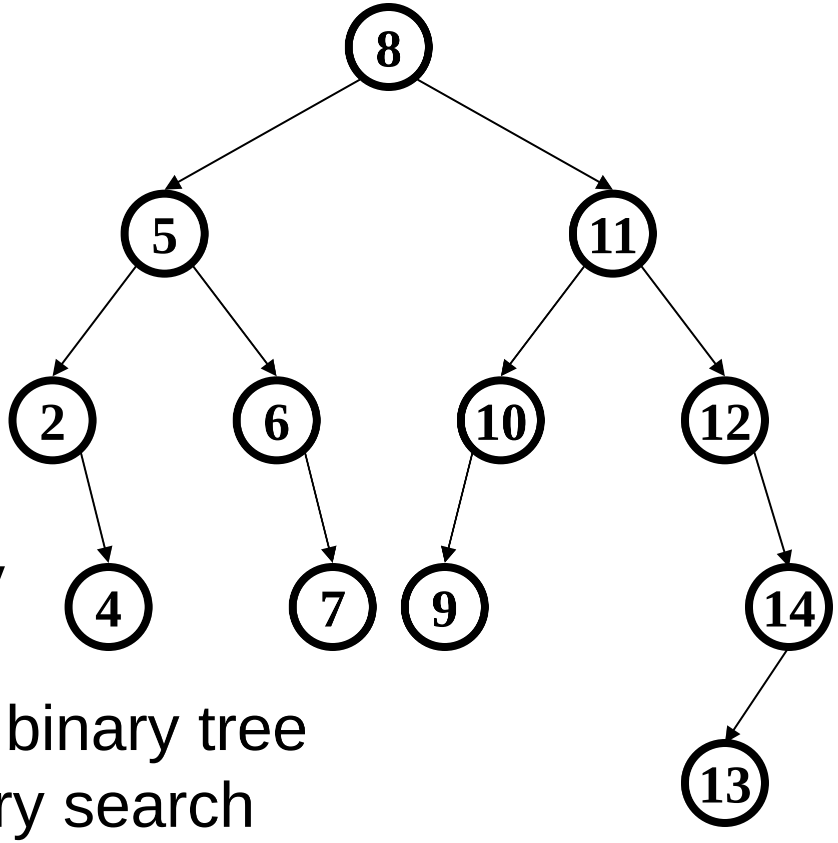
Sometimes order matters

- Example: evaluate an expression tree

Binary Search Tree (BST) Data Structure

- Structure property (binary tree)
  - Each node has ≤ 2 children
  - Result: keeps operations simple

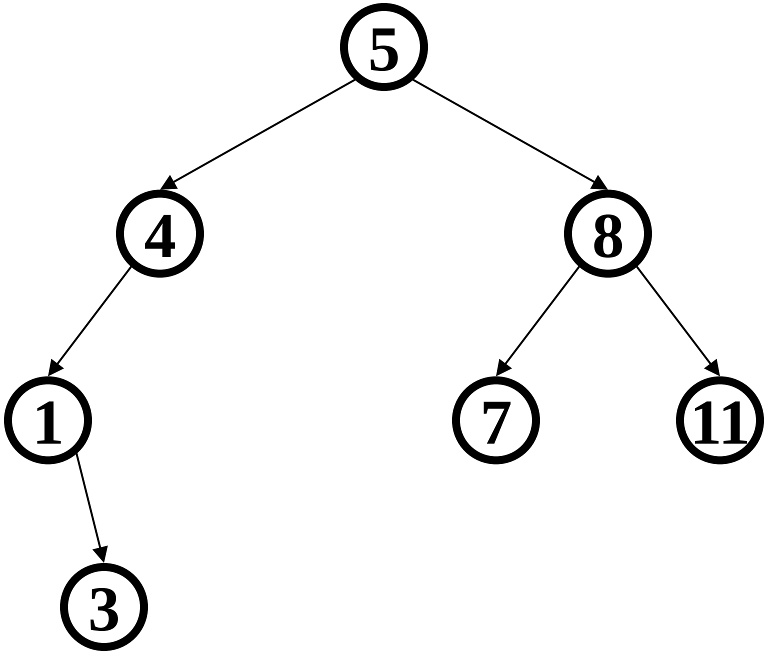
- Order property
  - All keys in left subtree smaller than node’s key
  - All keys in right subtree larger than node’s key
  - Result: easy to find any given key

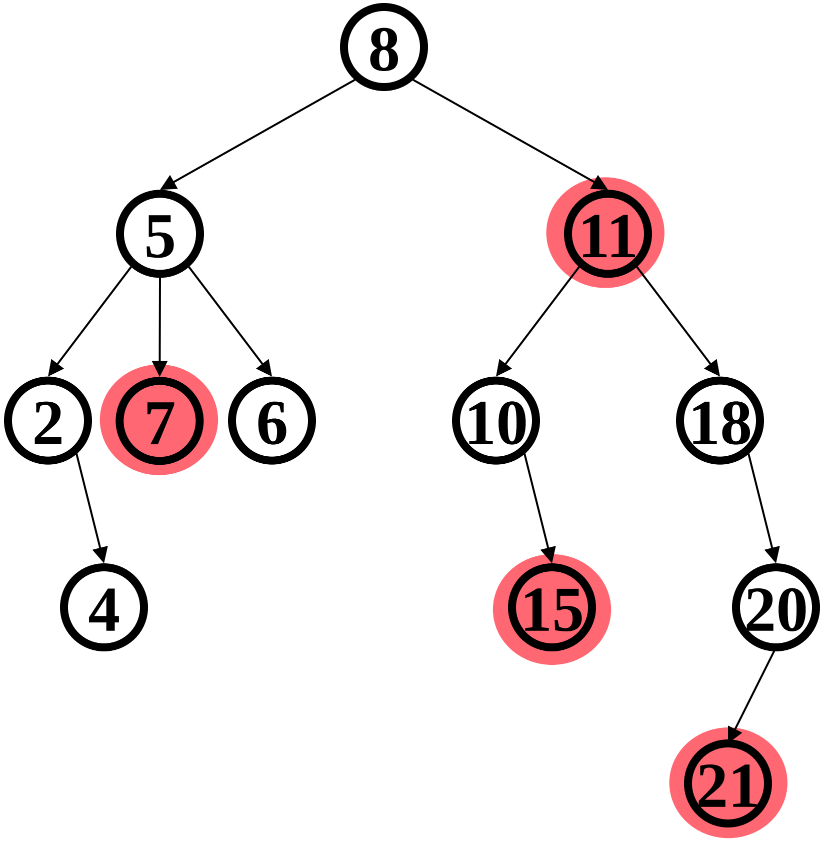


A binary search tree is a type of binary tree (but not all binary trees are binary search trees!)

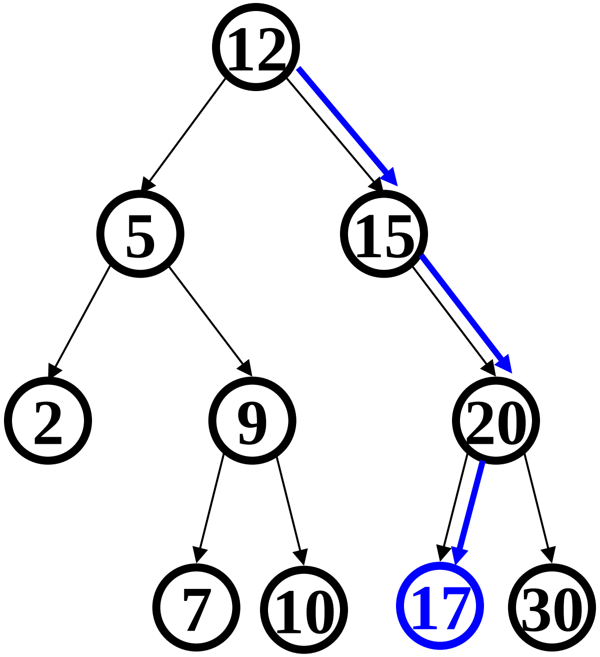
Are these BSTs?

Activity! What nodes violate the BST properties?





Find in BST, Recursive

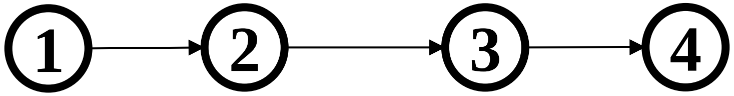


```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

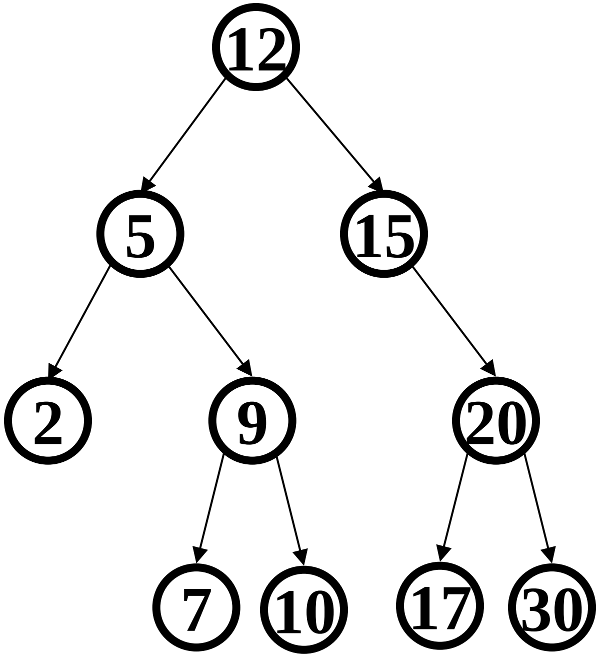
What is the time complexity? Worst case.

Worst case running time is O(n).

- Happens if the tree is very lopsided (e.g. list)



Find in BST, Iterative



Let's look for 16.

```
Data find(Key key, Node root){
 while(root != null
       && root.key != key) {
  if(key < root.key)
    root = root.left;
  else(key > root.key)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.data;
}
```

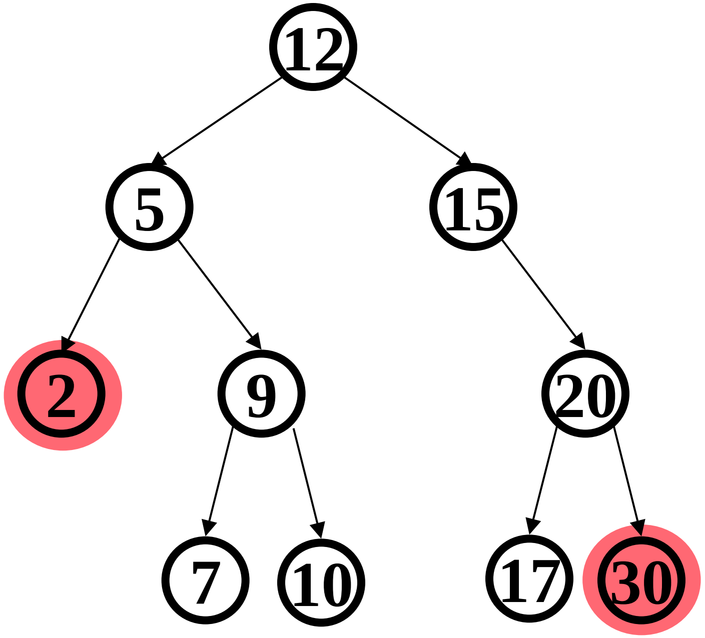
Worst case running time is O(n).

- Happens if the tree is very lopsided (e.g. list)

Bonus: Other BST "Finding" Operations

- **`FindMin`**: Find *minimum* node
  - Left-most node

- **`FindMax`**: Find *maximum* node
  - Right-most node



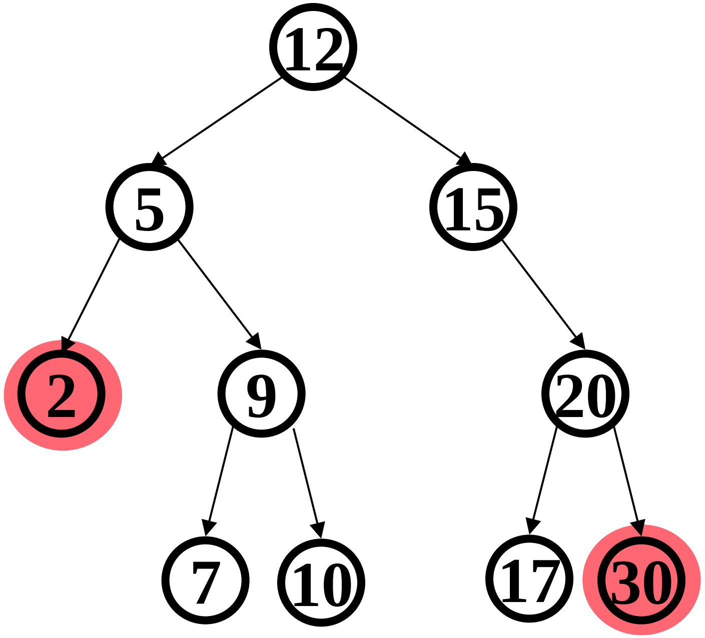
How would we implement?

Bonus: Other BST “Finding” Operations

- **`FindMin`**: Find *minimum* node
  - Left-most node

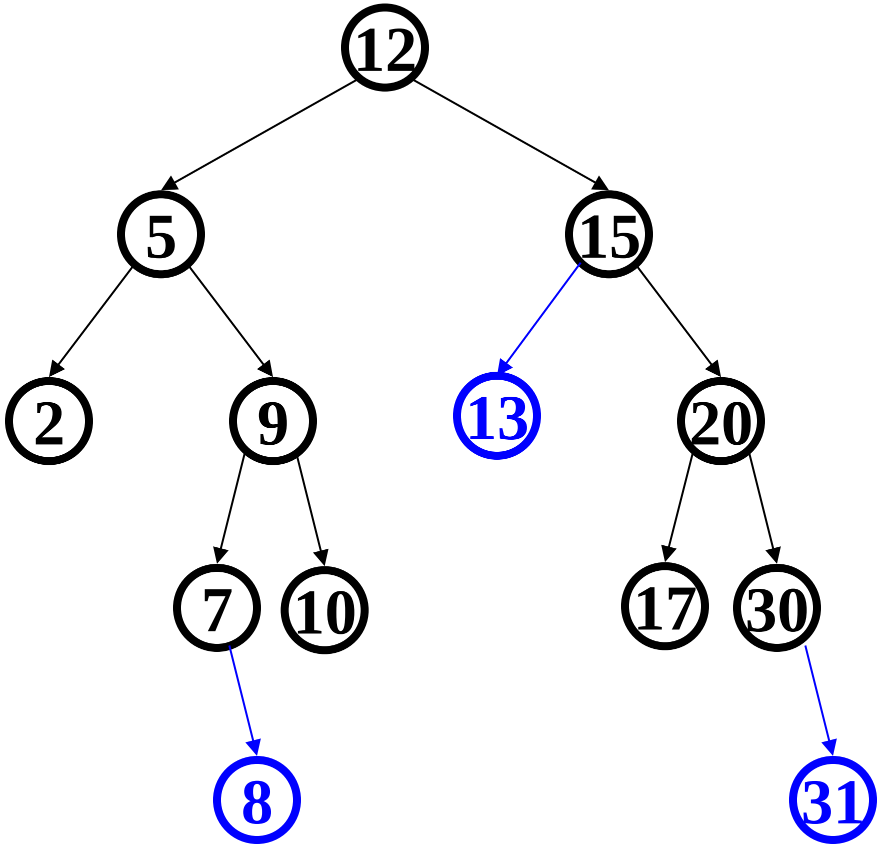
```
Node FindMin(Node root){
 if(root == null)
   return null;
 if(root.left==null)
  return root;
 return FindMin(root.left);
}
```

✓



Insert in BST

Find the right spot and hook on a new node.

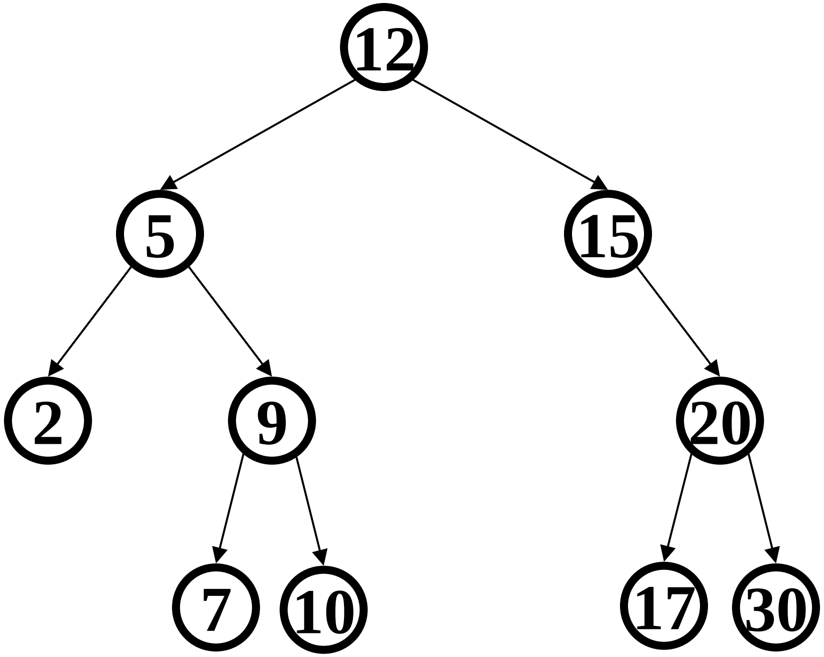


```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Again… worst case running time is O(n), which may happen if the tree is not balanced.

Deletion in BST



Why might deletion be harder than insertion?

Removing an item may disrupt the tree structure!

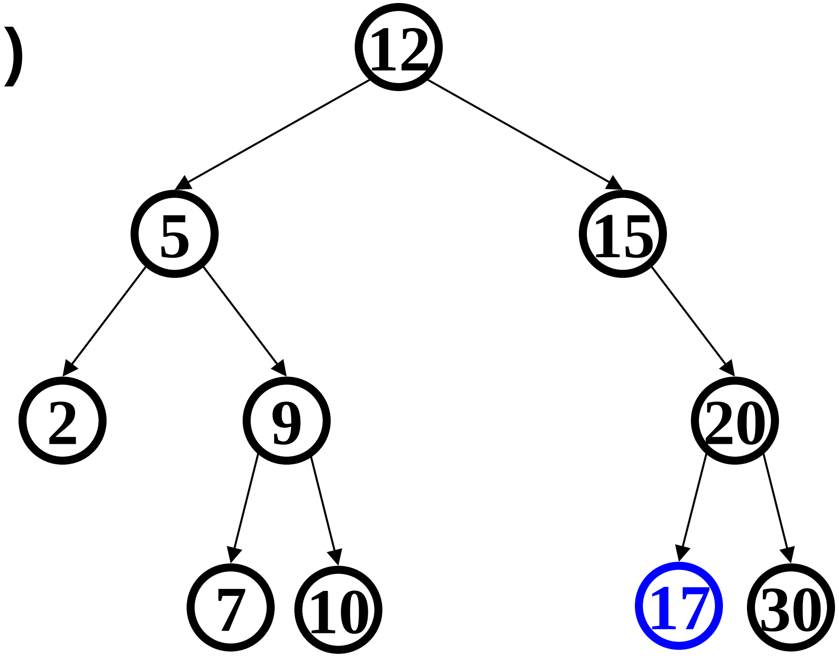
Deletion in BST

- Basic idea: `find` the node to be removed, then "fix" the tree so that it is still a binary search tree

- Three potential cases to fix:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

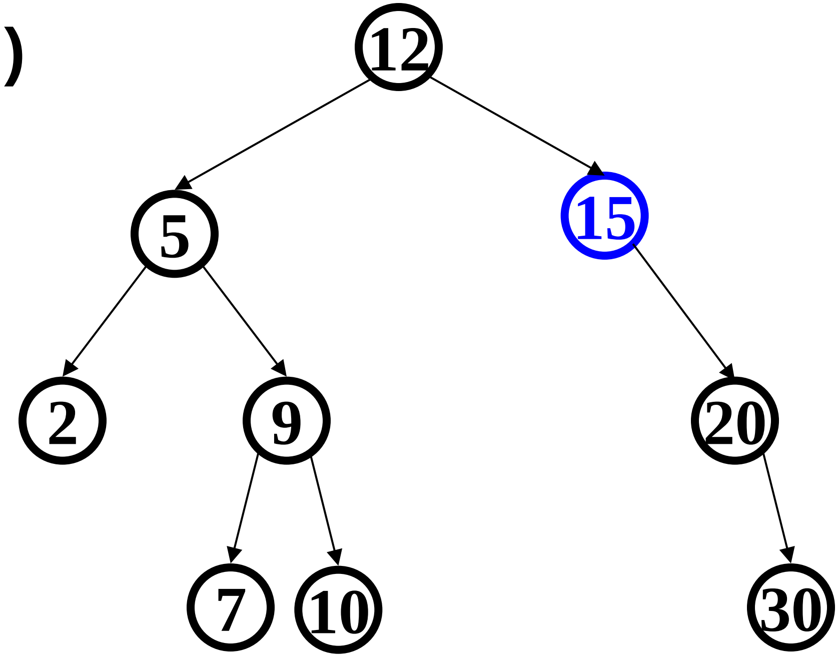
Deletion – The Leaf Case

`delete(17)`



Deletion – The One Child Case

`delete(15)`



Deletion – The One Child Case

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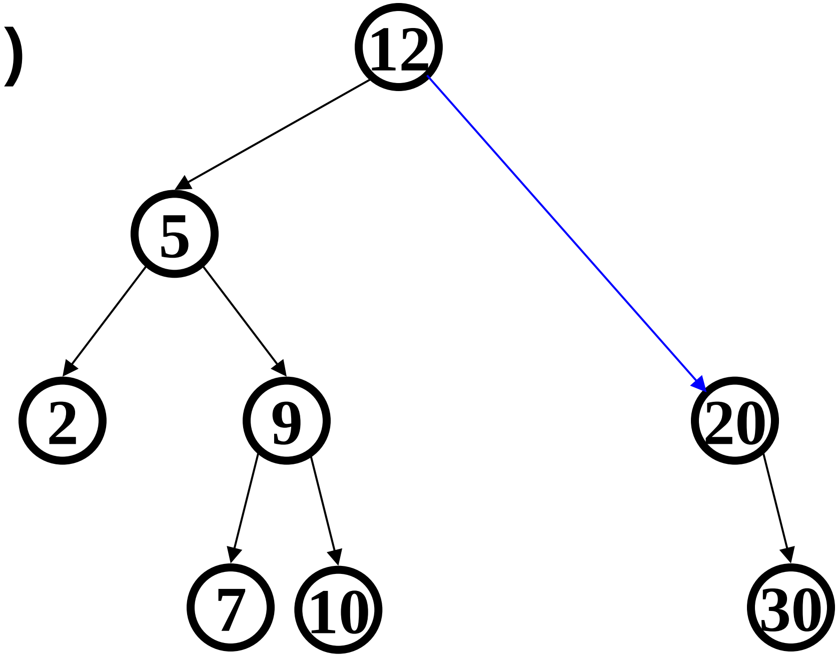
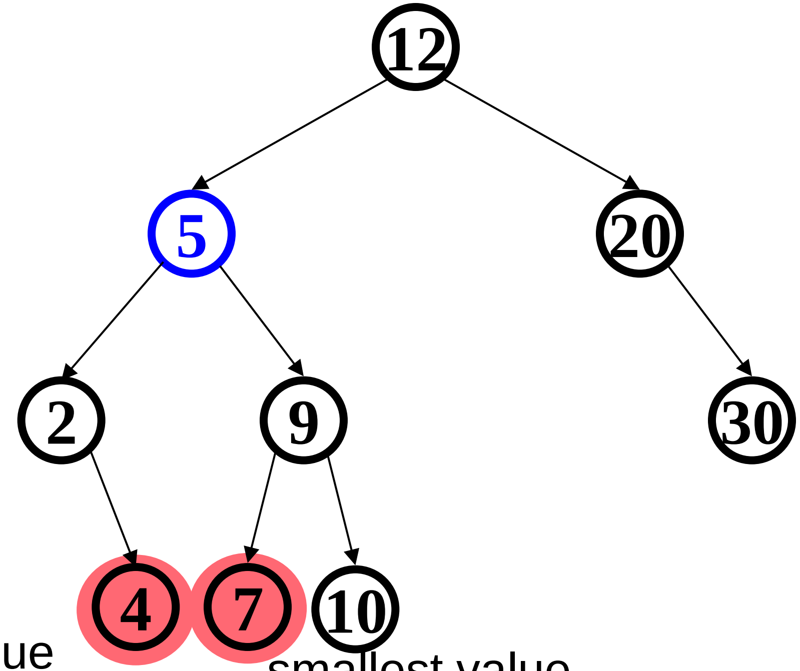


Deletion – The Two Child Case

`delete(5)`



largest value
on its left

smallest value
on its right

What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

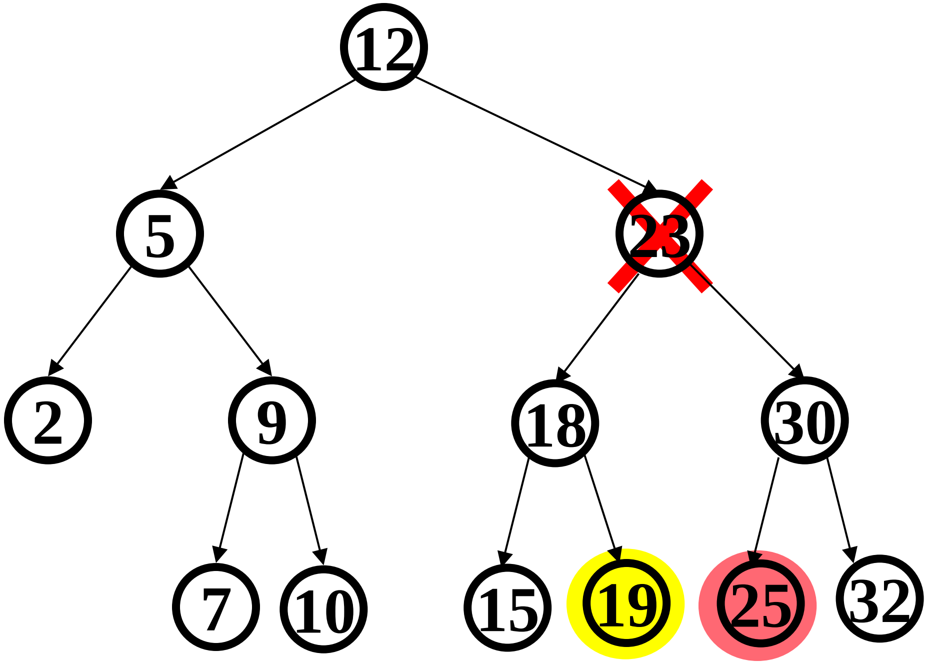
- *successor* minimum node from right subtree
  `findMin(node.right)` **`* the text does this`**

- *predecessor* maximum node from left subtree
  `findMax(node.left)`

Now delete the original node containing *successor* or *predecessor*

Deletion: The Two Child Case (example)

`delete(23)`



Deletion: The Two Child Case (example)

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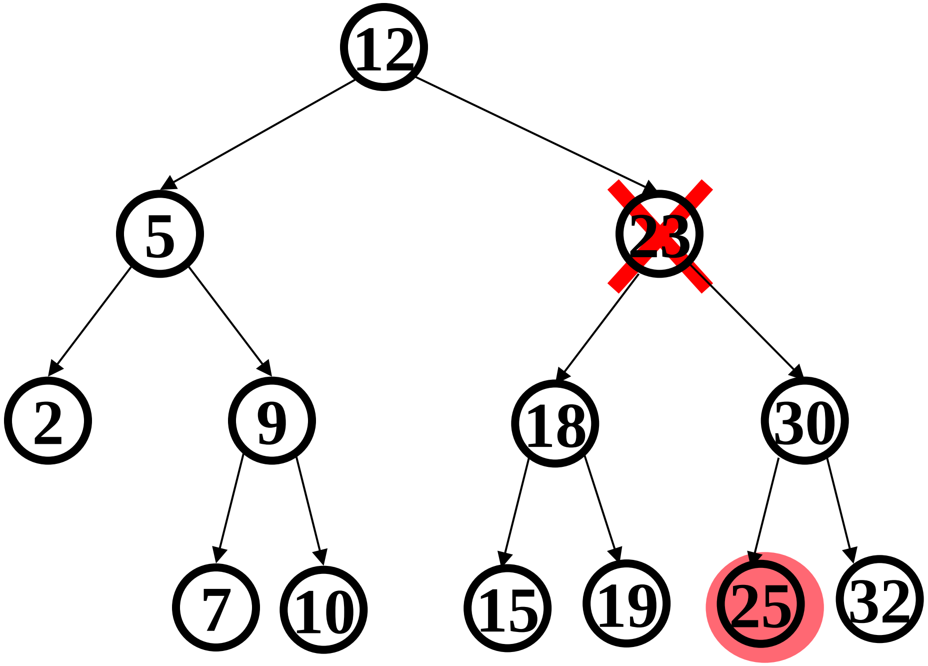
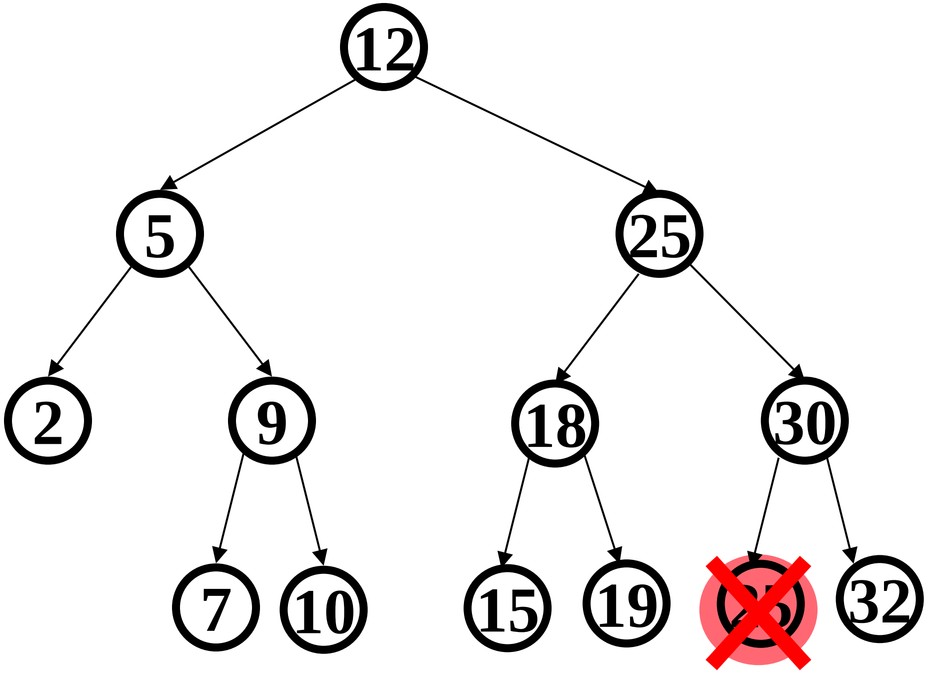


Deletion: The Two Child Case (example)

```
delete(23)
```



Deletion: The Two Child Case (example)

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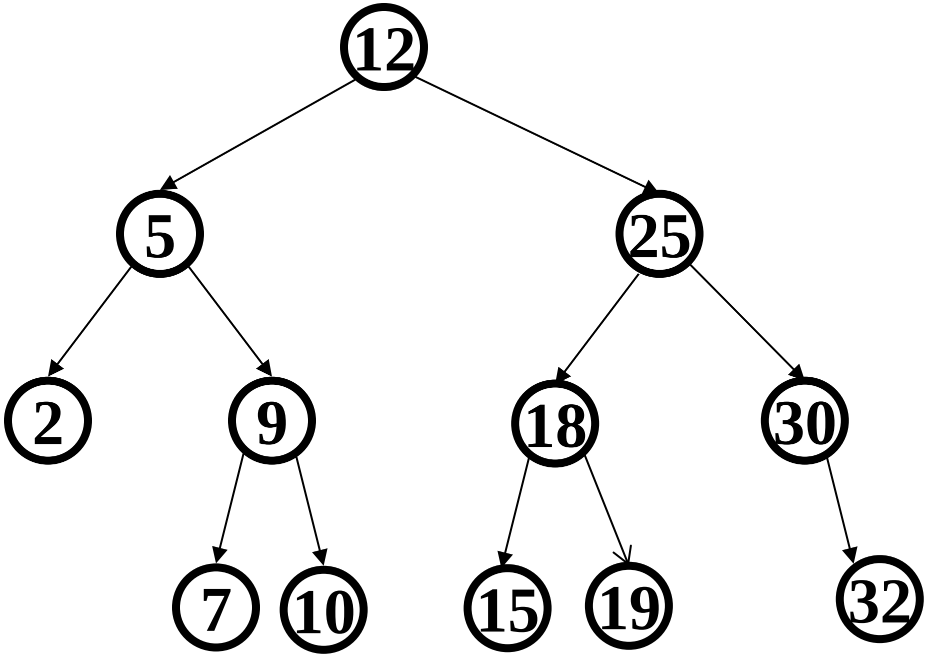
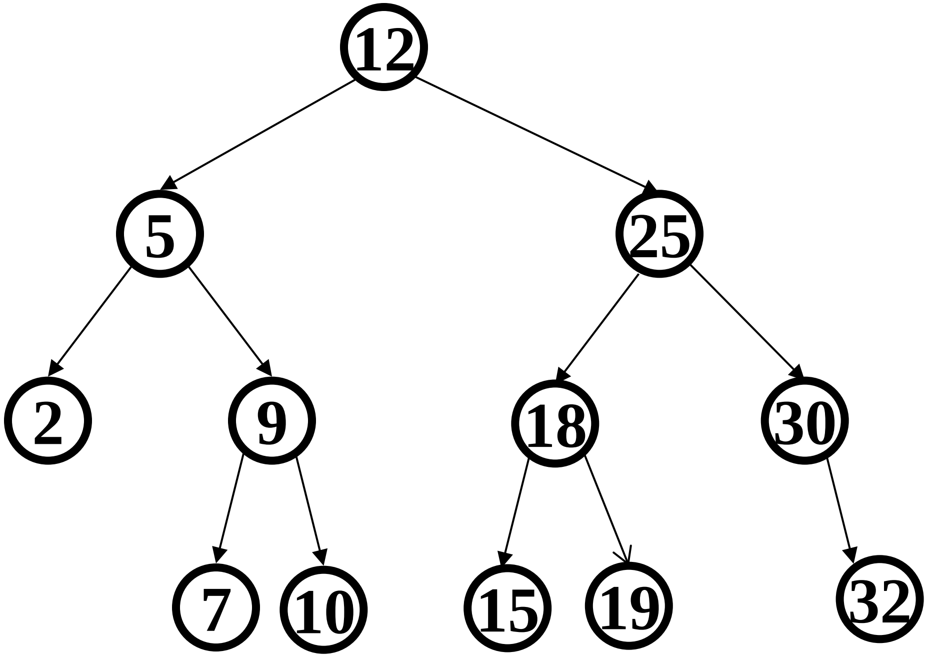


Success! ☺

Deletion: The Two Child Case (exercise)

`delete(12)`



Lazy Deletion

- Lazy deletion can work well for a BST
  - Simpler
  - Can do “real deletions” later as a batch
  - Some inserts can just “undelete” a tree node

- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - e.g., **`findMin`** and **`findMax`**?

BuildTree for BST

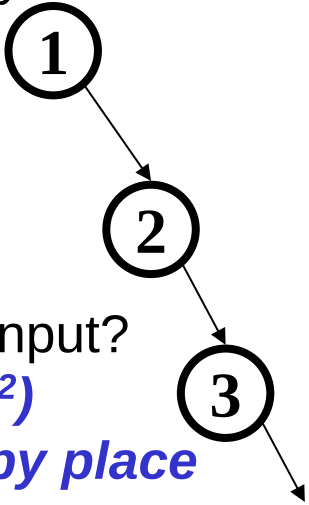
- Let's consider **`buildTree`**
  - Insert all, starting from an empty tree

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order,
    what is the tree?
  - What big-O runtime for this kind of sorted input?

    1 + 2 + 3 + . . . + n = n(n+1)/2

    O(n²)

    Not a happy place
  - Is inserting in the reverse order
    any better?



BuildTree for BST

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

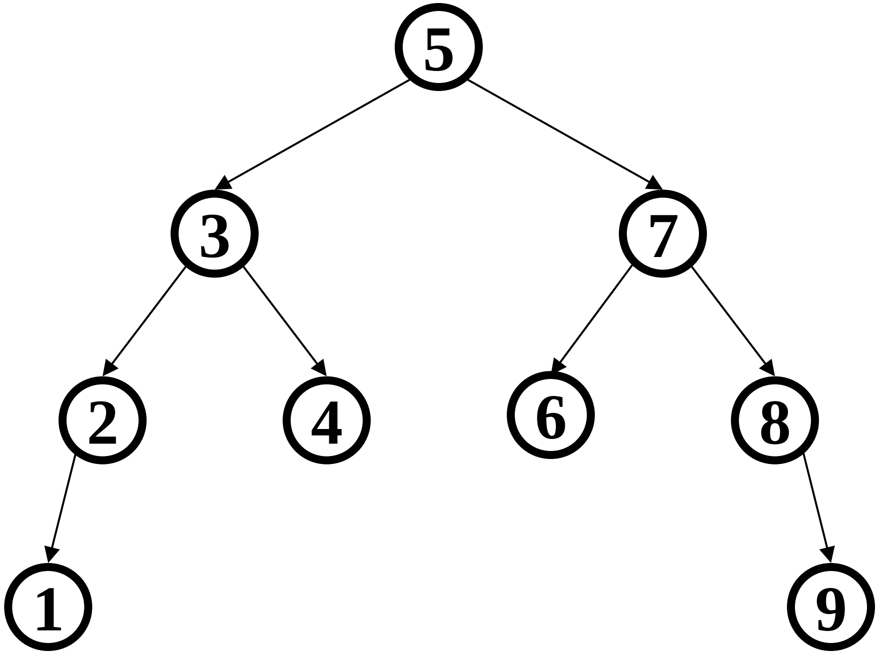
- What if we could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9

  - What tree does that give us?

  - What big-O runtime?

  O(n* log *n), definitely better

  - **So the order the values come in is important!**



Exercise

Build a binary search tree from the following ordered input.
If you get a duplicate, just ignore it as already there.

1. The month of your birthday.
2. The day of your birthday.
3. The number of siblings you have.
4. The number of courses you are taking.
5. Your age.
6. Your weight divided by 10 rounded down.
7. The rightmost digit of your social security number or student number.
8. The hour that your last class on Mondays ends.

What is the height of your tree?

Complexity of Building a Binary Search Tree

- Worst case: $O(n^2)$

- Best case: O(n log n)

- We do better by keeping the tree balanced.

- How are we going to do that?