Circle the section in which you are registered: BA BB BC BD BF

CSE 373–Autumn 2016 — Assignment 2

INSTRUCTIONS: We are planning to try using an online workflow-management system during the grading of this assignment. Thus we ask for your cooperation in following these instructions. Please print out this assignment, single-sided. Then put your answers to this assignment on the pages. Write your full name at the top of each page. On page 1, also circle your section number. Put all your answers inside the framed rectangular boxes that are provided. Write legibly and if you use a pencil, make sure that you write darkly enough that a normal scanner will pick up your writing. If you need more space, use the margins. Do not fold or crease any of the pages, or they may jam in our scanner. Turn in your 6-sheet, stapled hardcopy solutions at the start of lecture on the due date. Total possible points: 100.

1. Quick Evaluations (6 points)

Evaluate the following expressions, without using a calculator.

$\log_2 256 =$ $2^8 =$ $\log_2 16 =$

$\log_3 81 =$ $\log_4 256 =$ $\ln(e^n) =$

2. Calculations (6 points)

Evaluate the following expressions, using a calculator.

$\log_{10} 256 =$ $\log_2 10 =$ $\log_2 100 =$

$\log_3 10 =$ $\ln 10 =$ $\log_2 \log_2 256 =$

3. The Crossing Point (10 points)

a. Find the lowest integer value for n where $f(n) = n^2$ exceeds $g(n) = 100 \log_2 n$. Give the values of f and g at that value of n. (Use a calculator if needed).

$n =$ $f(n) =$ $g(n) =$

b. As n increases from 1, at what integer value will one of the functions $f(n) = 5^n$ and $g(n) = n!$ overtake the other?

$n =$

4. Shopping Plans (10 points)

a. Market Meandering

Suppose there are 13 farm stands at the farmers' market, and Dave wants to visit them all in a different order each week. How many weeks will it take him to try all possible orders? Will he live long enough? (Show your work)

b. Varying the Veges

Each time he visits the market, Dave buys vegetables from a farmer named Old MacDonald. It seems that each week, Old MacDonald has the same variety available (he uses greenhouses and produces all year 'round): 10 different kinds of vegetables, including artichokes, broccoli, carrots, daikon, endive, fennel, garlic, etc. Dave buys three kinds of vegetables each week. How many weeks will it take him to try all combinations of 3 vegetables from Old MacDonald? (Show your work)

5. From Floor to Ceiling (8 points)

Graph the functions $f(x) = \lfloor x \rfloor$ and $g(x) = \lceil x \rceil$ over the range $-3 \leq x \leq 3$. Please use red pencil, red ink, or red graphics for function f and blue pencil, ink, or graphics for g. Graph the functions together so it is easy to see where the function "curves" intersect each other.

6. Comparing Growth Rates (24 points)

The table below shows a function f_i in each of three rows and a function g_j in each of three columns. There are nine blank boxes where you should express the relationship(s) between f_i and g_j in terms of Big O, Big Omega (Ω) and/or Big Theta (Θ). In each box, give all that apply. If any do NOT apply, then don't write them.

Whenever $f_i(n) \in O(g_j(n))$, give values for c and n_0 that show the definition for Big O is satisfied, using $c = 1$ whenever possible.

Whenever $f_i(n) \in \Omega(g_j(n))$, give values for d and n_1 which show that $g_j(n) \in O(f_i(n))$, again, using $d = 1$ whenever possible. That is, give d and n_1 such that $g(n) \leq d \cdot f(n)$ for all $n \geq n_1$.

Clearly, whenever $f_i(n) \in \Theta(g_j(n))$ you will already have given values for all four of c, n_0, d, and n_1. The first box is done for you.

	$g_1(n) = 2n$	$g_2(n) = 0.1n^2$	$g_3(n) = 3^n$
$f_1(n) = n(n+1)/2$	$\Omega : d = 1, n_1 = 6$		
$f_2(n) = 2^n$			
$f_3(n) = 3^{n-1}$			

7. Growth Rate Comparison by Induction (8 points)

Prove by induction that $n! > 3^n$ for all $n > 6$. Your proof must be in the form of answers to parts a, b, and c below.

a. State and prove your base case.

b. State the induction hypothesis.

c. Prove the induction step.

8. Induction for Combinations (8 points)

Consider the following form of Pascal's Identity. Assuming that k is constant, prove by induction that this identity is valid for all $n \geq k$. Your proof must be in the form of answers to parts a, b, and c below.

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

a. State and prove your base case.

b. State the induction hypothesis.

c. Prove the induction step.

9. After the Apocalypse (10 points)

The apocalypse has taken place and almost all computers have been destroyed. A surviving human programmer, Sean, has a computer that almost works, but its multiplication operation doesn't work, due to damage to the CPU. He has been asked to deliver a program to compute the squares of non-negative numbers, which will be needed in rebuilding civilization's infrastructure. He comes up with the following algorithm. The boss is skeptical and demands that Sean prove that it is correct.

```
input n;
s = 0;
i = 0;
while i < n:
  s += n;
  i += 1;
return s;
```

Pretend you are Sean, and prove that the algorithm is correct, using the method for looping algorithms.

a. State your loop invariant

b. Show that it is true prior to the first iteration of the loop.

c. Show that if it is true before an iteration, then it will remain true after the iteration.

d. Show that the program terminates.

10. Overlapping Subsorts (10 points)

Prove that the following algorithm is correct, using the method for recursive algorithms.

```
Algorithm SortByTwoThirds:

Input array A[0]...A[n-1]
If n==2 and A[0] > A[1], swap A[0] and A[1];
If n > 2:
    Let k = ceiling(2n/3);
    Recursively sort A[0] through A[k-1];   // The first two-thirds.
    Recursively sort A[n-k] through A[n-1]; // The last two-thirds.
    Recursively sort A[0] through A[k-1];   // The first two-thirds.
Return A.
```

a. Prove the base case(s).

b. State your inductive hypothesis.

c. Prove the inductive step.

d. Show that the program terminates.