CSE373: Data Structures & Algorithms

Lecture 4: Dictionaries; Binary Search Trees

Kevin Quinn

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Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

1. Stack: **`push`**, **`pop`**, **`isEmpty`**, …
2. Queue: **`enqueue`**, **`dequeue`**, **`isEmpty`**, …

Next:

3. Dictionary (also known as a Map): associate keys with values
   – Extremely common

The Dictionary (a.k.a. Map) ADT

- Data:
  - *set* of (key, value) *pairs*
  - keys must be *comparable*

insert(cs373, ….)

- Operations:
  - **`insert(key,value)`**
  - **`find(key)`**
  - **`delete(key)`**
  - …

find(kanye)

Kanye West

Will tend to emphasize the keys;
don't forget about the stored values

- **cs373 → Data Structures**
- **dog → Labrador**
- **kanye → Kanye West**

Common Uses of Dictionaries

Counting frequency of words in a book: **Map<String, Integer>**

Storing a contact list: **Map<String, String>**

Making a Facebook-esque graph of friends: **Map<Person, Set<Person>>**

What happens when the keys aren't all the same type?

What about the values?

Comparison: The Set ADT

The *Set* ADT is like a Dictionary without any values

- A key is *present* or not (no duplicates)

For **`find`**, **`insert`**, **`delete`**, there is little difference

- In dictionary, values are "just along for the ride"
- So *same data-structure ideas* work for dictionaries and sets

But if your Set ADT has other important operations this may not hold

- **`union`**, **`intersection`**, **`is_subset`**
- Notice these are binary operators on sets

binary operation: a rule for combining two objects of a given type, to obtain another object of that type

Dictionary data structures

There are many good data structures for (large) dictionaries

1. AVL trees (Friday's class)
   – Binary search trees with *guaranteed balancing*
2. B-Trees
   – Also always balanced, but different and shallower
   – B ≠ Binary; B-Trees generally have large branching factor
3. Hashtables
   – Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

But first some applications and less efficient implementations…

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently. Lots of programs do that!

• Search:	inverted indexes, phone directories, …
• Networks:	router tables
• Operating systems:	page tables
• Compilers:	symbol tables
• Databases:	dictionaries with other nice properties
• Biology:	genome maps
• …	

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
Unsorted linked-list	*O(1)**	*O(n)*	*O(n)*
Unsorted array	*O(1)**	*O(n)*	*O(n)*
Sorted linked list	*O(n)*	*O(n)*	*O(n)*
Sorted array	*O(n)*	*O(logn)*	*O(n)*

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted

Plusses:

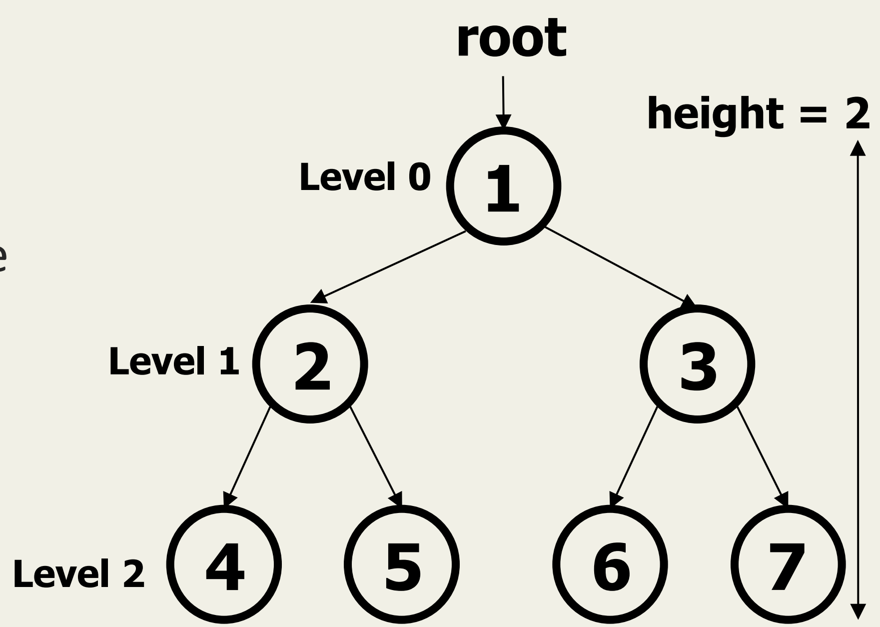
- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- **`find`** $O($**`log`** $m)$ *time* where m is data-structure size (okay)
- May complicate other operations

Tree Terminology

- **node**: an object containing a data value and left/right children
  - **root**: topmost node of a tree
  - **leaf**: a node that has no children
  - **branch**: any internal node (non-root)
  - **parent**: a node that refers to this one
  - **child**: a node that this node refers to
  - **sibling**: a node with a common
- **subtree**: the smaller tree of nodes on the left or right of the current node
- **height**: length of the longest path from the root to any node (count edges)
- **level** or **depth**: length of the path from a root to a given node



Some tree terms (mostly review)

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of “next” as the one child)

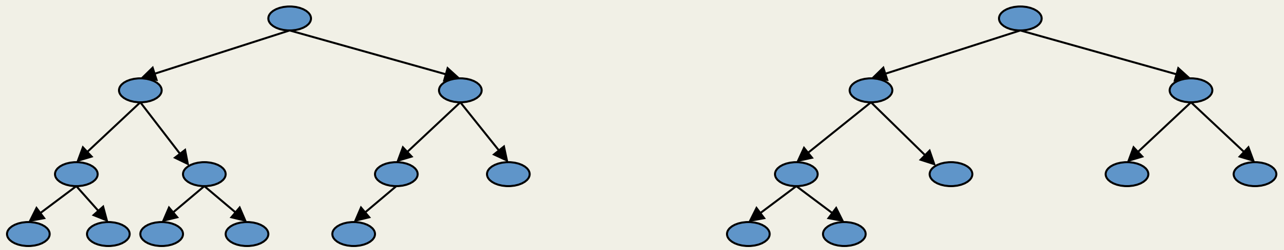
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree

- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of $O(\log n)$
  - Different tree data structures have different “balance conditions” to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right



What is the height of a perfect binary tree with n nodes?

A complete binary tree?

Tree terms (review?)

root(tree)

leaves(tree)

children(node)

parent(node)

siblings(node)

ancestors(node)

descendents(node)

subtree(node)

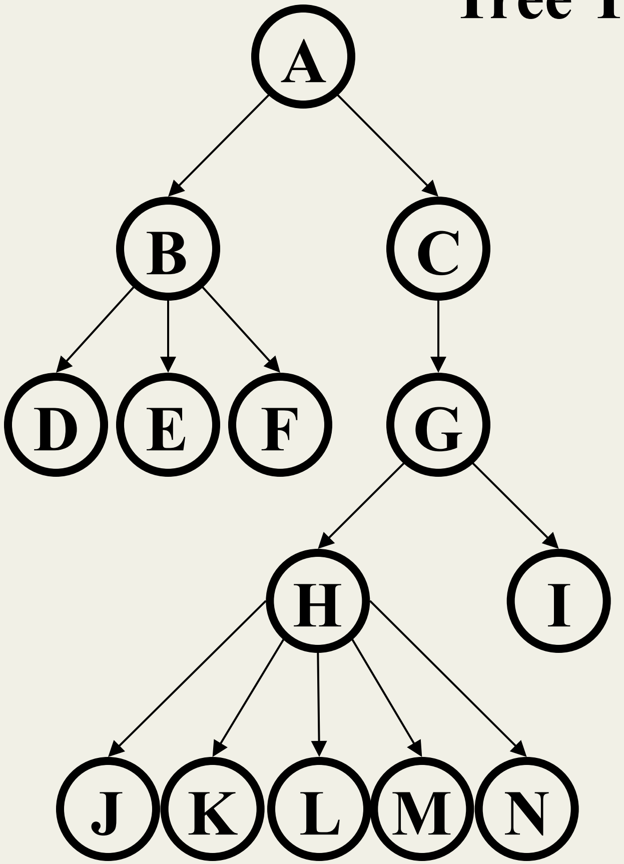
depth(node)

height(tree)

degree(node)

branching factor(tree)

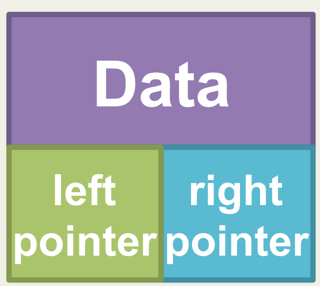
Tree T



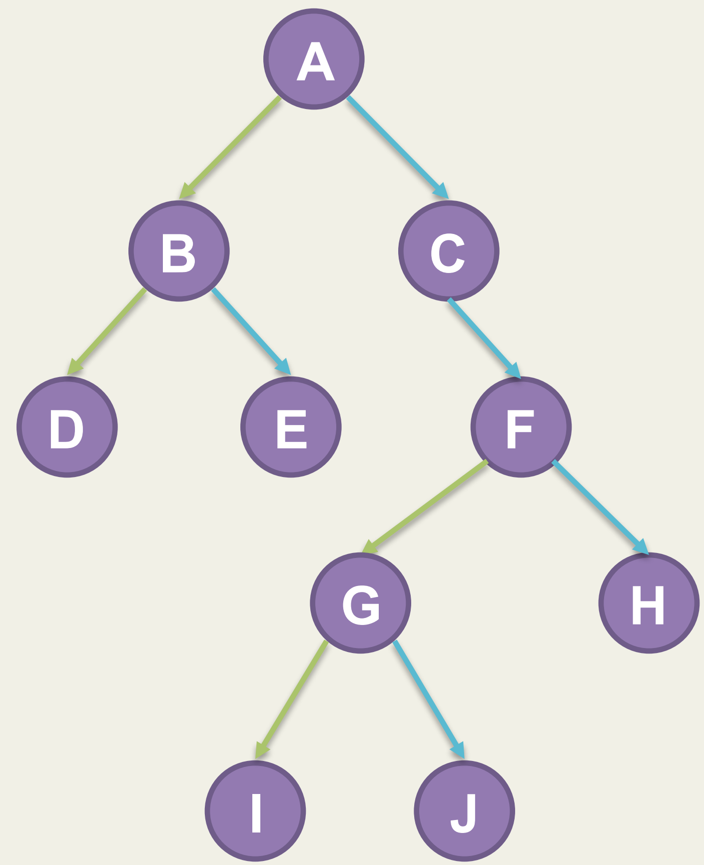
Binary Trees

- Binary tree is empty or
  - A **root** *(with data)*
  - A **left subtree** *(may be empty)*
  - A **right subtree** *(may be empty)*

- Representation:



- For a dictionary, data will include a key and a value



Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves: 2^{h}
- max # of nodes: $2^{(h + 1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(log n) height, and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

            ???


}
```

Calculating height

What is the height of a tree with root `root`?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

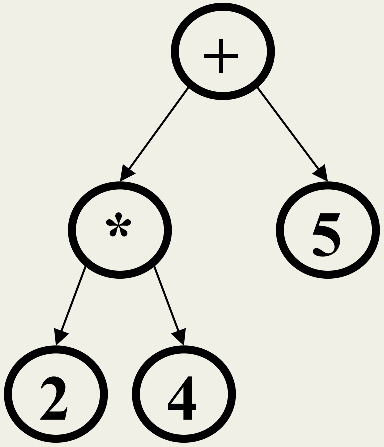
Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root

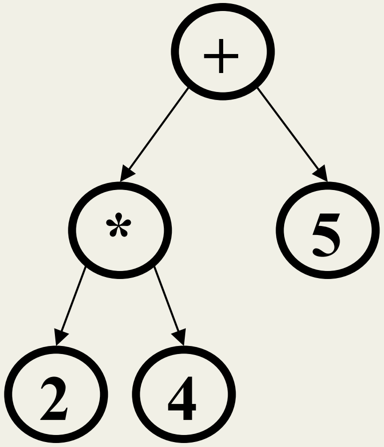


(an expression tree)

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

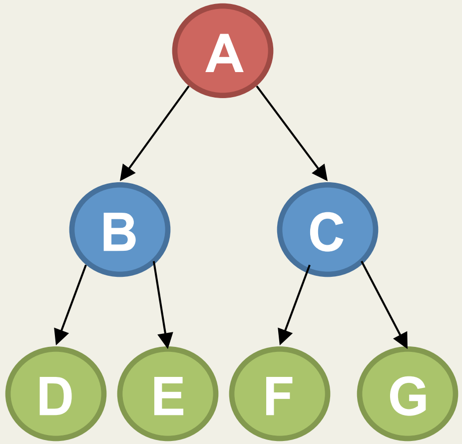
- ***Pre-order***: root, left subtree, right subtree
  + * 2 4 5
- ***In-order***: left subtree, root, right subtree
  2 * 4 + 5
- ***Post-order***: left subtree, right subtree, root
  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



Sometimes order doesn't matter

- Example: sum all elements

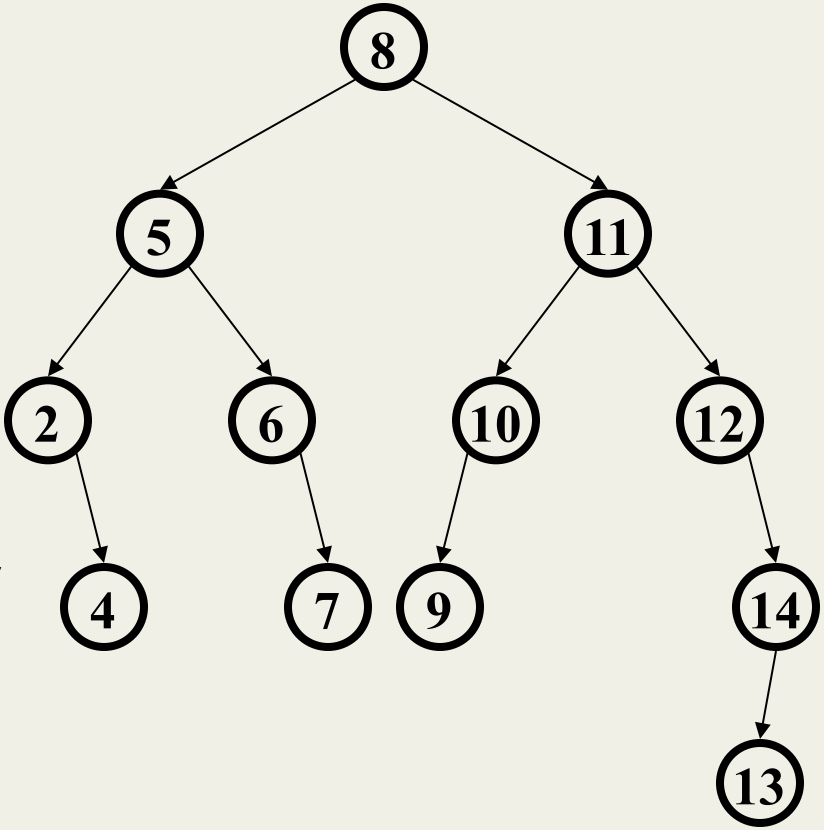
Sometimes order matters

- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

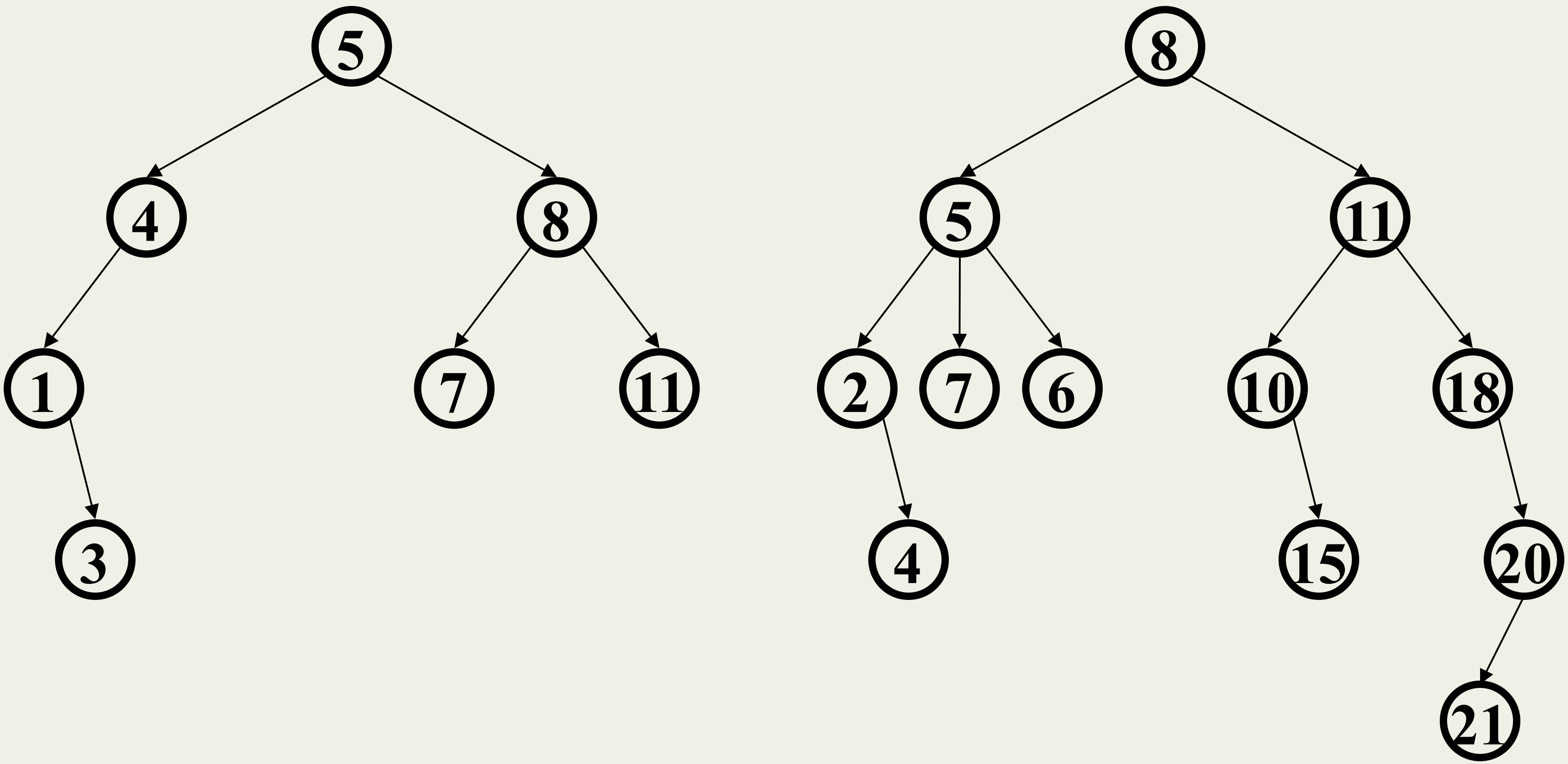
A
  B
    D
    E
  C
    F
    G

Binary Search Tree

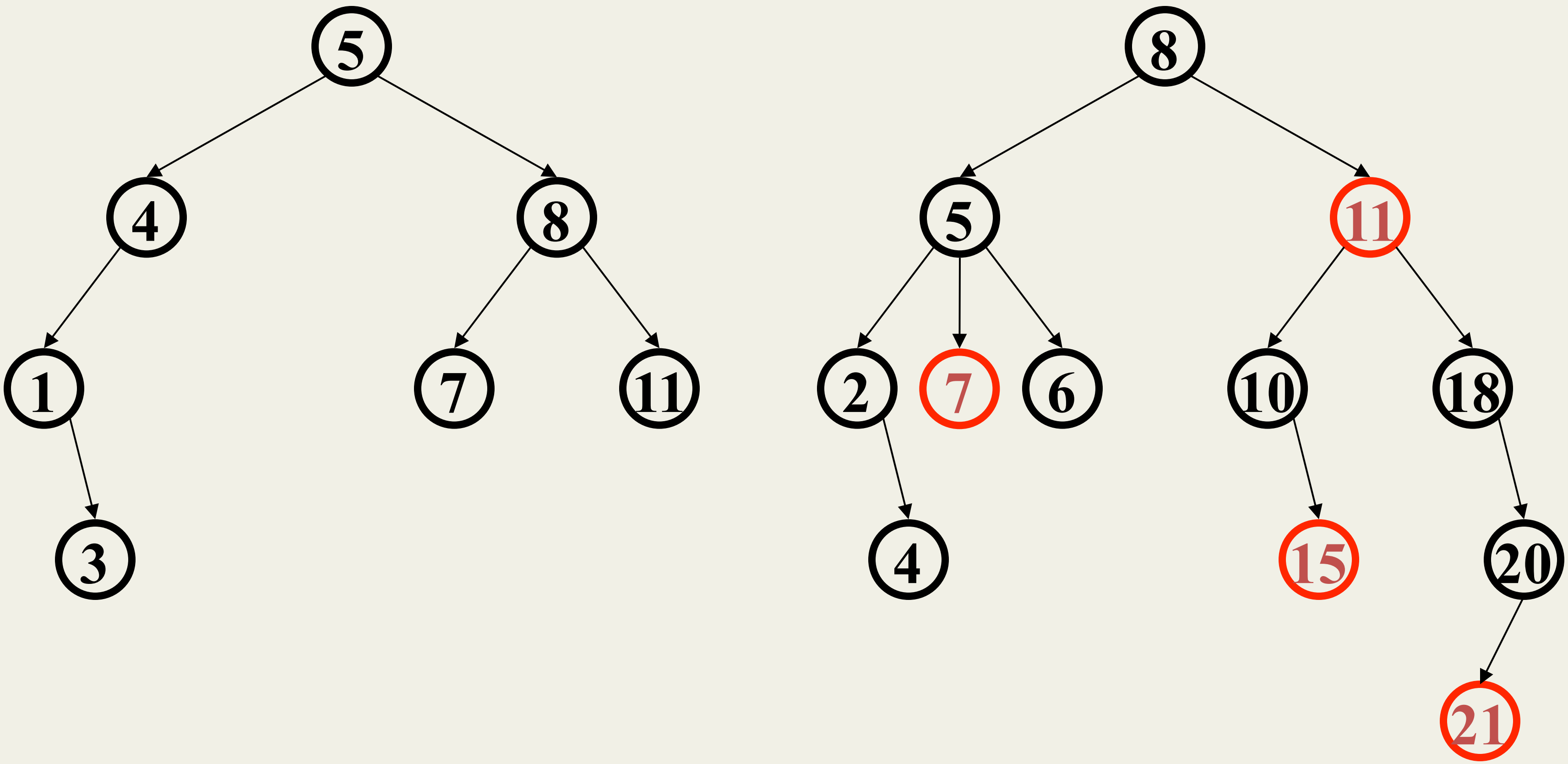
- Structure property ("binary")
  - Each node has ≤ 2 children
  - Result: keeps operations simple
- Order property
  - All keys in left subtree smaller than node's key
  - All keys in right subtree larger than node's key
  - Result: easy to find any given key



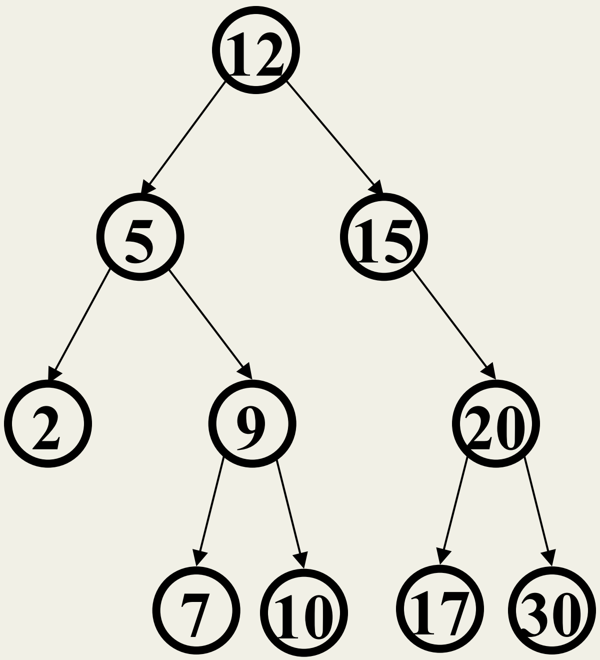
Are these BSTs?



Are these BSTs?

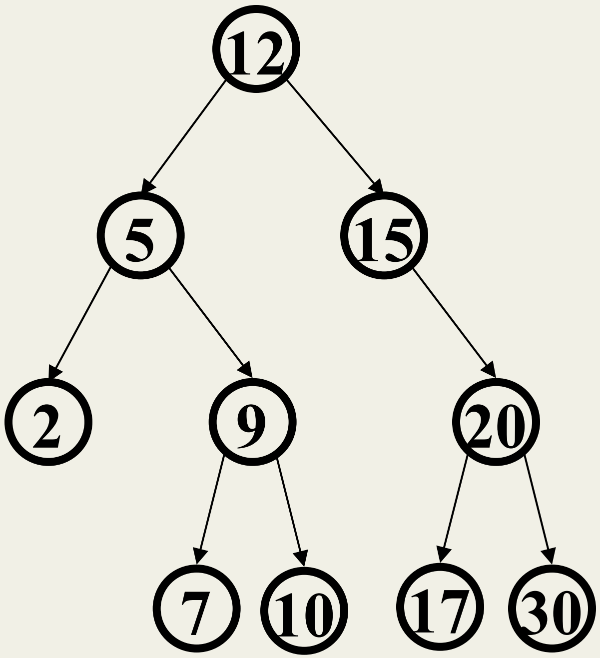


Find in BST, Recursive




```
int find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

Find in BST, Iterative

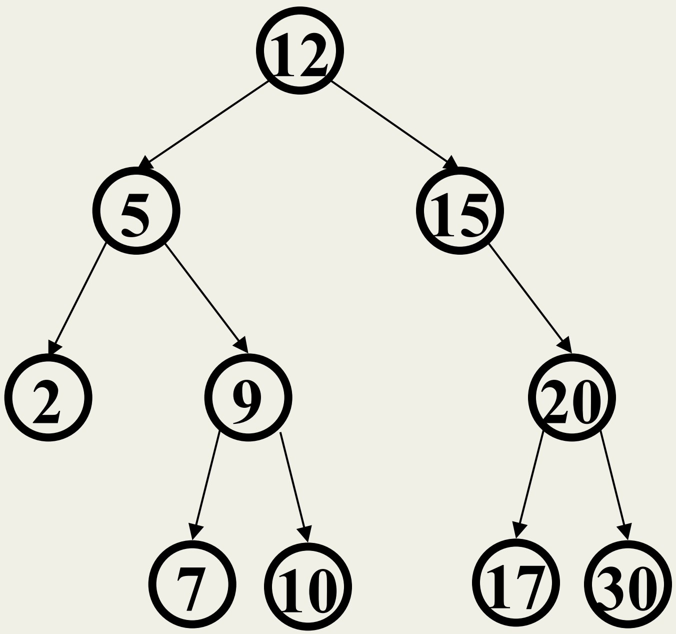



```
int find(Key key, Node root){
 while(root != null && root.key != key){
  if(key < root.key)
    root = root.left;
  else(key > root.key)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.data;
}
```

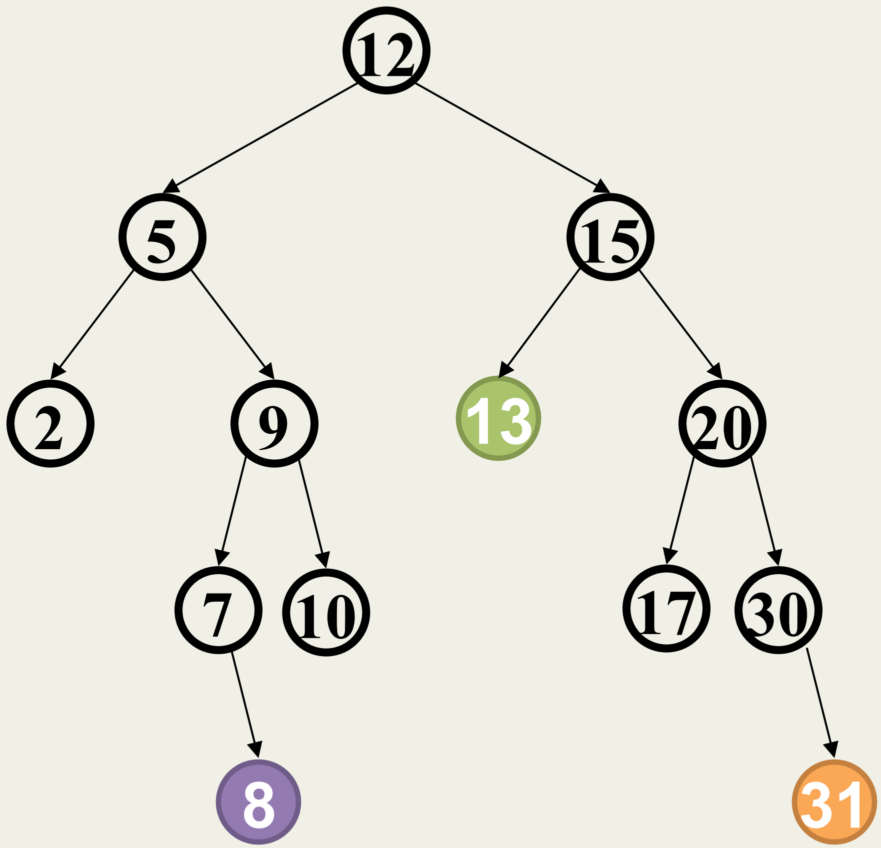
Other "Finding" Operations

- Find *minimum* node
  - "the Ralph Nader algorithm"
- Find *maximum* node
  - "the Zoolander algorithm"

- Find *predecessor* of a non-leaf
- Find *successor* of a non-leaf
- Find *predecessor* of a leaf
- Find *successor* of a leaf



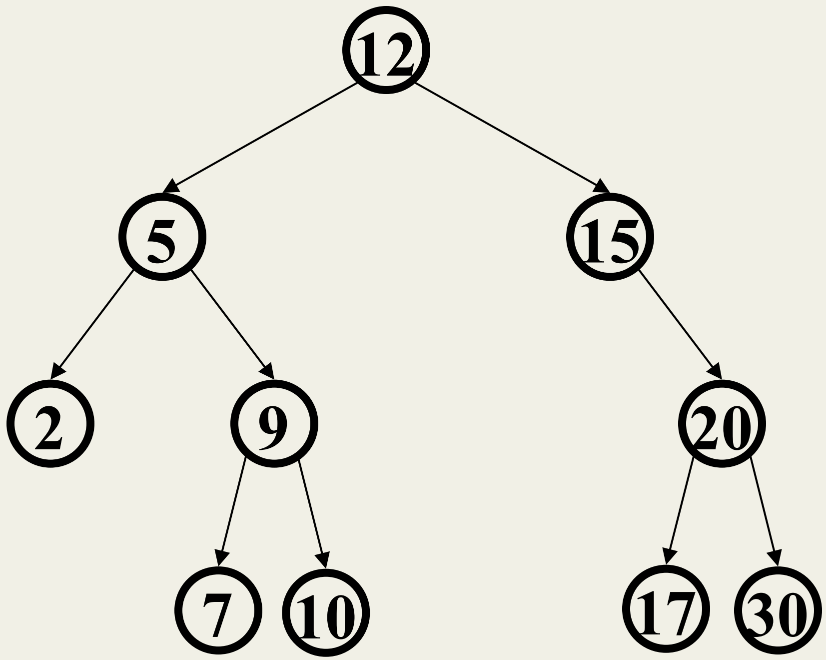
Insert in BST




```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Deletion in BST



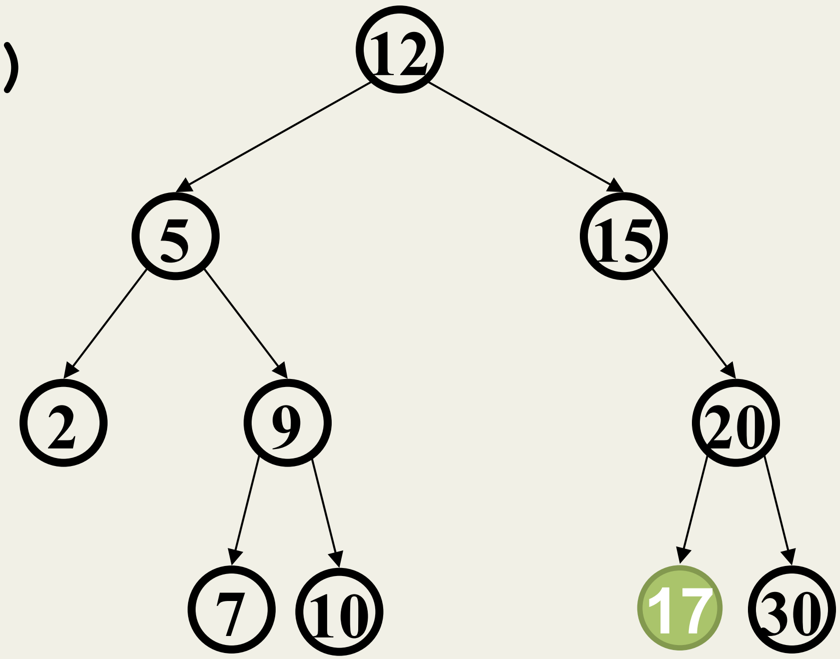
Why might deletion be harder than insertion?

Deletion

- Removing an item disrupts the tree structure
- Basic idea: **`find`** the node to be removed, then "fix" the tree so that it is still a binary search tree
- Three cases:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

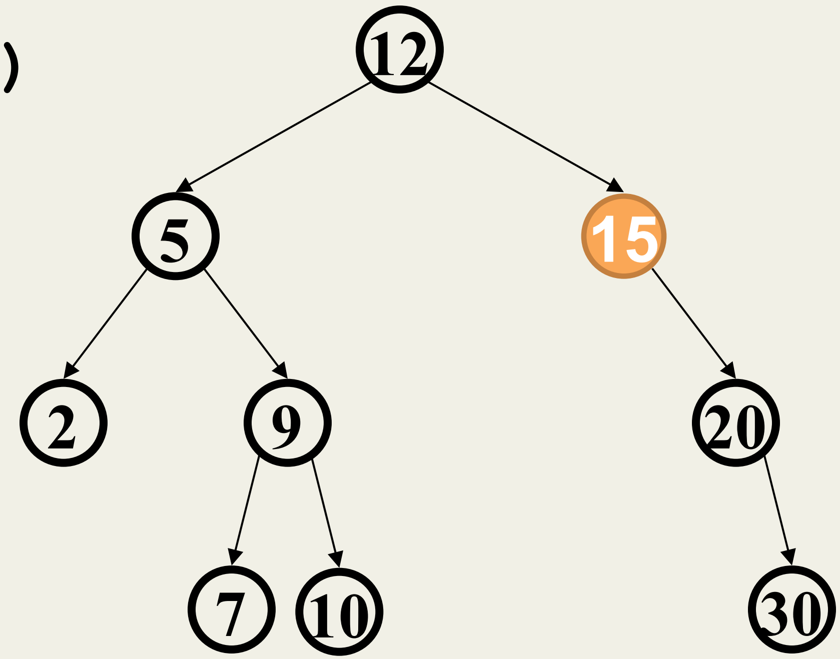
Deletion – The Leaf Case

`delete(17)`



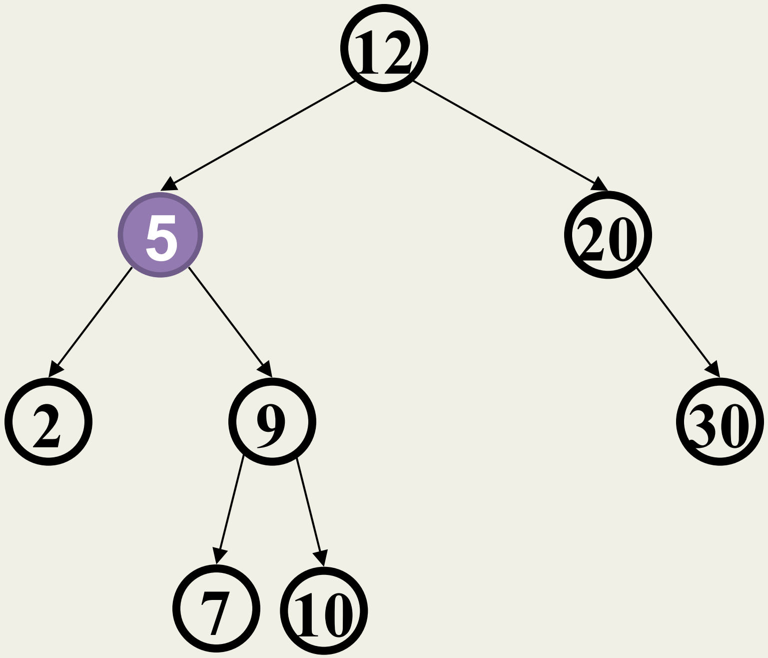
Deletion – The One Child Case

`delete(15)`



Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

- *successor* from right subtree: **`findMin(node.right)`**
- *predecessor* from left subtree: **`findMax(node.left)`**
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- Leaf or one child case – easy cases of delete!

Lazy Deletion

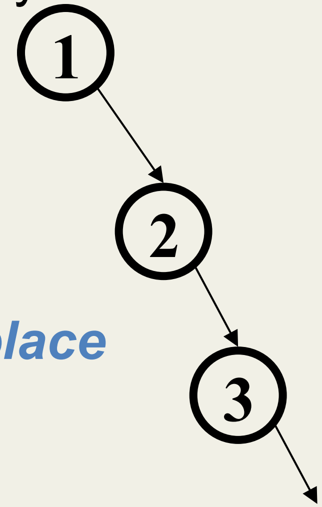
- Lazy deletion can work well for a BST
  - Simpler
  - Can do "real deletions" later as a batch
  - Some inserts can just "undelete" a tree node

- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - How would you change **`findMin`** and **`findMax`**?

BuildTree for BST

- Let's consider **`buildTree`**
  - Insert all, starting from an empty tree

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order, what is the tree?
  - What big-O runtime for this kind of sorted input?
  - Is inserting in the reverse order any better?



$O(n^2)$

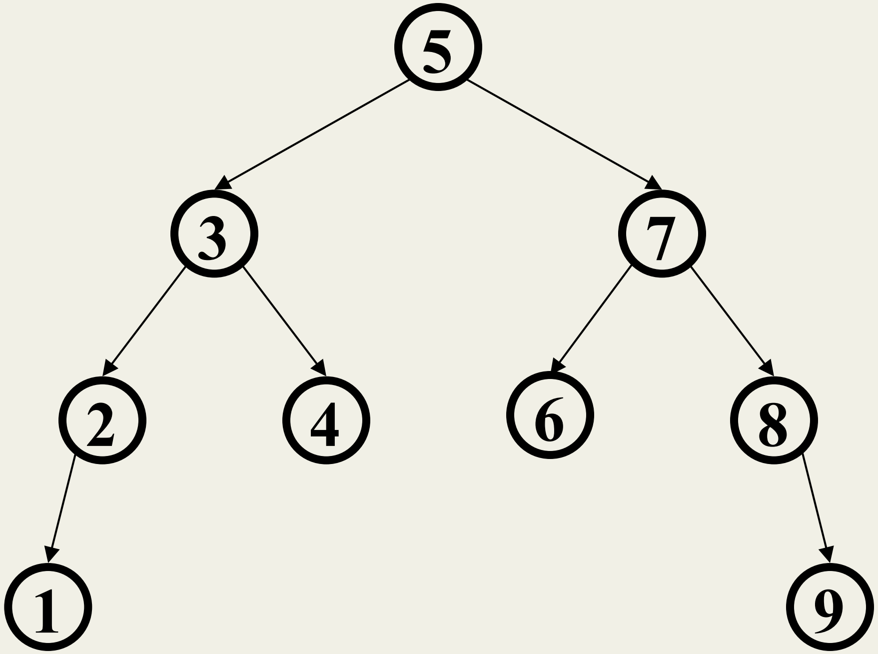
Not a happy place

BuildTree for BST

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

- What we if could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9
  - What tree does that give us?
  - What big-O runtime?

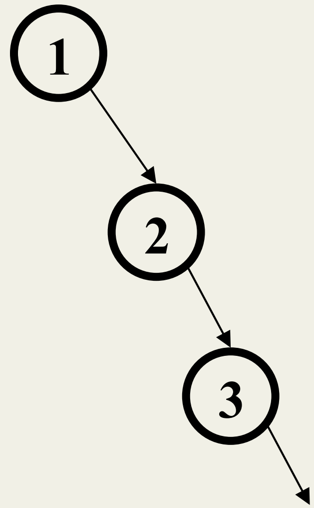
  O(n log n), definitely better



Unbalanced BST

- Balancing a tree at build time is insufficient, as sequences of operations can eventually transform that carefully balanced tree into the dreaded list

- At that point, everything is *O(n)* and nobody is happy
  - **`find`**
  - **`insert`**
  - **`delete`**



Balanced BST

Observation

- BST: the shallower the better!
- For a BST with n nodes inserted in arbitrary order
  - Average height is $O(\log n)$ – see text for proof
  - Worst case height is $O(n)$
- Simple cases, such as inserting in key order, lead to the worst-case scenario

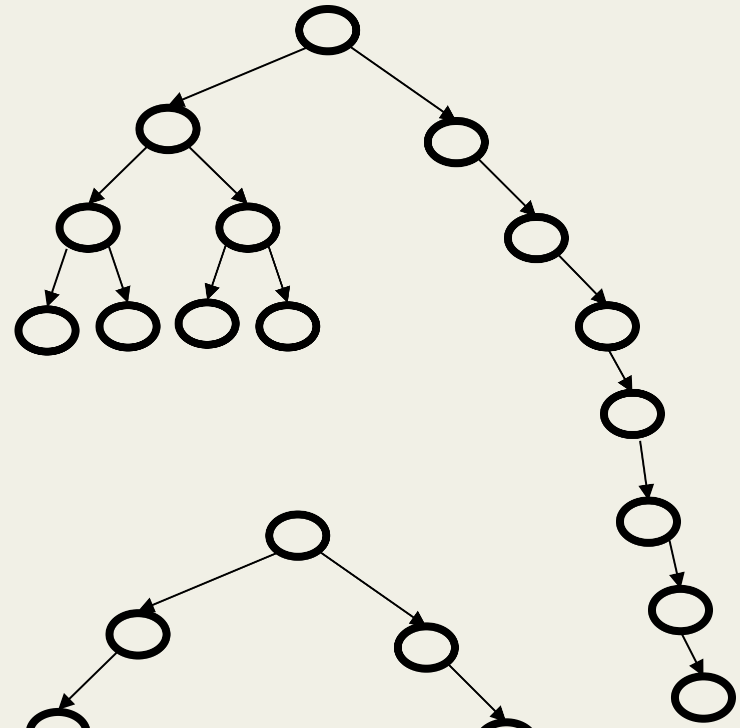
Solution: Require a **Balance Condition** that

1. Ensures depth is always $O(\log n)$ – strong enough!
2. Is efficient to maintain – not too strong!

Potential Balance Conditions

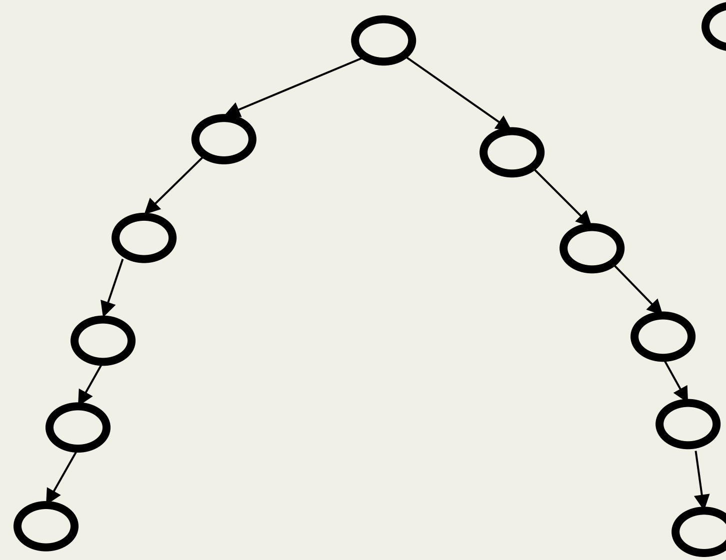
1. Left and right subtrees of the *root* have equal number of nodes

   Too weak!
   Height mismatch example:



2. Left and right subtrees of the *root* have equal *height*

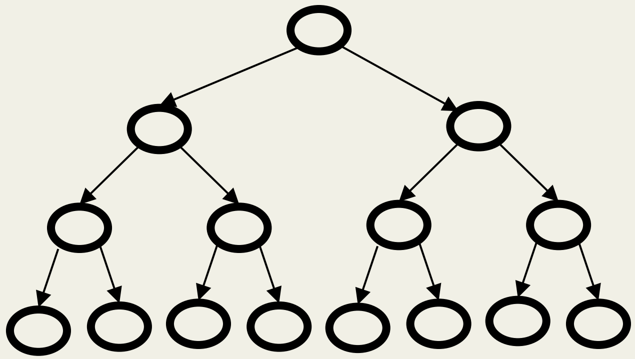
   Too weak!
   Double chain example:



Potential Balance Conditions

3. Left and right subtrees of every node have equal number of nodes

Too strong!
Only perfect trees (2^n – 1 nodes)



4. Left and right subtrees of every node have equal *height*

Too strong!
Only perfect trees (2^n – 1 nodes)

The AVL Balance Condition

Left and right subtrees of *every node*
have *heights* **differing by at most 1**

Definition: **balance**(*node*) = height(*node*.left) – height(*node*.right)

AVL *property*: **for every node *x*, –1 ≤ balance(*x*) ≤ 1**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height *h* must have a number of nodes *exponential* in *h*

- Efficient to maintain
  - Using single and double rotations