CSE373: Data Structures & Algorithms

Lecture 6: Binary Search Trees

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Summer 2015

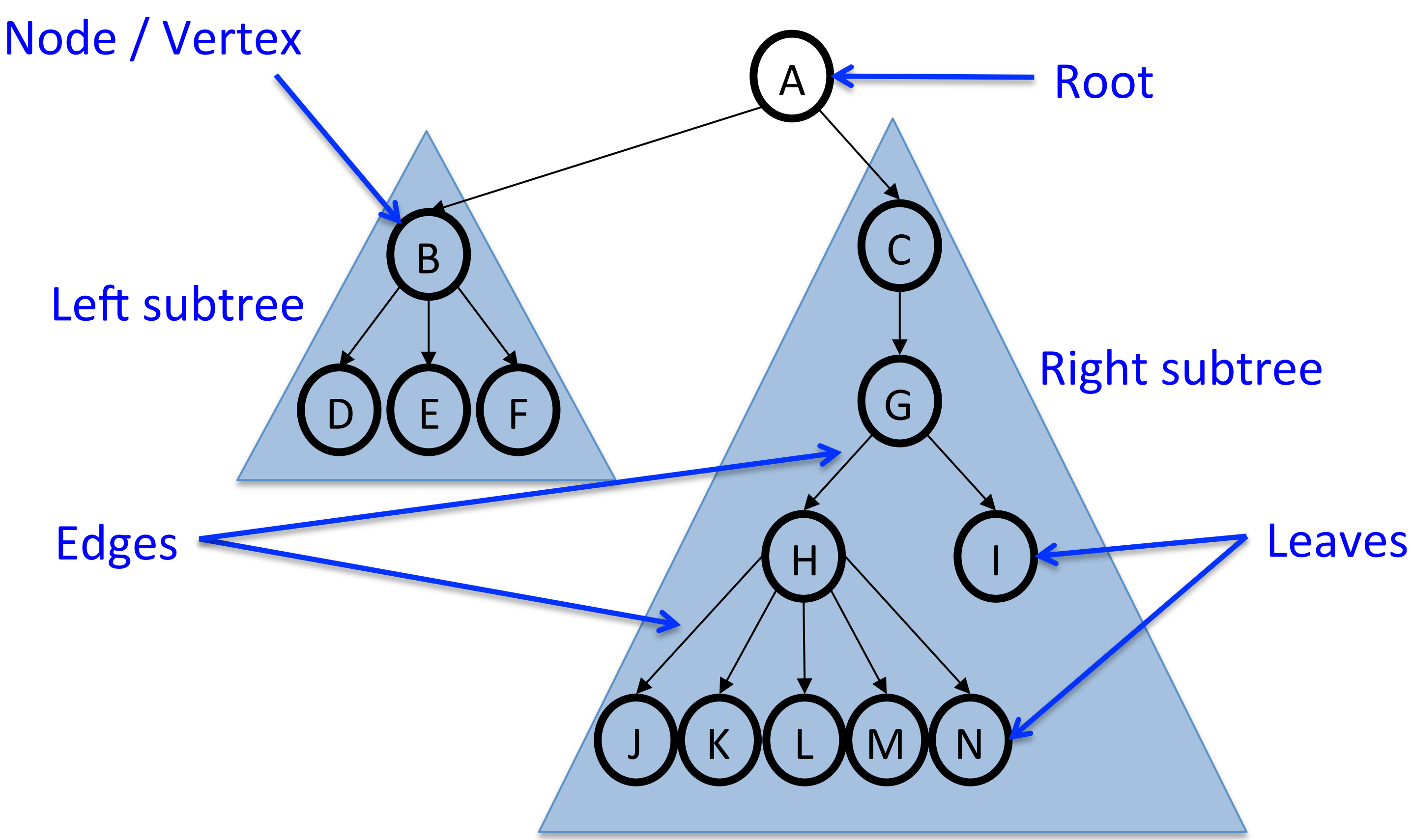
Announcements

- HW2 due 10:59 PM Friday
- Going to try to rearrange session times.

Previously on CSE 373

- Dictionary ADT
  - stores (key, value) pairs
  - **`find`**, **`insert`**, **`delete`**
- Trees
  - Terminology
  - Binary Trees

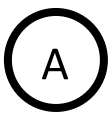
Reminder: Tree terminology



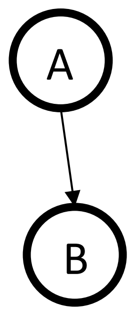
Example Tree Calculations

Recall: Height of a tree is the maximum number of edges from the root to a leaf.

What is the height of this tree?

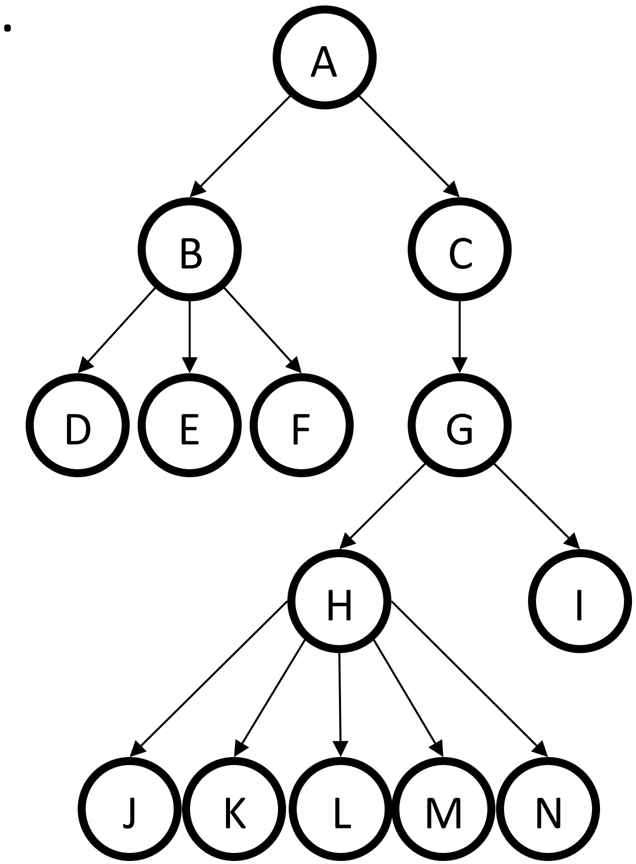


Height = 0



Height = 1

Height = 4



What is the depth of node G?

Depth = 2

What is the depth of node L?

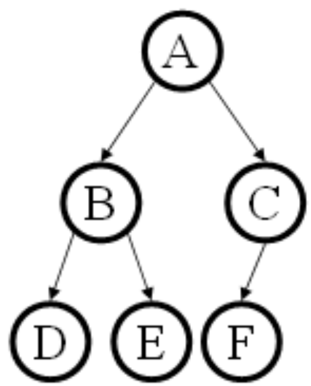
Depth = 4

Binary Trees

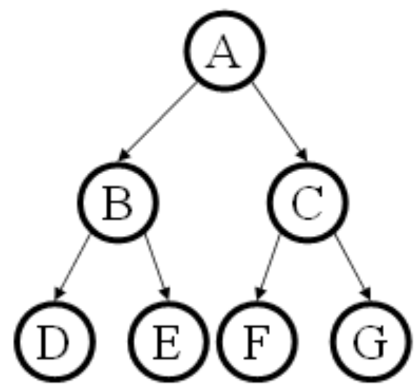
- Binary tree: Each node has at most 2 children (branching factor 2)

- Binary tree is
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*

- Special Cases



Complete Tree

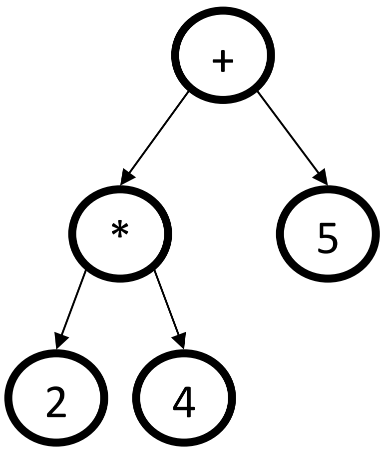


Perfect Tree

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

(an expression tree)



- *Pre-order*: root, left subtree, right subtree

  + * 2 4 5

- *In-order*: left subtree, root, right subtree

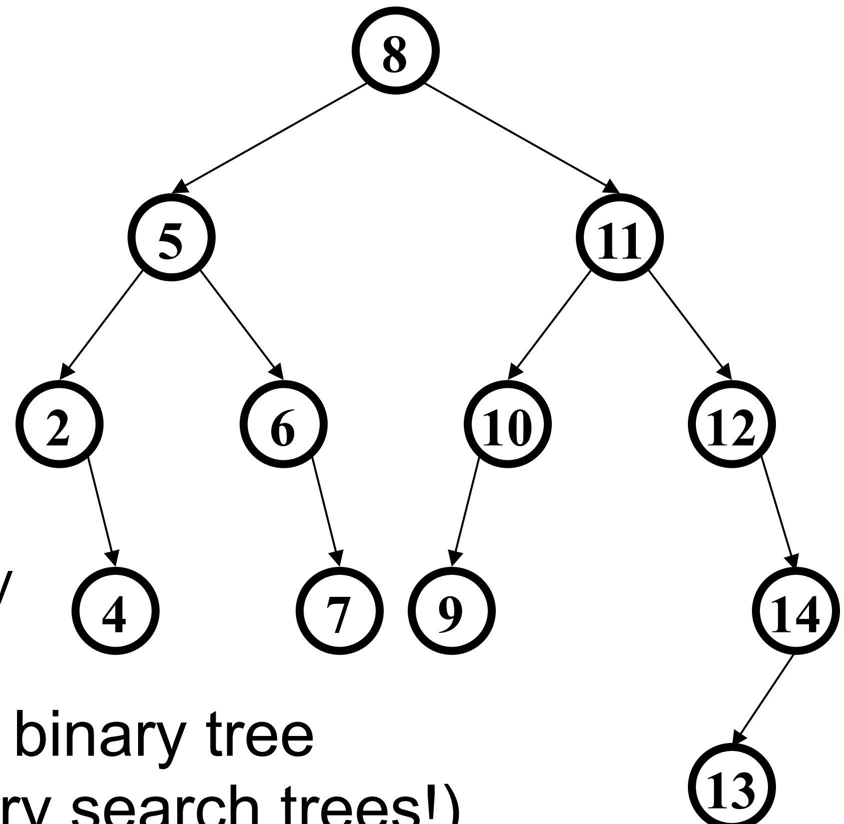
  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

  2 4 * 5 +

Binary Search Tree (BST) Data Structure

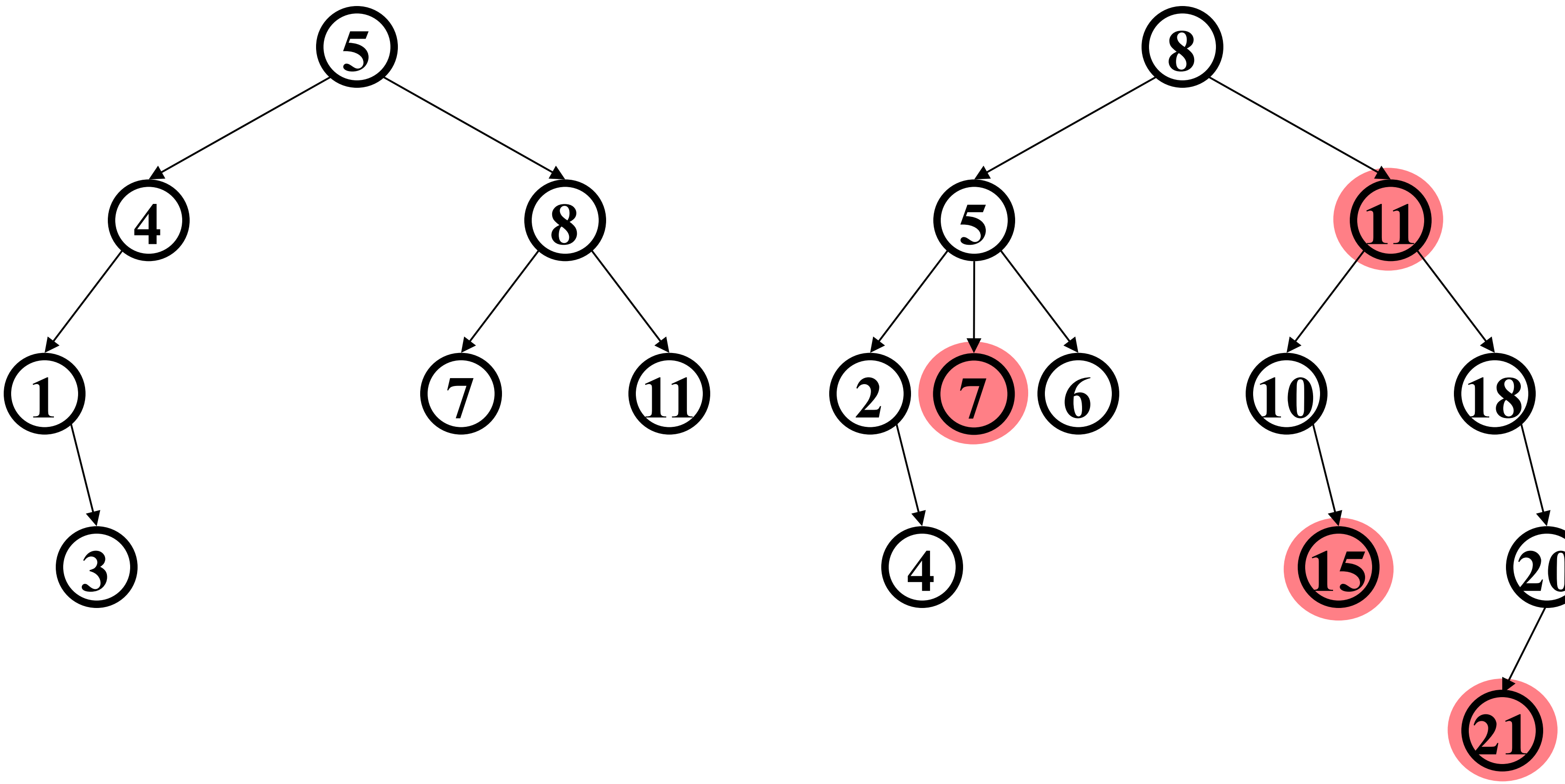
- Structure property (binary tree)
  - Each node has ≤ 2 children
  - Result: keeps operations simple



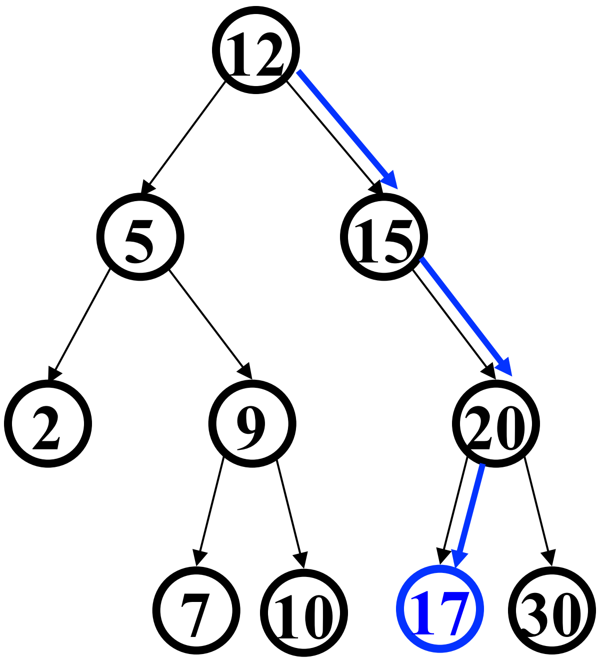
- Order property
  - All keys in left subtree smaller than node's key
  - All keys in right subtree larger than node's key
  - Result: easy to find any given key

A binary search tree is a type of binary tree
(but not all binary trees are binary search trees!)

Are these BSTs?



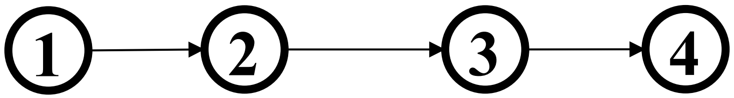
Find in BST, Recursive



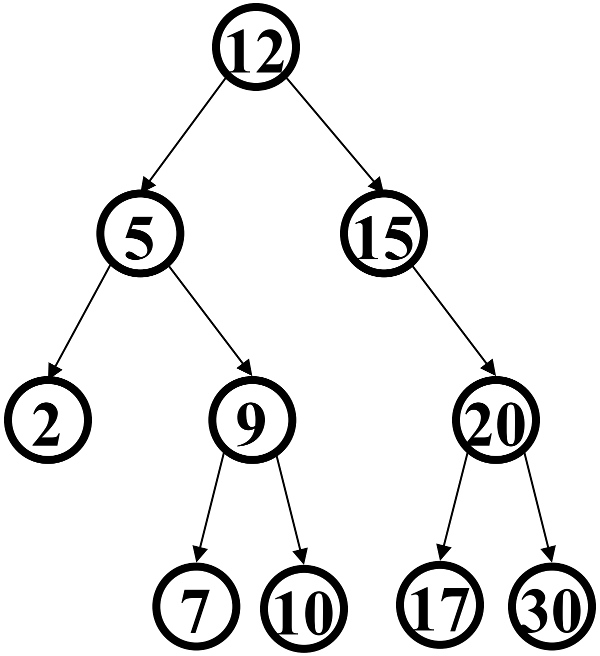

```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

What is the running time?

Worst case running time is O(n).
- Happens if the tree is very lopsided (e.g. list)



Find in BST, Iterative

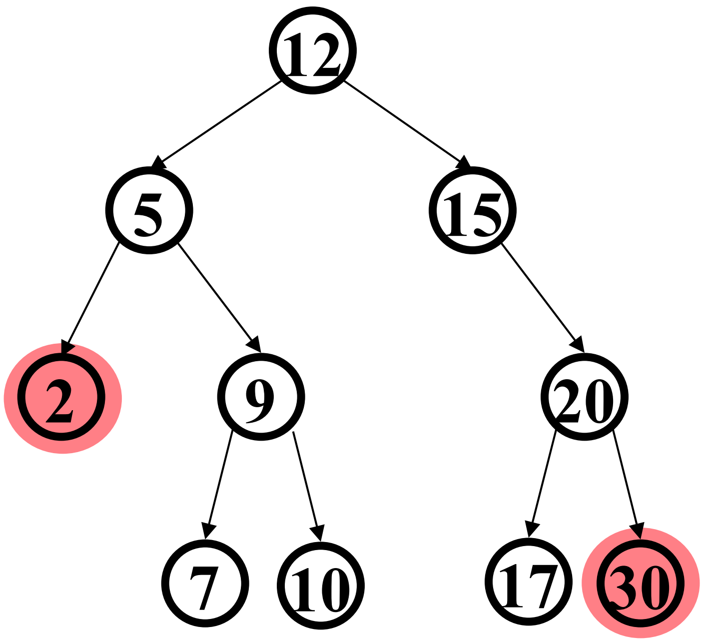



```
Data find(Key key, Node root){
 while(root != null
         && root.key != key) {
  if(key < root.key)
     root = root.left;
  else(key > root.key)
     root = root.right;
 }
 if(root == null)
     return null;
 return root.data;
}
```

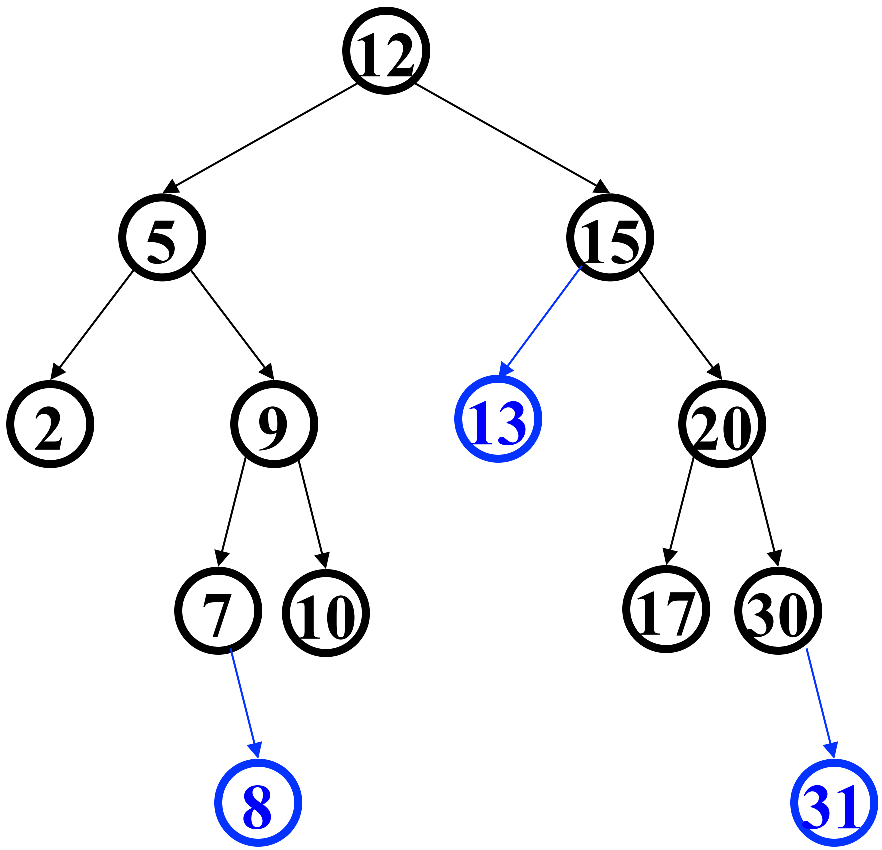
Worst case running time is O(n).
- Happens if the tree is very lopsided (e.g. list)

Bonus: Other BST "Finding" Operations

- **`FindMin`**: Find *minimum* node
  - Left-most node
- **`FindMax`**: Find *maximum* node
  - Right-most node



Insert in BST

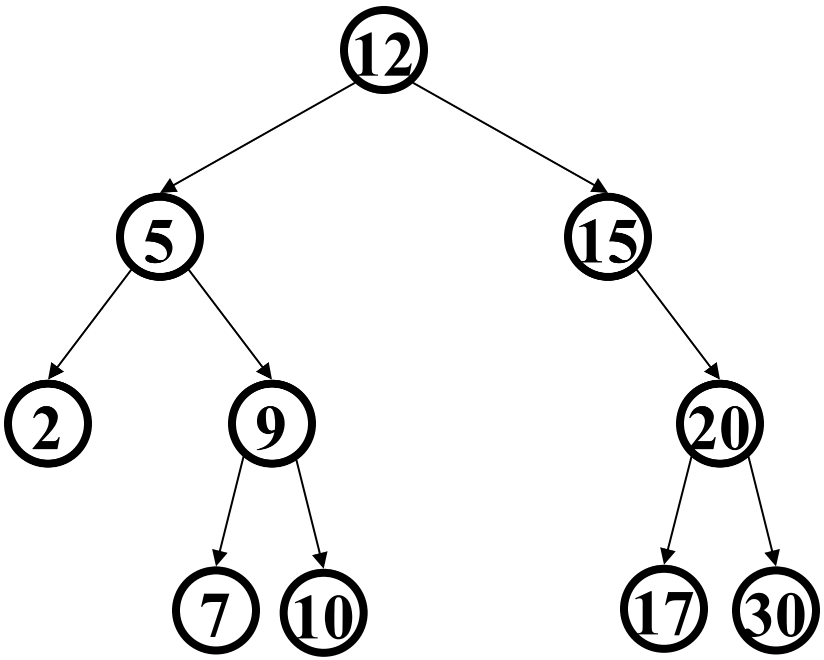



```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Again… worst case running time is O(n), which may happen if the tree is not balanced.

Deletion in BST



Why might deletion be harder than insertion?

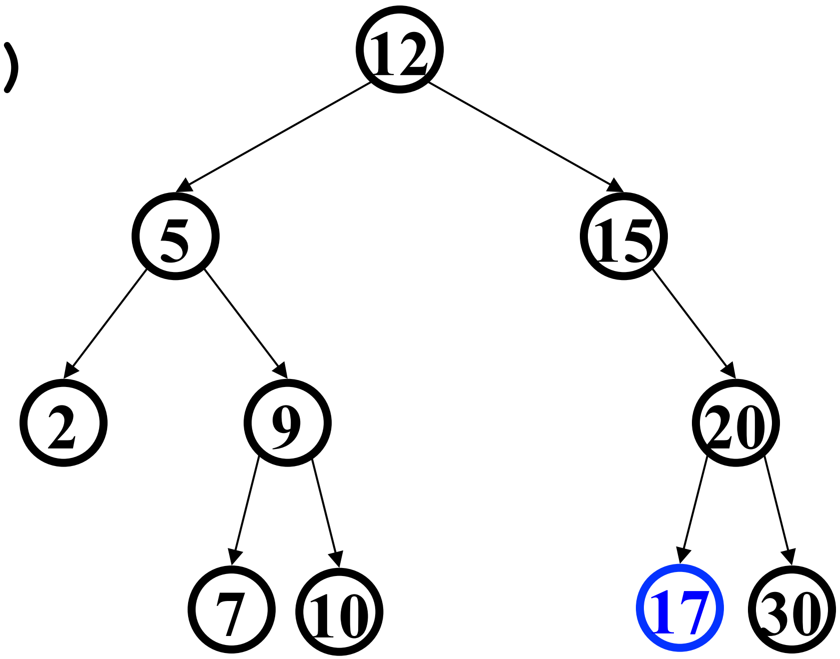
Deletion in BST

- Basic idea: `find` the node to be removed, then "fix" the tree so that it is still a binary search tree

- Three potential cases to fix:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

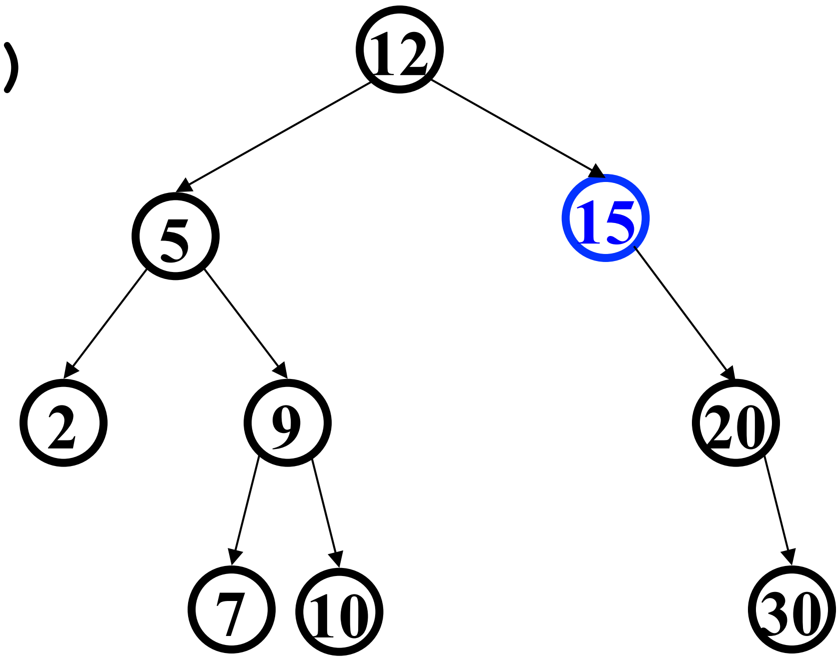
Deletion – The Leaf Case

`delete(17)`



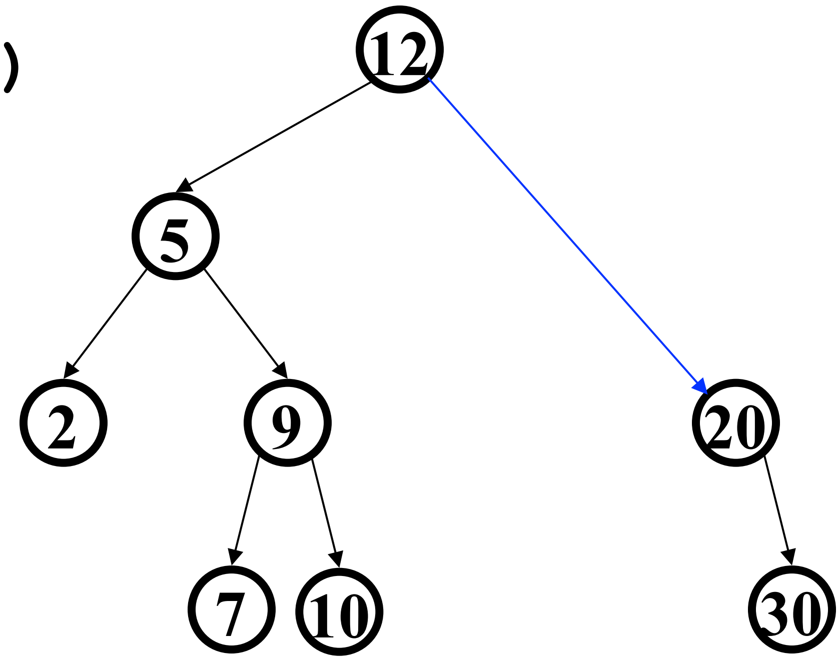
Deletion – The One Child Case

`delete(15)`



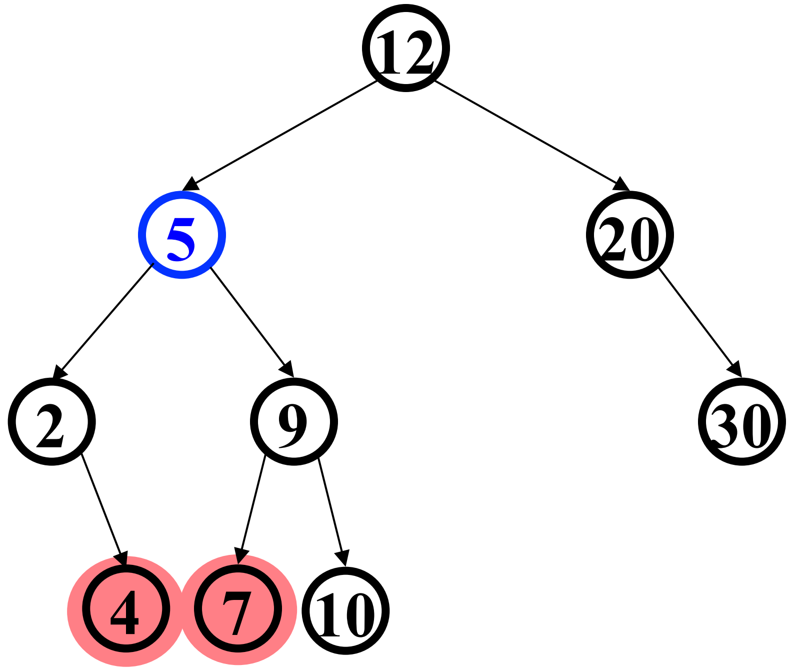
Deletion – The One Child Case

`delete(15)`



Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

What can we replace the node with?

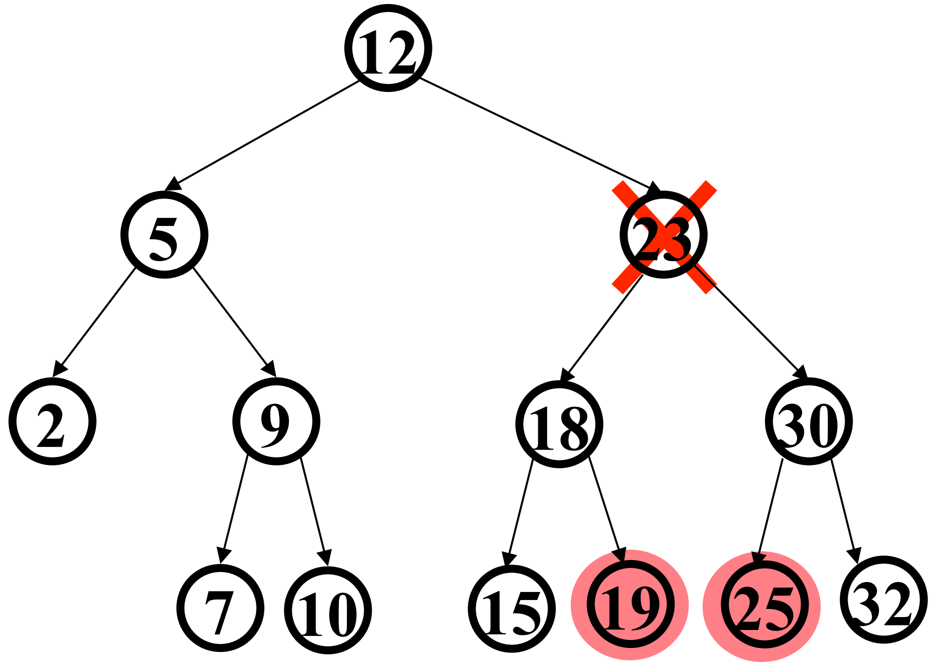
Options:

- *successor* minimum node from right subtree
  `findMin(node.right)`

- *predecessor* maximum node from left subtree
  `findMax(node.left)`

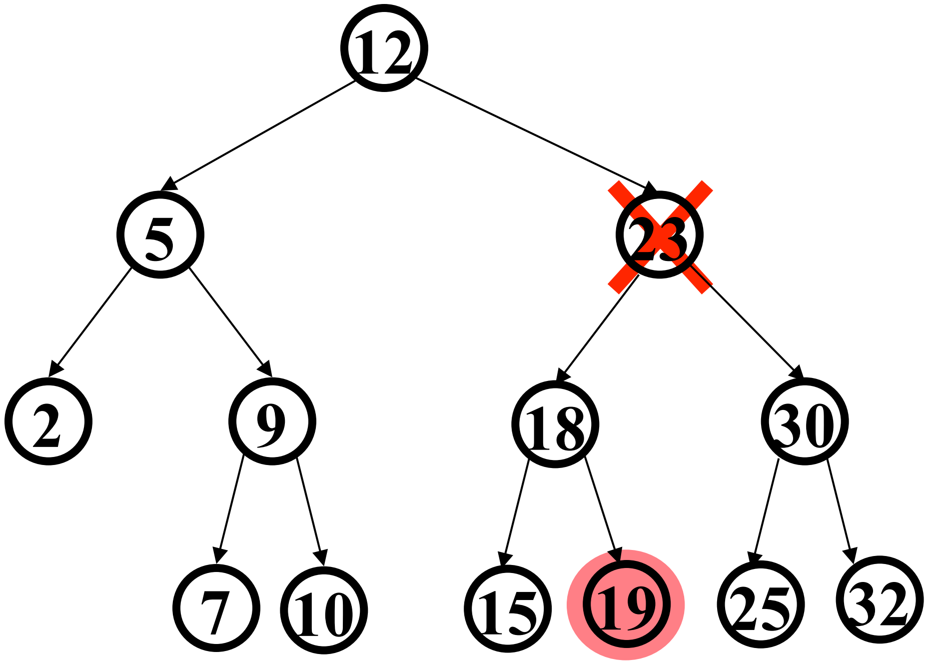
Deletion: The Two Child Case (example)

`delete(23)`



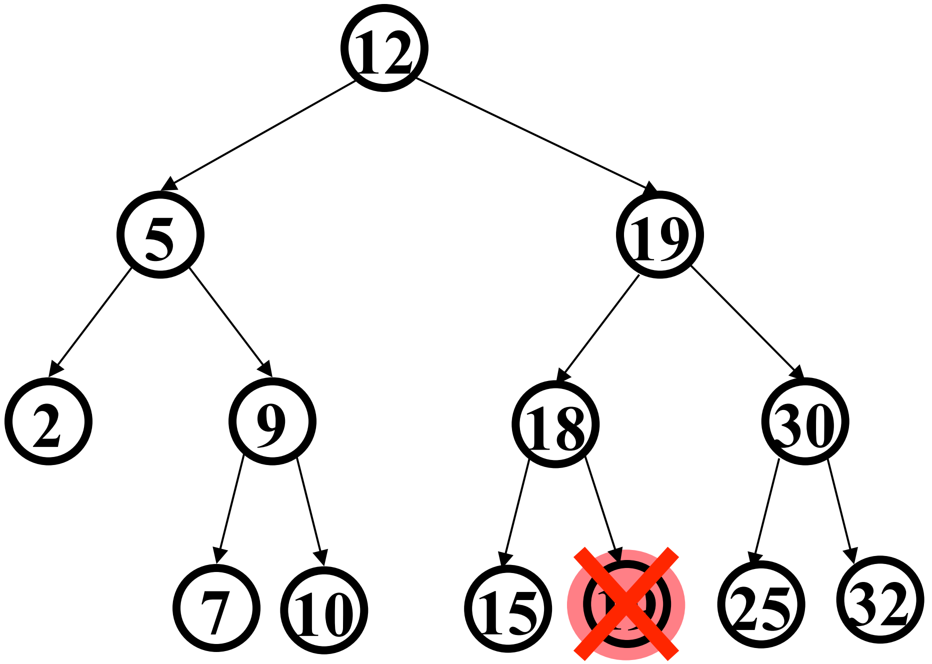
Deletion: The Two Child Case (example)

`delete(23)`



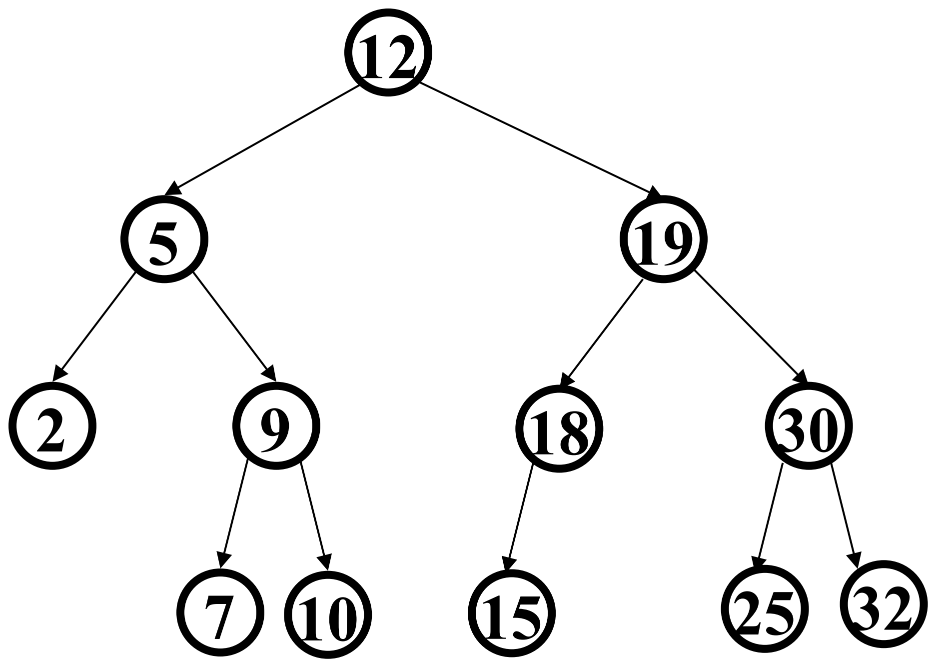
Deletion: The Two Child Case (example)

`delete(23)`



Deletion: The Two Child Case (example)

`delete(23)`



Success! ☺

Lazy Deletion

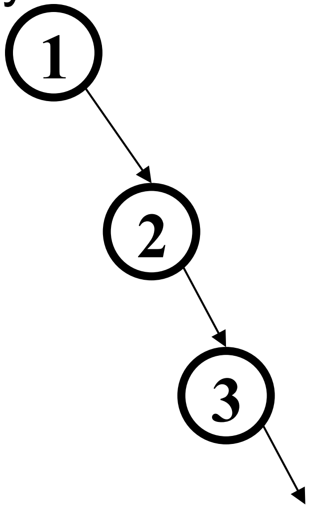
- Lazy deletion can work well for a BST
  - Simpler
  - Can do “real deletions” later as a batch
  - Some inserts can just “undelete” a tree node

- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - e.g., **`findMin`** and **`findMax`**?

BuildTree for BST

- Let's consider **`buildTree`**
  - Insert all, starting from an empty tree

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order, what is the tree?
  - What big-O runtime for buildTree on this sorted input?
  - Is inserting in the reverse order any better?



$O(n^2)$

BuildTree for BST

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
- What we if could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9
  - What tree does that give us?
  - What big-O runtime?

O(n* log *n), definitely better

  - **So the order the values come in is important!**