



CSE373: Data Structure & Algorithms

Lecture 21: Comparison Sorting

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Announcements

Why Study Sorting in this Class?

- Unlikely you will ever need to reimplement a sorting algorithm yourself
 - Standard libraries will generally implement one or more (Java implements 2)
- You will almost certainly use sorting algorithms
 - Important to understand relative merits and expected performance
- Excellent set of algorithms for practicing analysis and comparing design techniques
 - Classic part of a data structures class, so you'll be expected to know it

The main problem, stated carefully

For now, assume we have n comparable elements in an array and we want to rearrange them to be in increasing order

Input:

- An array \mathbf{A} of data records
- A key value in each data record
- A comparison function (consistent and total)

85 | 96 | 3 | 18 | 15 | 25

3 | 15 | 18 | 25 | 85 | 96



Effect:

- Reorganize the elements of \mathbf{A} such that for any i and j , if $i < j$ then $\mathbf{A}[i] \leq \mathbf{A}[j]$
- (Also, \mathbf{A} must have exactly the same data it started with)
- Could also sort in reverse order, of course

An algorithm doing this is a **comparison sort**

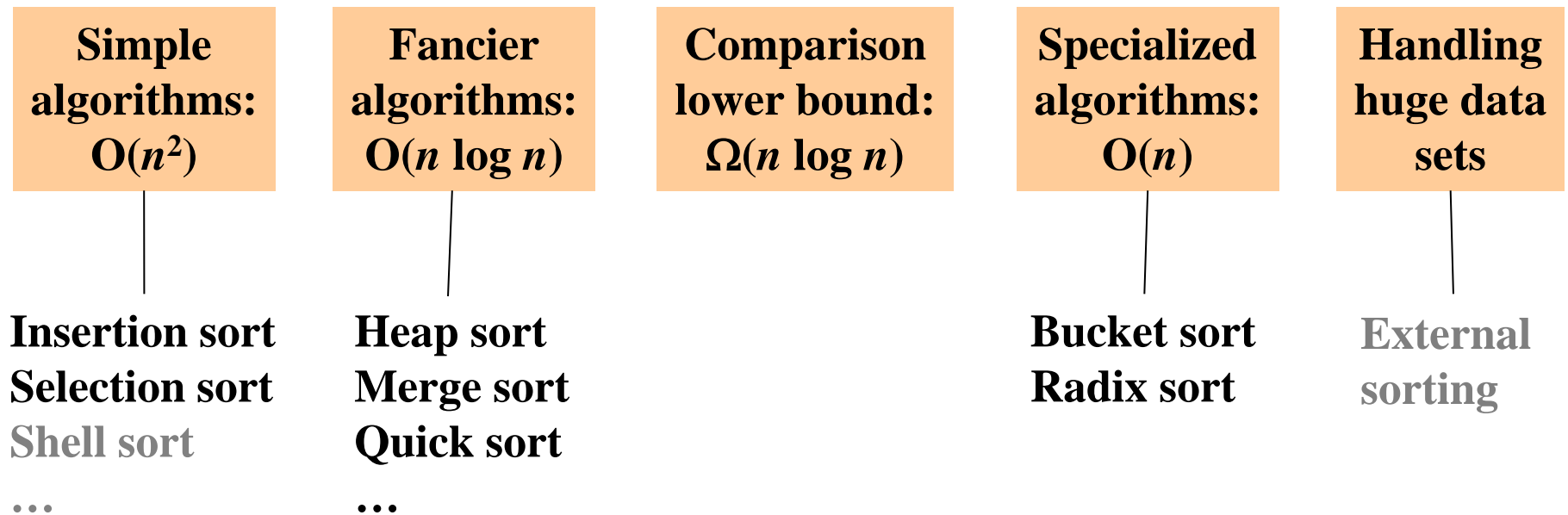
Variations on the Basic Problem

1. Maybe elements are in a **linked list** (could convert to array and back in linear time, but some algorithms needn't do so)
2. Maybe **ties** need to be resolved by “original array position”
 - Sorts that do this naturally are called **stable sorts**
 - Others could tag each item with its original position and adjust comparisons accordingly (non-trivial constant factors)
3. Maybe we must not use more than $O(1)$ “**auxiliary space**”
 - Sorts meeting this requirement are called **in-place sorts**
4. Maybe we can do more with elements than just compare
 - Sometimes leads to **faster** algorithms
5. Maybe we have **too much data** to fit in memory
 - Use an “**external sorting**” algorithm



Sorting: The Big Picture

Surprising amount of neat stuff to say about sorting:



Insertion Sort

- Idea: At step k , put the k^{th} element in the correct position among the first k elements
- Alternate way of saying this:
 - Sort first two elements
 - Now insert 3rd element in order
 - Now insert 4th element in order
 - ...
- “Loop invariant”: when loop index is i , first i elements are sorted
- Let’s see a visualization (<http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html>)
- Time?
Best-case _____ Worst-case _____ “Average” case _____

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- Time?
 - Best-case $O(n)$ Worst-case $O(n^2)$ “Average” case $O(n^2)$
start sorted start reverse sorted (see text)

Try one

9 | 5 | 4 | 1

5 | 9 | 4 | 1

4 | 5 | 9 | 1

1 | 4 | 5 | 9

Swap the first two elements.

Insert the 4 into the sorted part.

Insert the 1 into the sorted part.

Selection sort

- Idea: At step k , find the smallest element among the not-yet-sorted elements and put it at position k
- Alternate way of saying this:
 - Find smallest element, put it 1st
 - Find next smallest element, put it 2nd
 - Find next smallest element, put it 3rd ...
- “Loop invariant”: when loop index is i , first i elements are the i smallest elements in sorted order
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- Time?
 - Best-case $O(n^2)$ Worst-case $O(n^2)$ “Average” case $O(n^2)$
 - Always* $T(1) = 1$ and $T(n) = n + T(n-1)$

Try one

9 | 5 | 4 | 1

1 | 9 | 5 | 4

1 | 4 | 9 | 5

1 | 4 | 5 | 9

1 | 4 | 5 | 9

Find smallest element and put it first.

Find next smallest element and put it second.

Find next smallest element and put it third.

Insertion Sort vs. Selection Sort

- Different algorithms
- Solve the same problem
- Have the same worst-case and average-case asymptotic complexity
 - Insertion-sort has better best-case complexity; preferable when input is “mostly sorted”
- Other algorithms are more efficient for *large* arrays that are *not already almost sorted*
 - Insertion sort may do well on small arrays

Aside: We Will Not Cover Bubble Sort

- It is usually taught in introductory courses
- It doesn't have good asymptotic complexity: $O(n^2)$
- It's not particularly efficient with respect to constant factors

Basically, almost everything it is good at some other algorithm is at least as good at

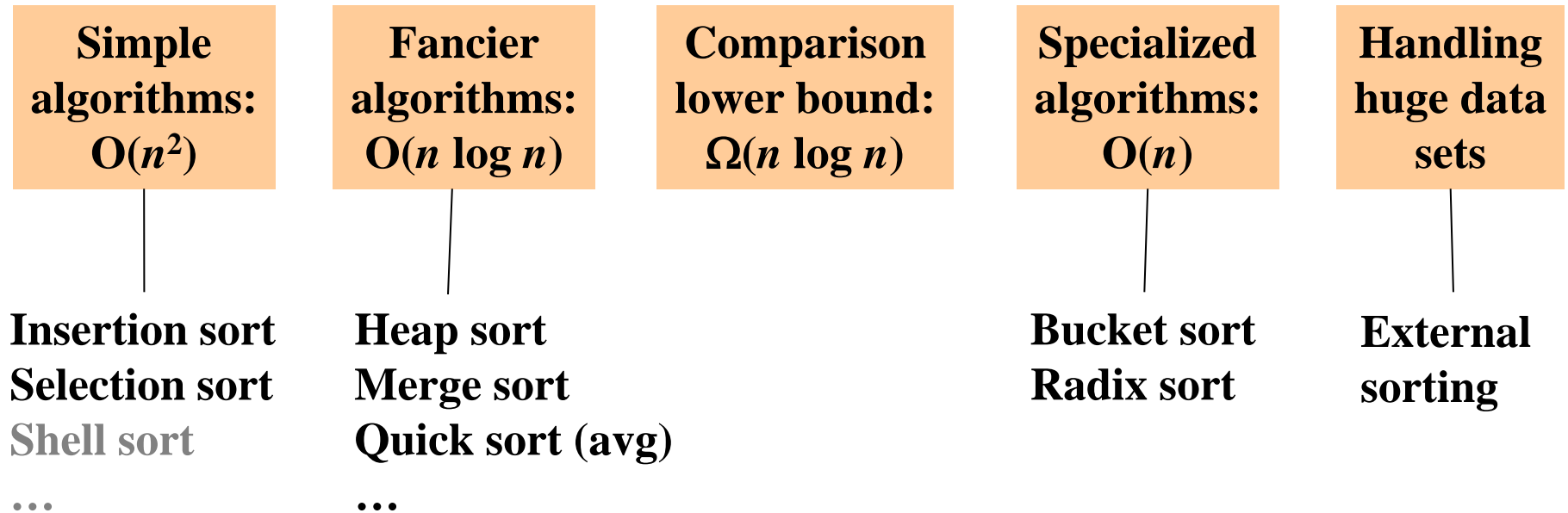
- Perhaps people teach it just because someone taught it to them?

Fun, short, optional read:

Bubble Sort: An Archaeological Algorithmic Analysis, Owen Astrachan, SIGCSE 2003, <http://www.cs.duke.edu/~ola/bubble/bubble.pdf>

The Big Picture

Surprising amount of juicy computer science: 2-3 lectures...



Heap sort

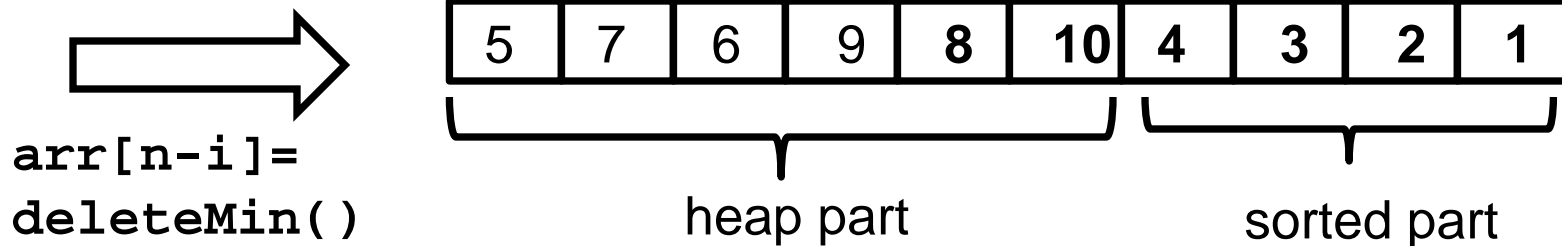
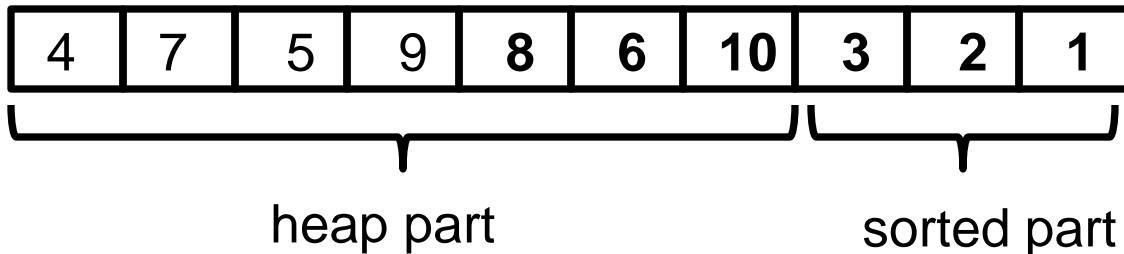
- Sorting with a heap is easy:
 - insert each `arr[i]` into a separate Heap, or better yet use **buildHeap**
 - `for(i=0; i < arr.length; i++)`
 `arr[i] = deleteMin();`
- Worst-case running time: $O(n \log n)$
- We have the array-to-sort (`arr`) and the heap (Heap)
 - So this is not an in-place sort
 - There's a trick to make it in-place...

In-place heap sort

But this reverse sorts –
how would you fix that?

Max Heap

- Treat the initial array as a heap (via `buildHeap`)
- When you delete the i^{th} element, put it at `arr[n-i]`
 - That array location isn't needed for the heap anymore!



Let's try it with a max heap

8 | 5 | 6 | 2 | 1

delete the 8 and move to end

6 | 5 | 2 | 1 | 8

delete the 6 and move to end-1

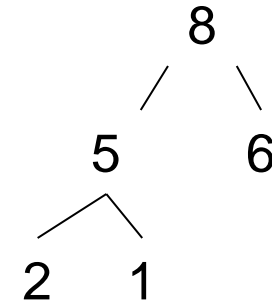
5 | 2 | 1 | 6 | 8

delete the 5 and move to end-2

2 | 1 | 5 | 6 | 8

delete the 2 and move to end-3

1 | 2 | 5 | 6 | 8



Divide and conquer

Very important technique in algorithm design

1. Divide problem into smaller parts
2. Independently solve the simpler parts
 - Think recursion
 - Or potential parallelism
3. Combine solution of parts to produce overall solution

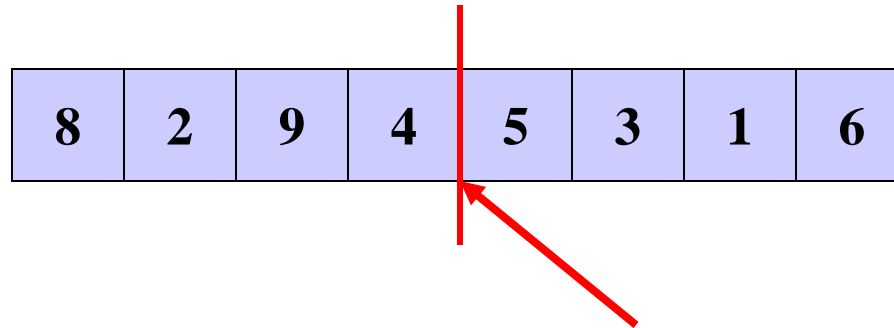
(This technique has a *long* history.)

Divide-and-Conquer Sorting

Two great sorting methods are fundamentally divide-and-conquer

- 1. Mergesort:** Sort the left half of the elements (recursively)
Sort the right half of the elements (recursively)
Merge the two sorted halves into a sorted whole
- 2. Quicksort:** Pick a “pivot” element
Divide elements into less-than pivot
and greater-than pivot
Sort the two divisions (recursively on each)
Answer is
sorted-less-than **then** pivot **then** sorted-greater-than

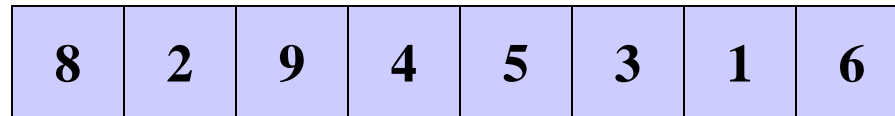
Merge sort



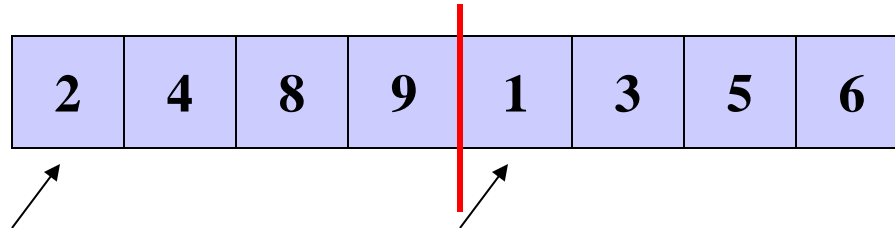
- To sort array from position lo to position hi :
 - If range is 1 element long, it is already sorted! (Base case)
 - Else:
 - Sort from lo to $(hi+lo)/2$
 - Sort from $(hi+lo)/2$ to hi
 - Merge the two halves together
- Merging takes two sorted parts and sorts everything
 - $O(n)$ but requires auxiliary space...

Example, focus on merging

Start with:

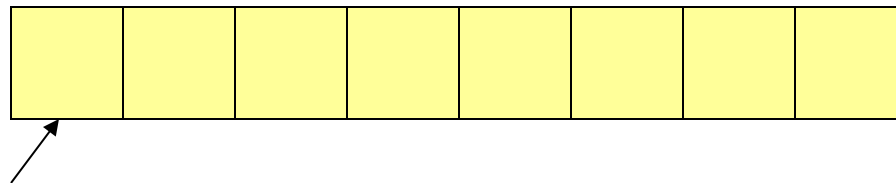


After recursion:
(not magic 😊)



Merge:

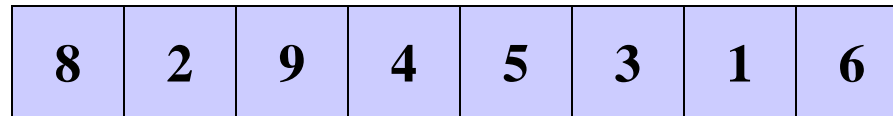
Use 3 pointers
and 1 more array



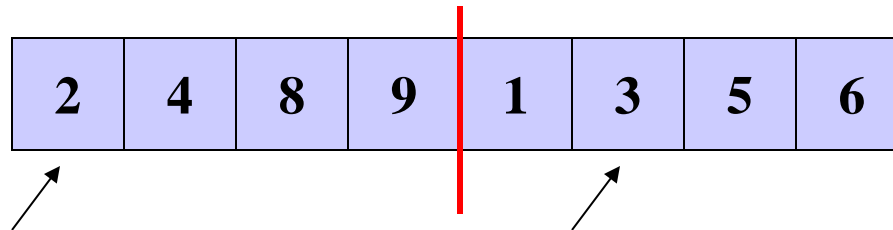
(After merge,
copy back to
original array)

Example, focus on merging

Start with:

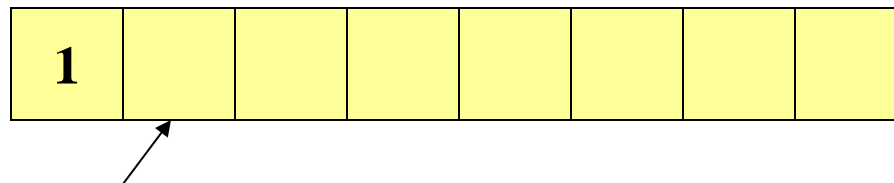


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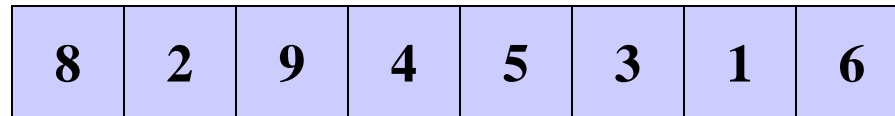
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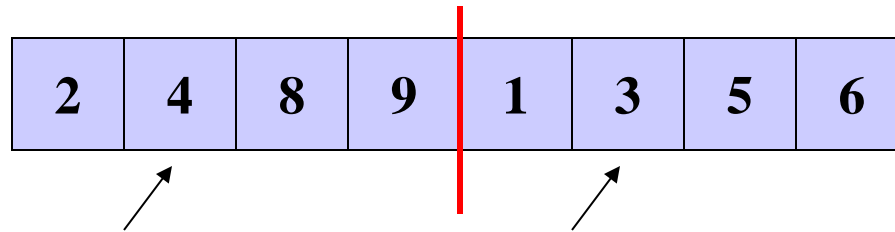
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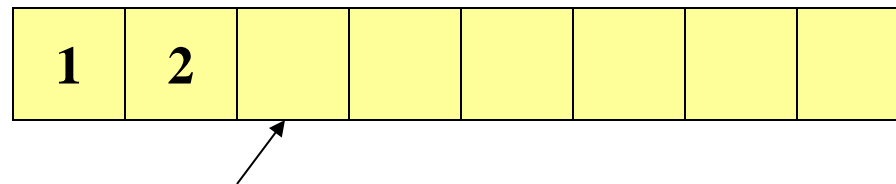


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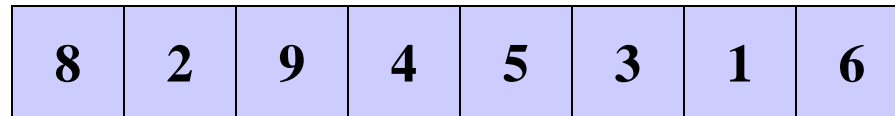
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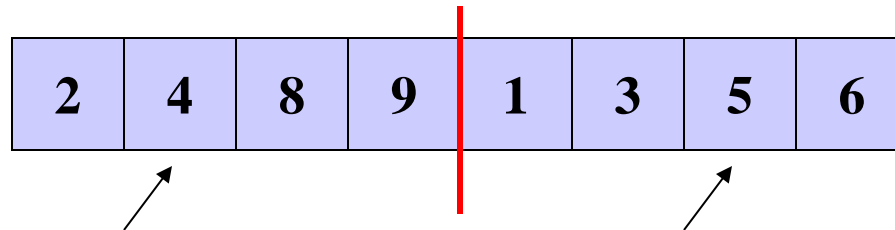
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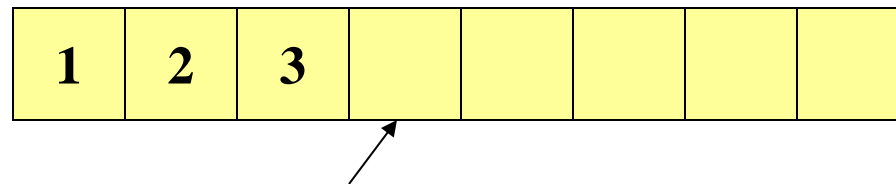


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Merge:

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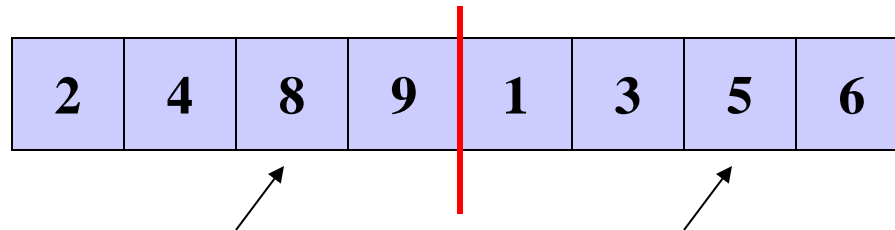
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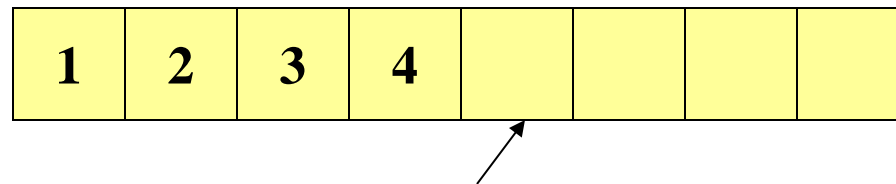


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Merge:

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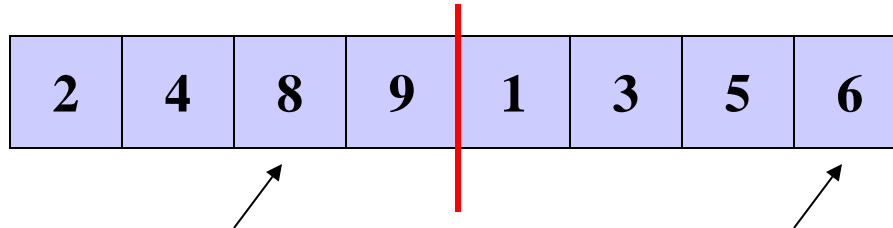
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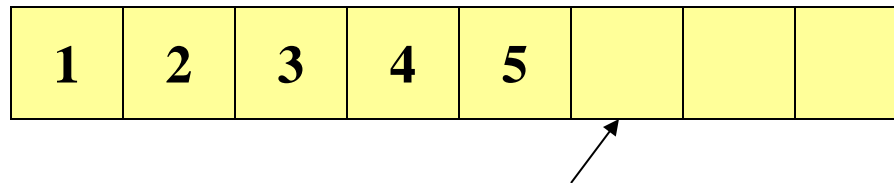


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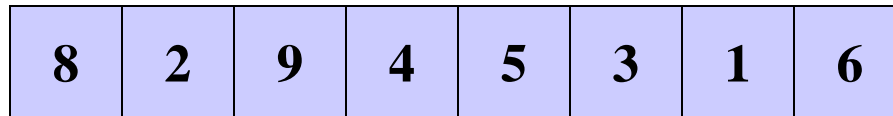
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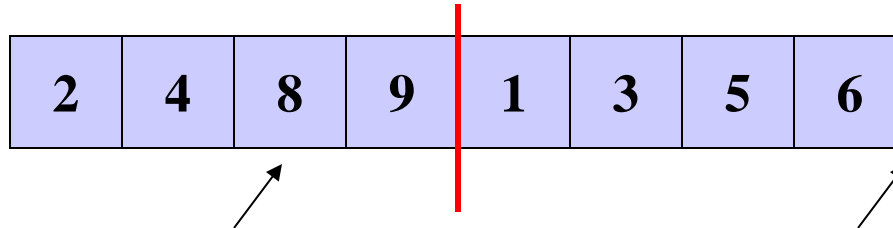
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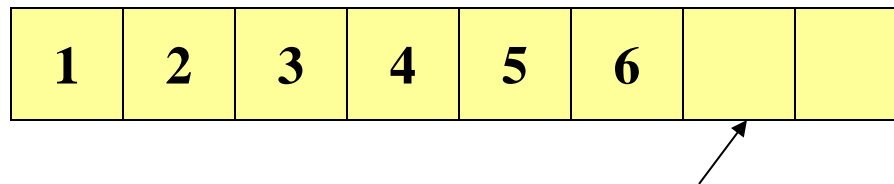


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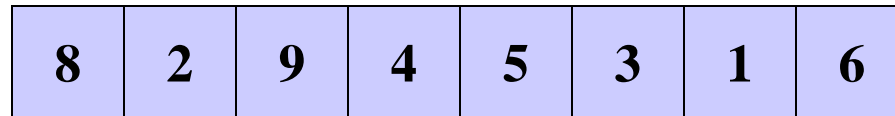
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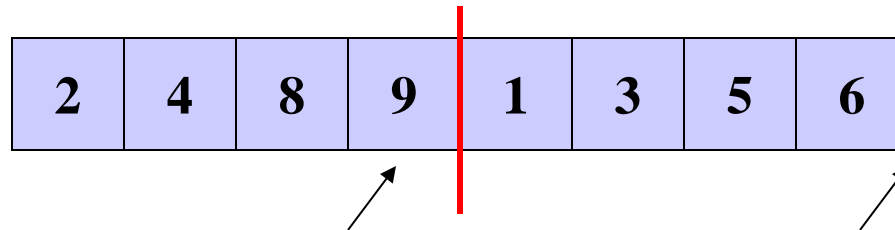
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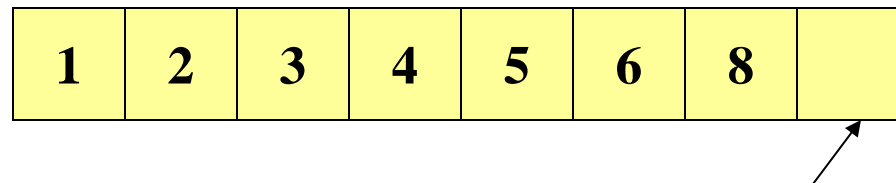


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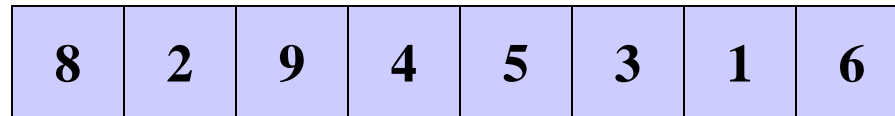
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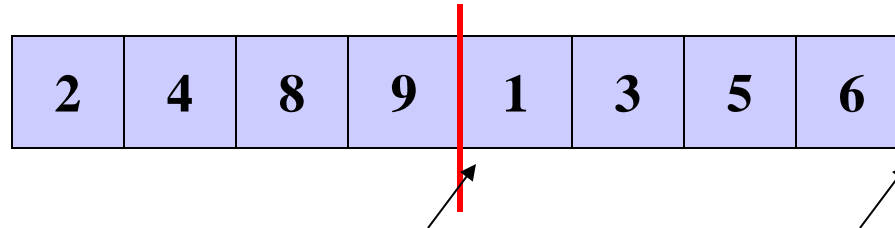
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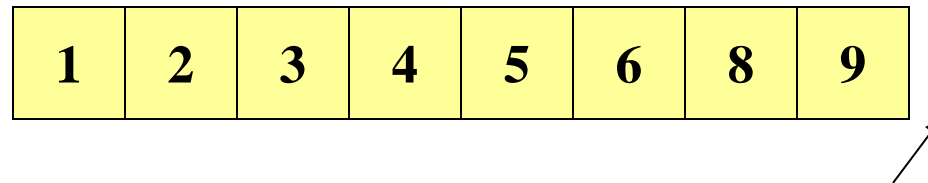


After recursion:
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Merge:

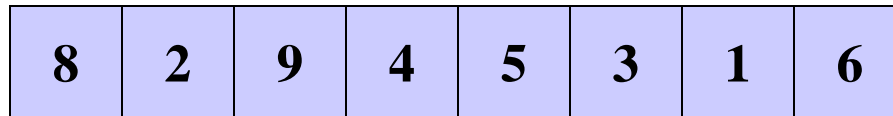
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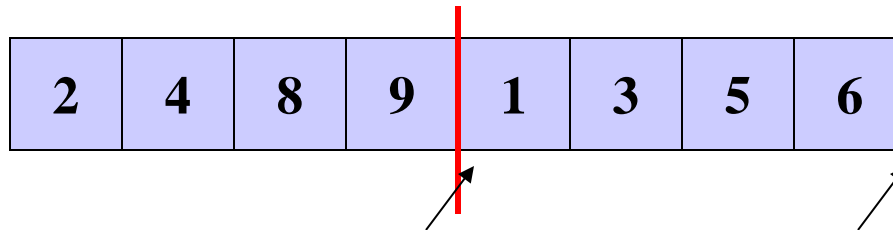
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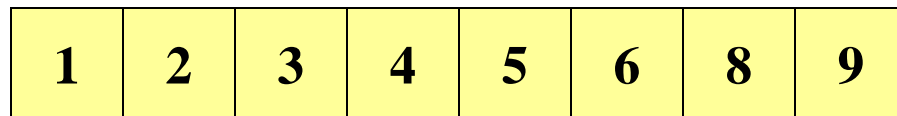


After recursion:
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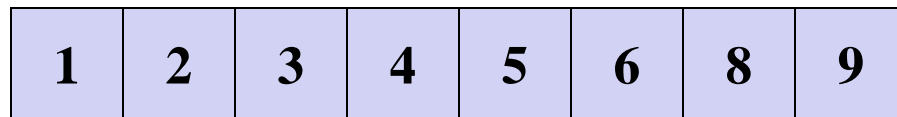


Merge:

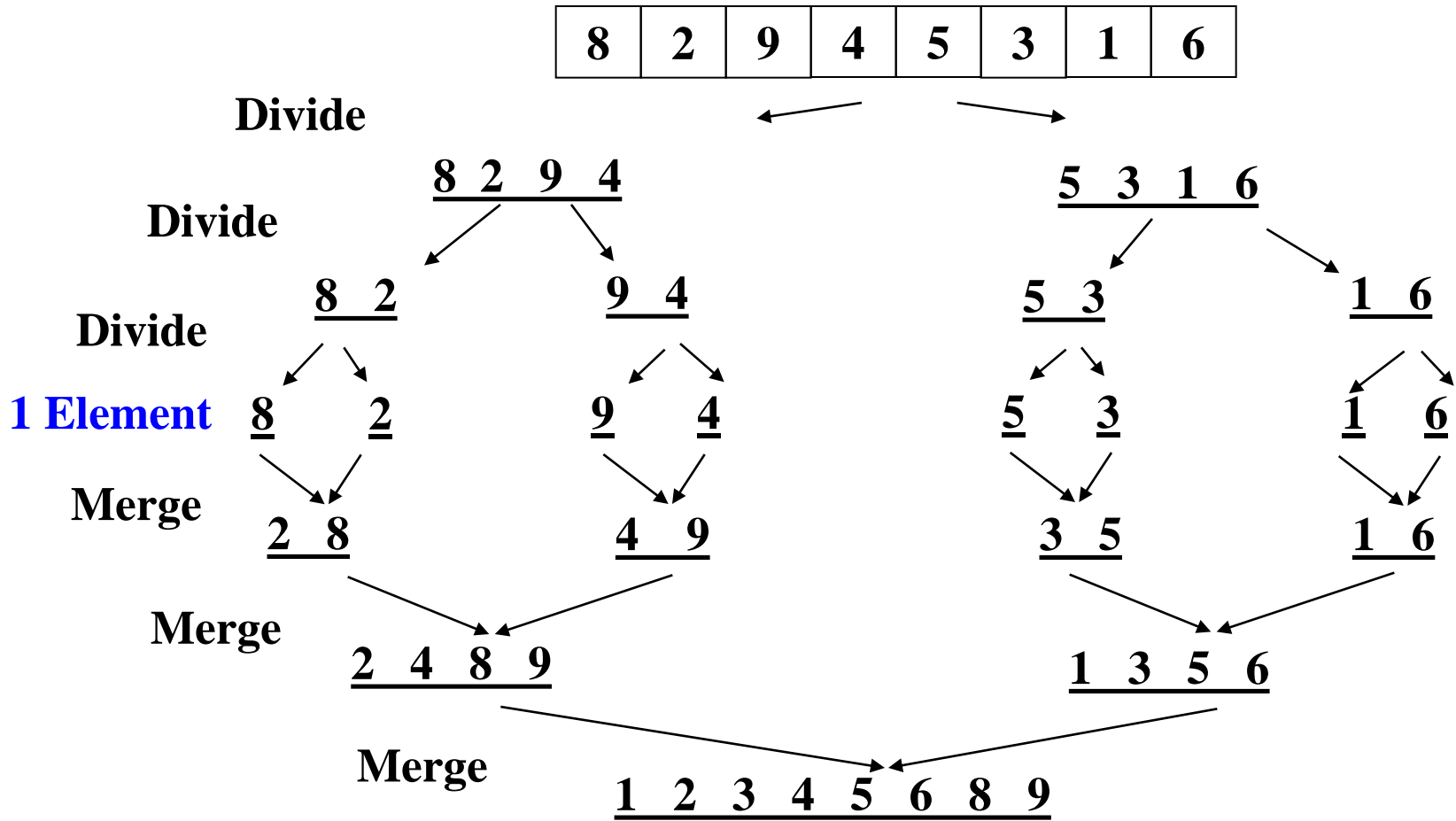
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(After merge,
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Example, Showing Recursion

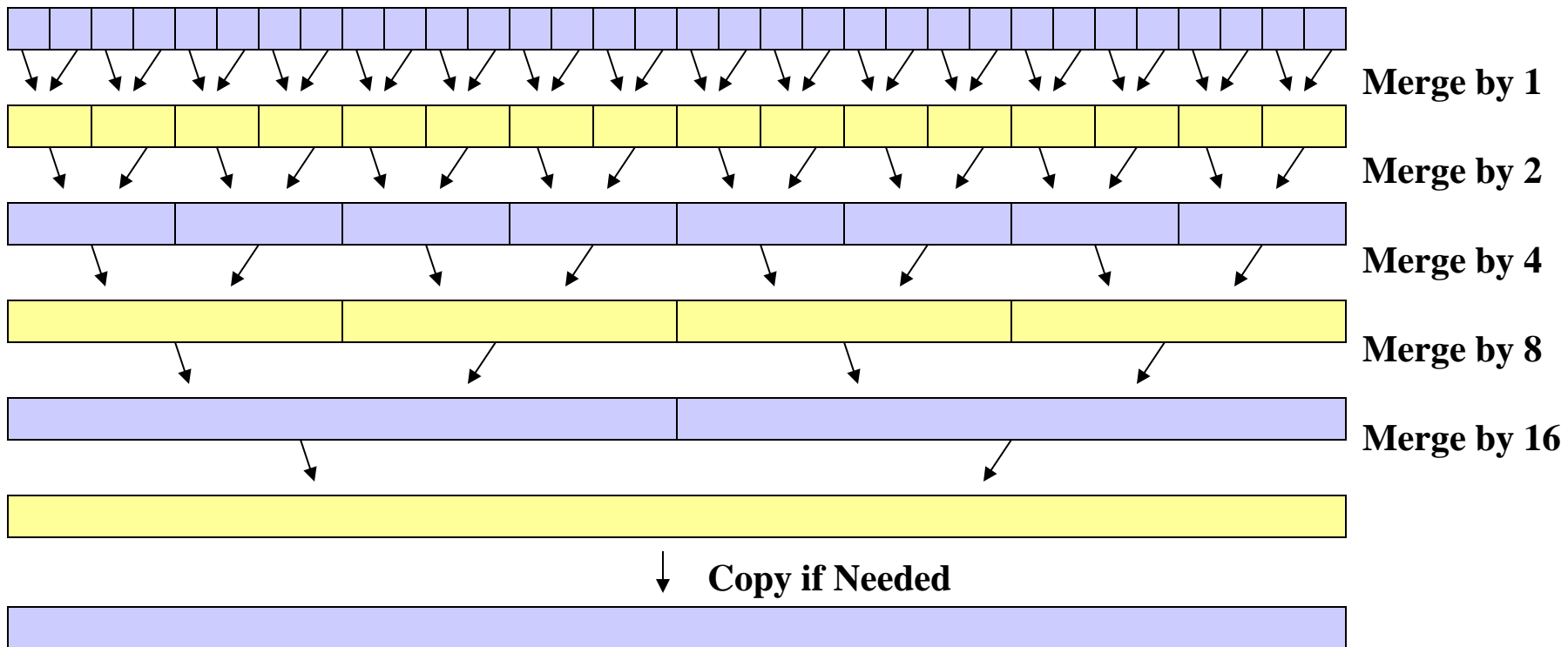


Merge sort visualization

- <http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html>

Swapping Original / Auxiliary Array (“best”)

- First recurse down to lists of size 1
- As we return from the recursion, swap between arrays



(Possible to code up without recursion at all)

Linked lists and big data

We defined sorting over an array, but sometimes you want to sort linked lists

One approach:

- Convert to array: $O(n)$
- Sort: $O(n \log n)$
- Convert back to list: $O(n)$

Or **merge sort works very nicely on linked lists directly (HOW?)**

- Heapsort and quicksort do not
- Insertion sort and selection sort do but they're slower

Merge sort is also the sort of choice for external sorting

- Linear merges minimize disk accesses
- And can leverage multiple disks to get streaming accesses

Analysis

Having defined an algorithm and argued it is correct, we should analyze its running time and space:

To sort n elements, we:

- Return immediately if $n=1$
- Else do 2 subproblems of size $n/2$ and then an $O(n)$ merge

Recurrence relation:

$$T(1) = c_1$$

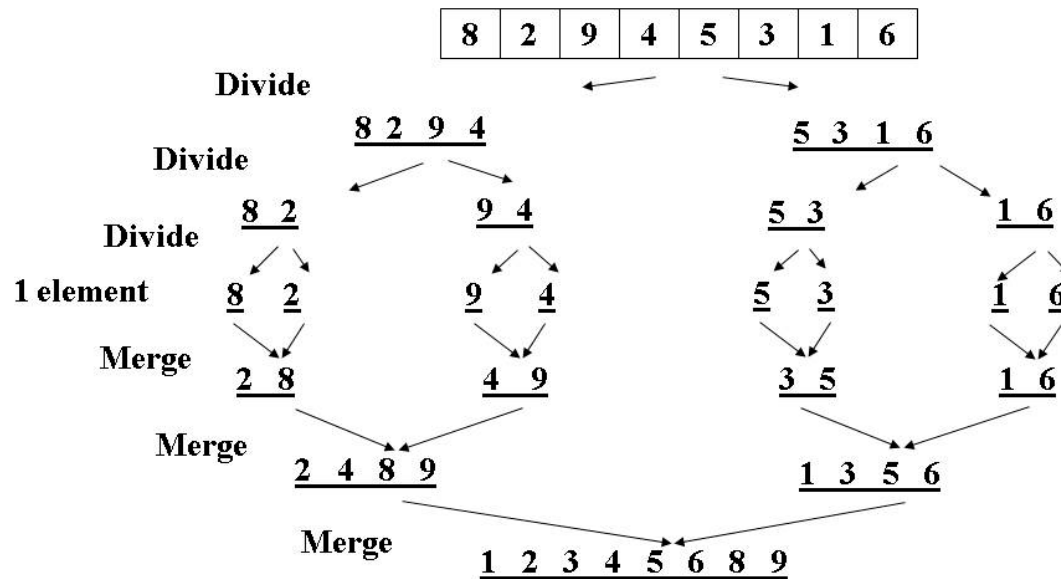
$$T(n) = 2T(n/2) + c_2n$$

Analysis intuitively

This recurrence is common you just “know” it’s $O(n \log n)$

Merge sort is relatively easy to intuit (best, worst, and average):

- The recursion “tree” will have **$\log n$ height**
- At each level we do a *total* amount of **merging equal to n**



Next lecture

- Quick sort 😊

Practice Problem: Heapsort/Mergesort

9	5	4	1
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9	5	4	1
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Practice Problem: Heapsort/Mergesort

9 | 5 | 4 | 1

5 | 4 | 1 | 9

4 | 1 | 5 | 9

1 | 4 | 5 | 9

1 | 4 | 5 | 9

