CSE373: Data Structures & Algorithms

Lecture 5: Dictionary ADTs; Binary Trees

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Spring 2016

Today's Outline

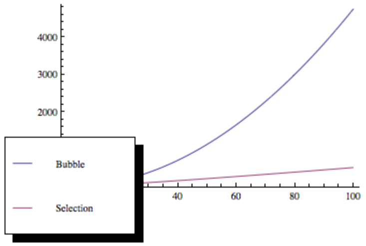
Announcements

- Homework 1 due TODAY at 11:59 pm ☺
- Homework 2 out (paper and pencil assignment)
  - Due in class Wednesday April 13 at the START of class

Today's Topics

- Finish Asymptotic Analysis
- Dictionary ADT (a.k.a. Map): associate keys with values
  – Extremely common
- Binary Trees

Summary of Asymptotic Analysis



Analysis can be about:

- The problem or the algorithm (usually algorithm)
- Time or space (usually time)
  - Or power or dollars or …
- Best-, worst-, or average-case (usually worst)
- Upper-, lower-, or tight-bound (usually upper)

g(n) ≤ f(n) O

g(n) ≥ f(n) Ω

both θ

- The most common thing we will do is give an O upper bound to the worst-case running time of an algorithm.

How to apply the definition easily

- Theory (mine)
- Let g(n) = c1*g1(n) + c2*g2(n) + ... + ck*gk(n) + c0
- Suppose the functions g1, g2, ... gk are already arranged in order with highest complexity at the far left.
- Select the LARGEST of the constants C = max(c1, c2, ..ck, c0)
- Then g(n) ≤ C*g1(n) + C*g2(n) + ... + C*gk(n) + C
- Or g(n) ≤ C(g1(n) + g2(n) + ... + gk(n) + 1)
- But g1(n) is bigger than g2(n) and all the others beyond some known n0.
- So g(n) ≤ C(g1(n) + g1(n) + ... +g1(n) + g1(n))
- Or g(n) ≤ C(k+1)*g1(n) = C'*g1(n) for all n greater than n0.

Example

$g(n) = 25\, n^4 + 30\, n^2 + 100\, n\log n + 54$

$g(n) \leq 100\, n^4 + 100n^2 + 100\, n\log n + 100$

$g(n) \leq 100(n^4 + n^2 + n\log n + 1)$

$g(n) \leq 100(n^4 + n^4 + n^4 + n^4)$

$g(n) \leq 100*4*n^4$

$g(n) \leq 400*n^4$ for all $n \geq 1$

Big-Oh Caveats

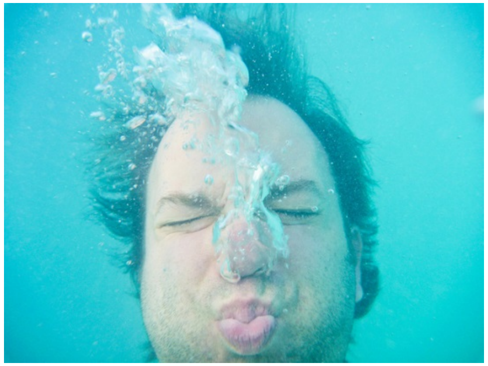
- Asymptotic complexity focuses on behavior for large n and is independent of any computer / coding trick
- But you can "abuse" it to be misled about trade-offs
- Example: $n^{1/10}$ vs. `log` n
  - Asymptotically $n^{1/10}$ grows more quickly
  - But the "cross-over" point is around $5 * 10^{17}$
  - So if you have input size less than 2^{58}, prefer $n^{1/10}$
- For *small n*, an algorithm with worse asymptotic complexity might be faster
  - If you care about performance for small n then the constant factors can matter

Addendum: Timing vs. Big-Oh Summary

- Big-oh is an essential part of computer science's mathematical foundation
  - Examine the algorithm itself, not the implementation
  - Reason about (even prove) performance as a function of *n*

- Timing also has its place
  - Compare implementations
  - Focus on data sets you care about (versus worst case)
  - Determine what the constant factors "really are"

Let's take a breath



- So far we've covered
  - Some simple ADTs: stacks, queues, lists
  - Some math (proof by induction)
  - How to analyze algorithms
  - Asymptotic notation (Big-Oh)

- Coming up….
  - Many more ADTs
    - Starting with dictionaries

The Dictionary (a.k.a. Map) ADT

- Data:
  - set of (key, value) pairs
  - keys must be comparable
- Operations:
  - **`insert(key,value)`**
  - **`find(key)`**
  - **`delete(key)`**
  - …

insert(ezgi, ….) →

find(bran) ←

Bran Hagger, …

- **ezgi**
  Ezgi Mercan
  OH: Thurs 10.30-11.30
  …
- **mert**
  Mert Sagalm
  OH: TTH 3.30-4.30
  …
- **bran**
  Bran Hagger
  OH: Mon 10.00-11.00
  …

*Will tend to emphasize the **keys**;*
don't forget about the stored values

A Modest Few Uses

key	attr1	attr2	attr3
k1	v11	v12	v13
k2	v21	v22	v23

Any time you want to store information according to some key and be able to retrieve it efficiently

- Lots of programs do that!

- Search: inverted indexes, phone directories, …
- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Biology: genome maps
- …

Possibly the most widely used ADT

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$*	$O(n)$	$O(n)$
• Unsorted array	$O(1)$*	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better

but not in the worst case (unless we keep it balanced)

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- May complicate other operations

Better dictionary data structures

There are many good data structures for (large) dictionaries

1. Binary trees
2. AVL trees
   - Binary search trees with *guaranteed balancing*
3. B-Trees
   - Also always balanced, but different and shallower
   - B-Trees are not the same as Binary Trees
     - B-Trees generally have large branching factor
4. Hash Tables
   - Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

Tree terms (review?)

Root **(tree)**

Leaves **(tree)**

Children **(node)**

Parent **(node)**

Siblings **(node)**

Ancestors **(node)**

Descendents **(node)**

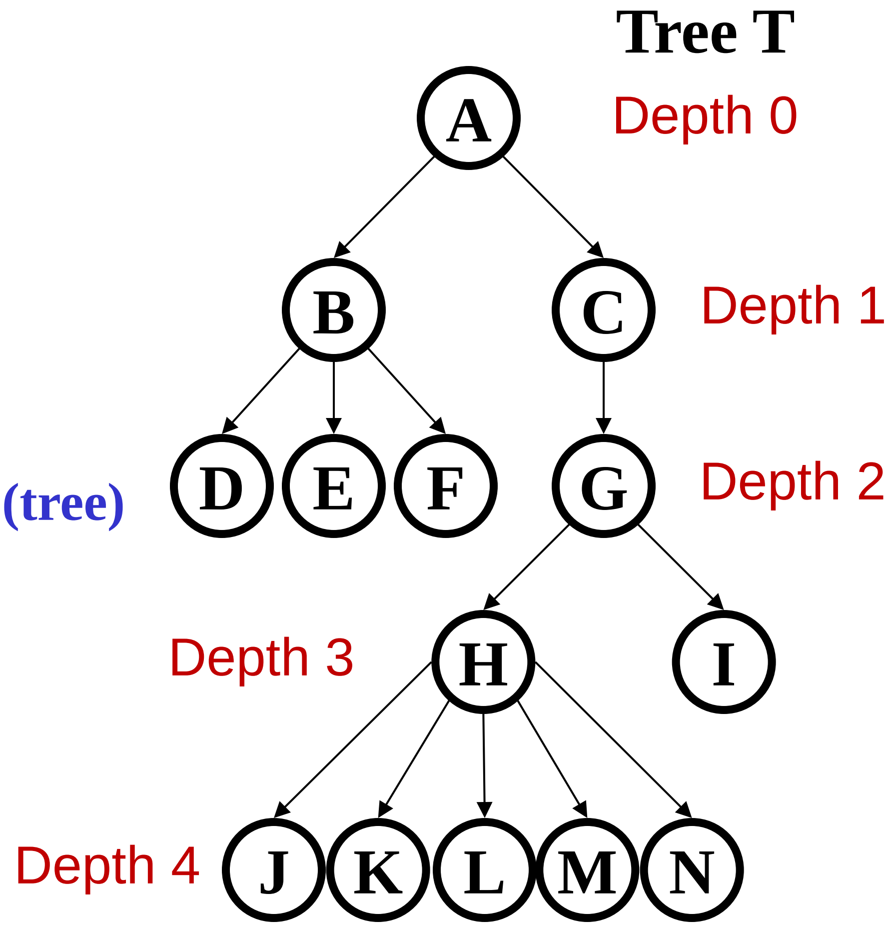
Subtree **(node)**

Depth **(node)**

Height **(tree)** 4

Degree **(node)**

Branching factor **(tree)**



More tree terms

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of "next" as the one child)

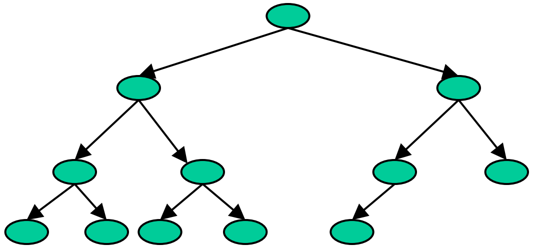
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree

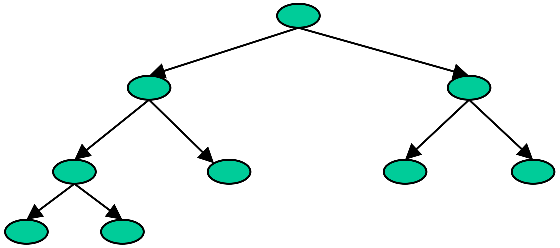
- A tree can be balanced or not
  - A balanced tree with n nodes has a height of $O(\mathbf{log}\ n)$
  - Different tree data structures have different "balance conditions" to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right





What is the height of a perfect binary tree with n nodes? $\lfloor \log_2 n \rfloor$

A complete binary tree?

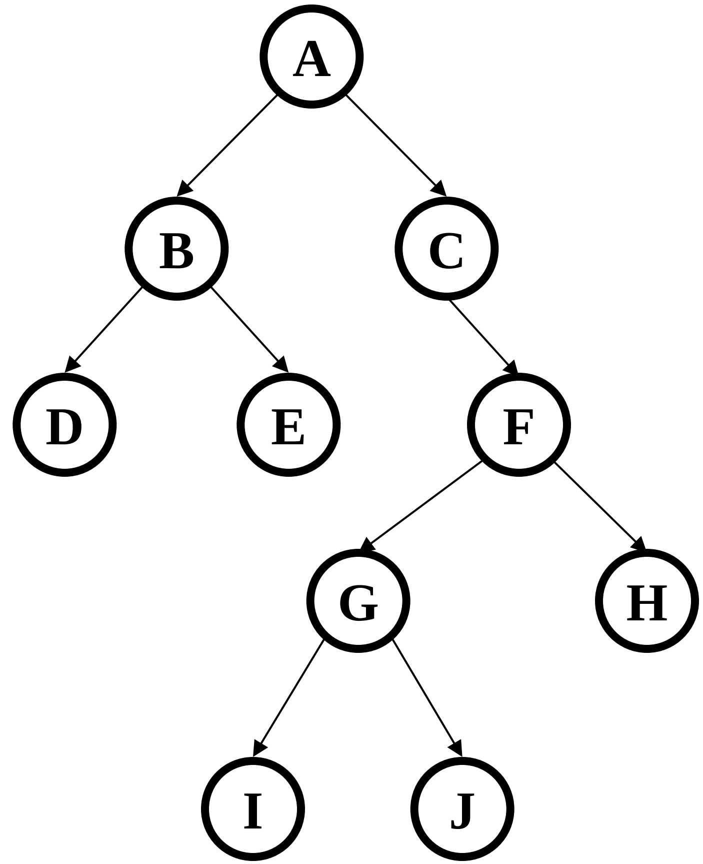
Binary Trees

- Binary tree: Each node has at most 2 children (branching factor 2)

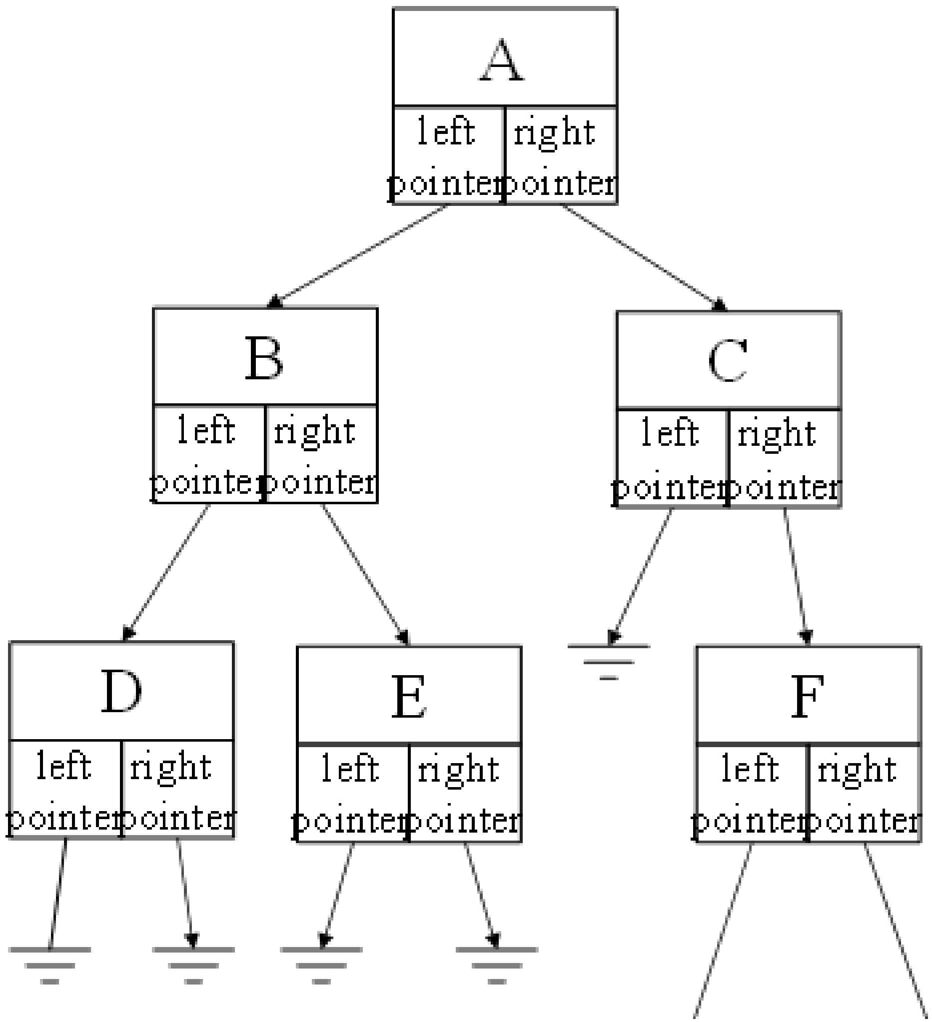
- Binary tree is
  - A root *(with data)*
  - A left subtree that's a binary tree
  - A right subtree that's a binary tree
- *These subtrees may be empty.*
- Representation:

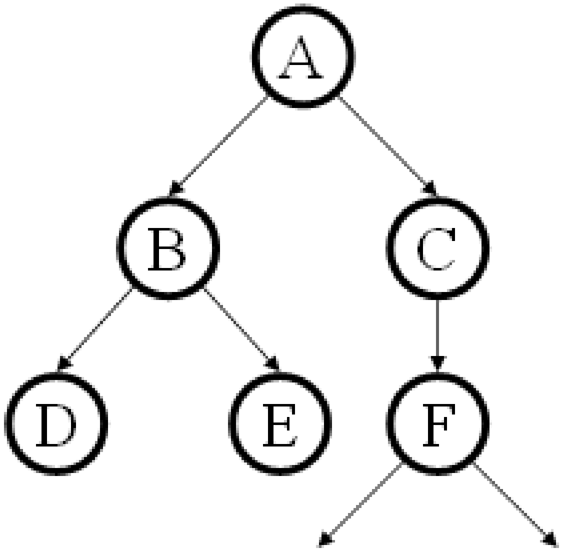
Data	
left pointer	right pointer

- For a dictionary, data will include a key and a value



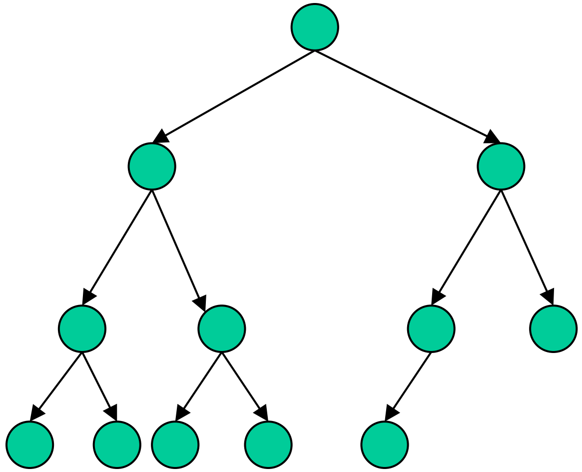
Binary Tree Representation





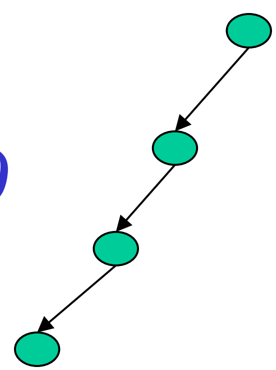
Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)



For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{(h + 1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$



For n nodes, we cannot do better than O(`log` n) height and we want to avoid O(n) height

Calculating height

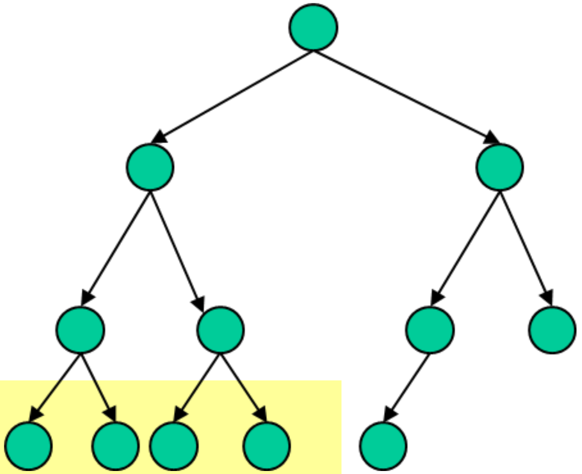
What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

            ???


}
```

Calculating height



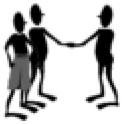
What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals



A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree

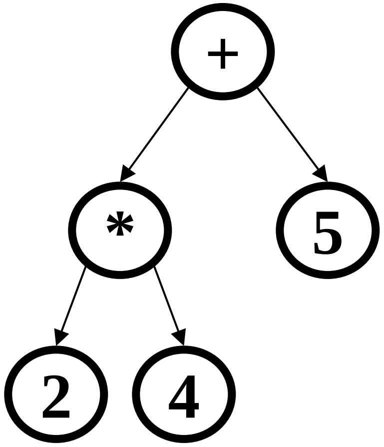
  + * 2 4 5

- *In-order*: left subtree, root, right subtree

  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

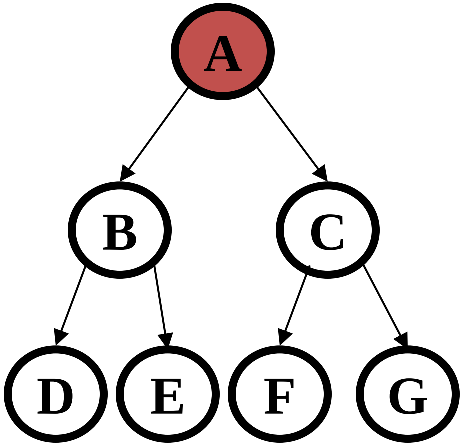
  2 4 * 5 +



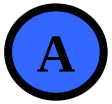
(an expression tree)

More on traversals

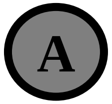
```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



= current node



= processing (on the call stack)



= completed node

✓= element has been processed

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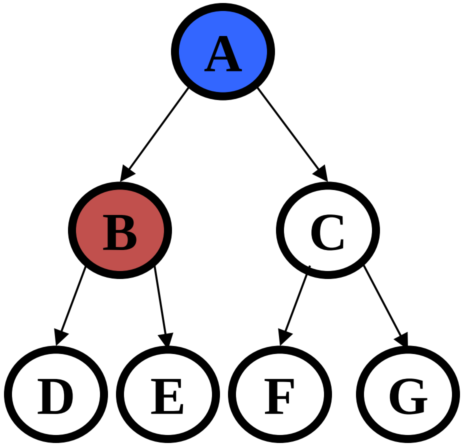

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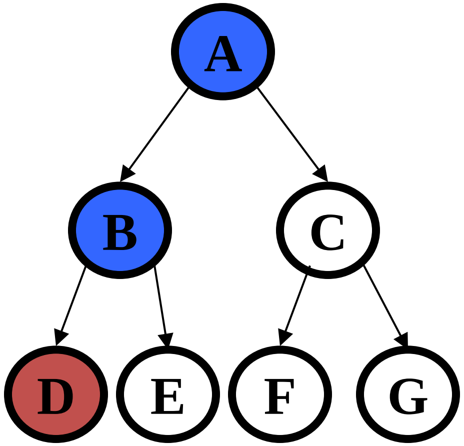

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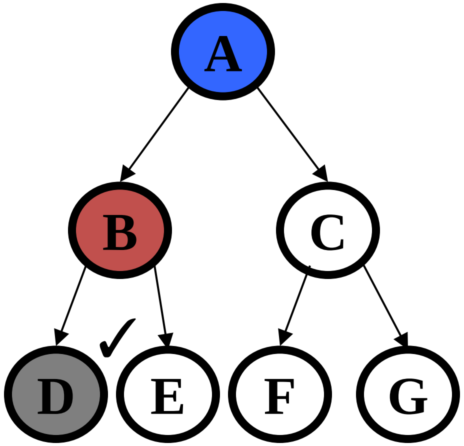

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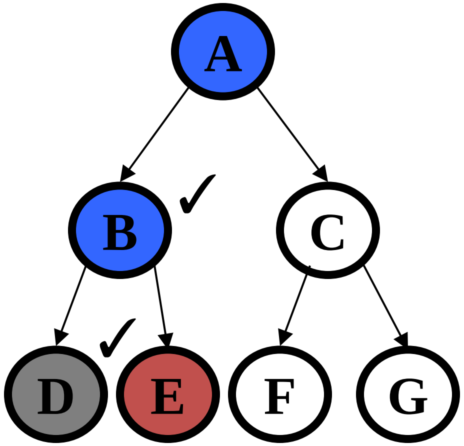

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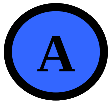

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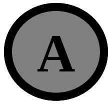

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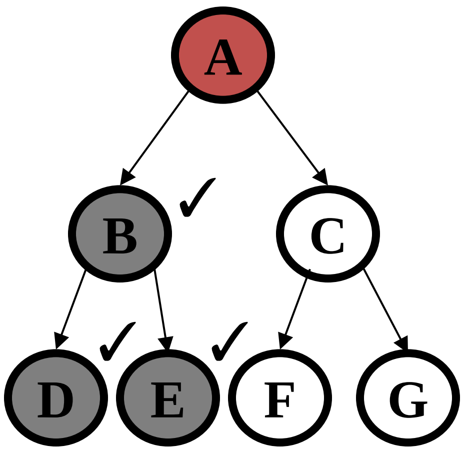

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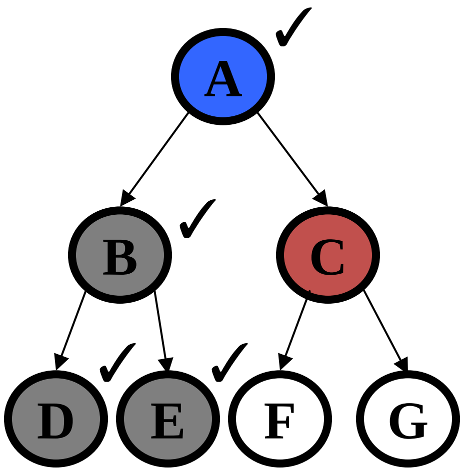

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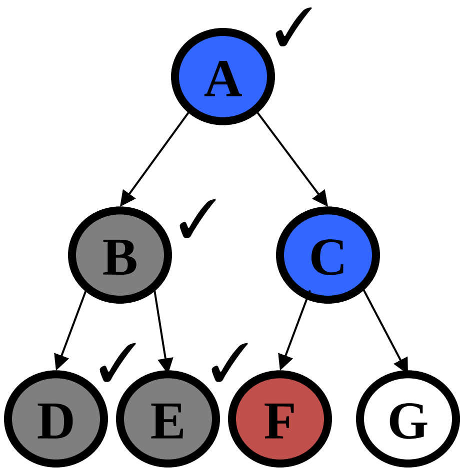

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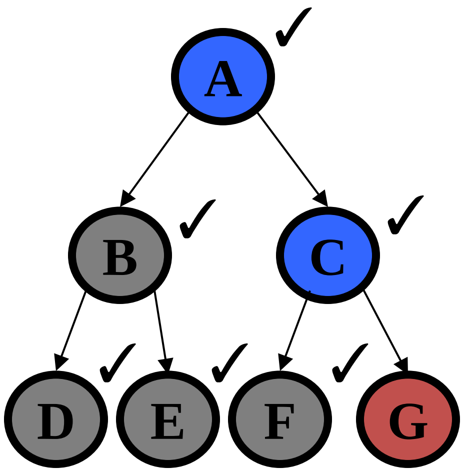

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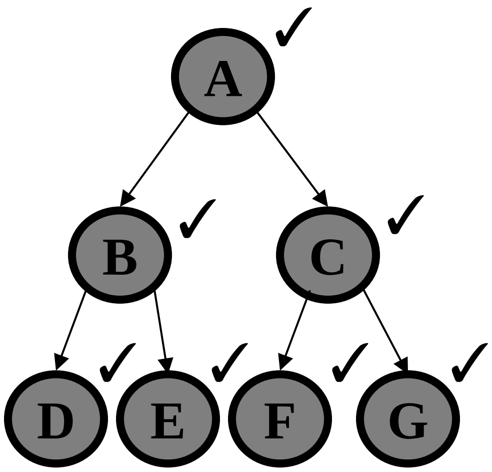
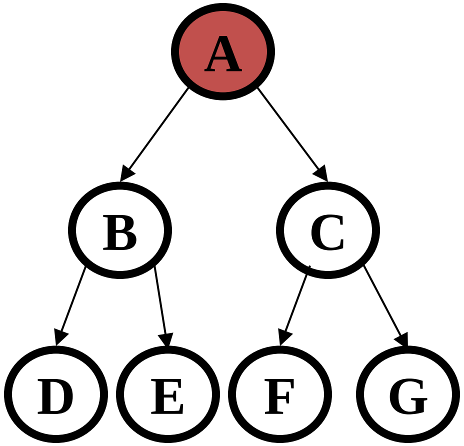


A = current node A = processing (on the call stack)

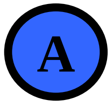
A = completed node ✓= element has been processed

More on traversals

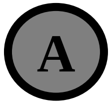
```
void preOrderTraversal(Node t){
  if(t != null) {
    process(t.element);
    preOrderTraversal(t.left);
    preOrderTraversal(t.left)
  }
}
```



= current node



= processing (on the call stack)



= completed node

✓= element has been processed

Preorder Exercise