CSE 373

Graphs 3: Implementation

reading: Weiss Ch. 9

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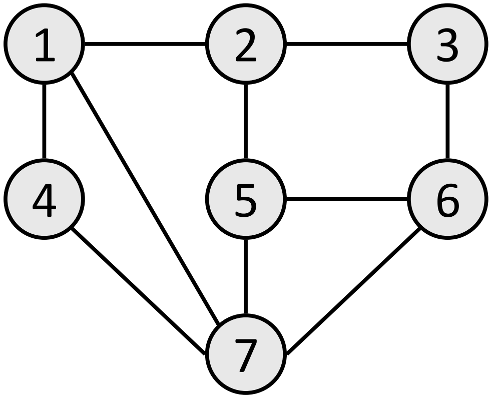
Implementing a graph

- If we wanted to program an actual data structure to represent a graph, what information would we need to store?
  - for each vertex? for each edge?

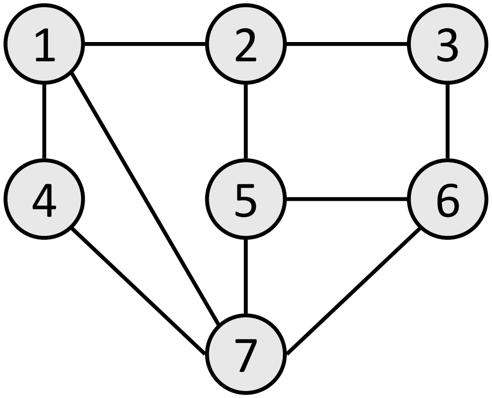


- What kinds of questions would we want to be able to answer quickly:
  - about a vertex?
  - about edges / neighbors?
  - about paths?
  - about what edges exist in the graph?

- We'll explore three common graph implementation strategies:
  - *edge list*, *adjacency list*, *adjacency matrix*

Edge list

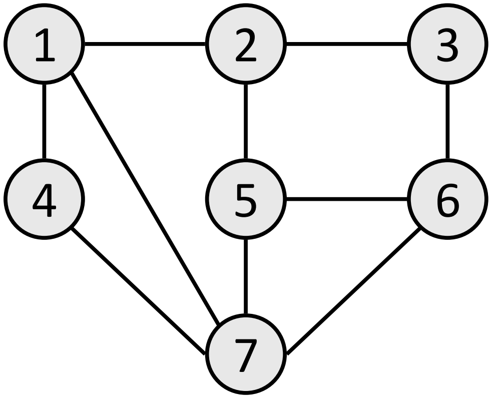
- **edge list**: An unordered list of all edges in the graph.
    - an array, array list, or linked list
- *advantages*:
    - easy to loop/iterate over all edges
- *disadvantages*:
    - hard to quickly tell if an edge exists from vertex *A* to *B*
    - hard to quickly find the degree of a vertex (how many edges touch it)



0	1	2	3	4	5	6	7	8
(1, 2)	(1, 4)	(1, 7)	(2, 3)	2, 5)	(3, 6)	(4, 7)	(5, 6)	(6, 7)

Graph operations

- Using an edge list, how would you find:
    - all neighbors of a given vertex?
    - the degree of a given vertex?
    - whether there is an edge from *A* to *B*?
    - whether there are any loops (self-edges)?
        - What is the Big-Oh of each operation?

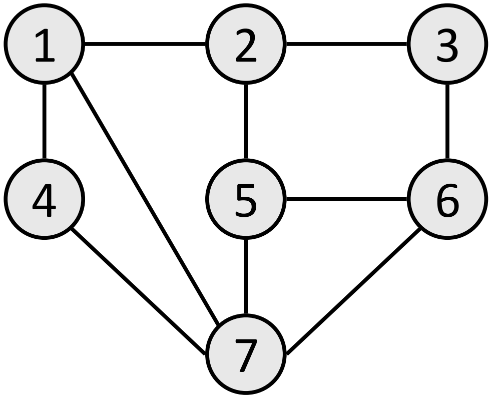


0	1	2	3	4	5	6	7	8
(1, 2)	(1, 4)	(1, 7)	(2, 3)	2, 5)	(3, 6)	(4, 7)	(5, 6)	(6, 7)

Adjacency matrix

- **adjacency matrix**: An *N* × *N* matrix where:
  - the non-diagonal entry *a*[*i*,*j*] is the number of edges joining vertex *i* and vertex *j* (or the weight of the edge joining vertex *i* and vertex *j*).
  - the diagonal entry *a*[*i*,*i*] corresponds to the number of loops (self-connecting edges) at vertex *i* *(often disallowed)*.
  - in an undirected graph, *a*[*i*,*j*] = a[*j*,*i*] for all *i*, *j*. *(diagonally symmetric)*

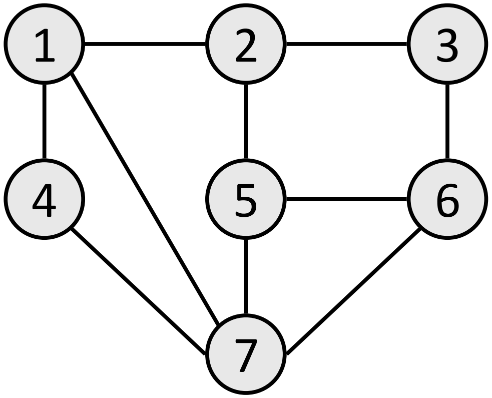
	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0



Graph operations

- Using an *adjacency matrix*, how would you find:
  - all neighbors of a given vertex?
  - the degree of a given vertex?
  - whether there is an edge from *A* to *B*?
  - whether there are any loops (self-edges)?
    - What is the Big-Oh of each operation?

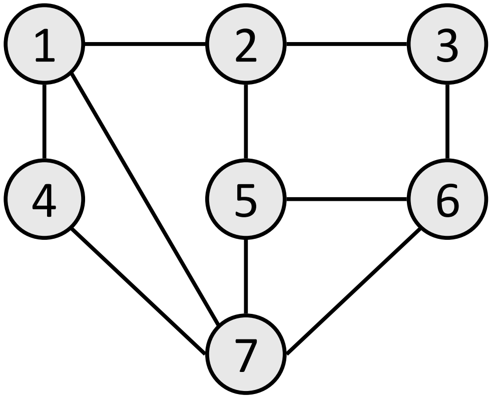
	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0



Adj matrix pros / cons

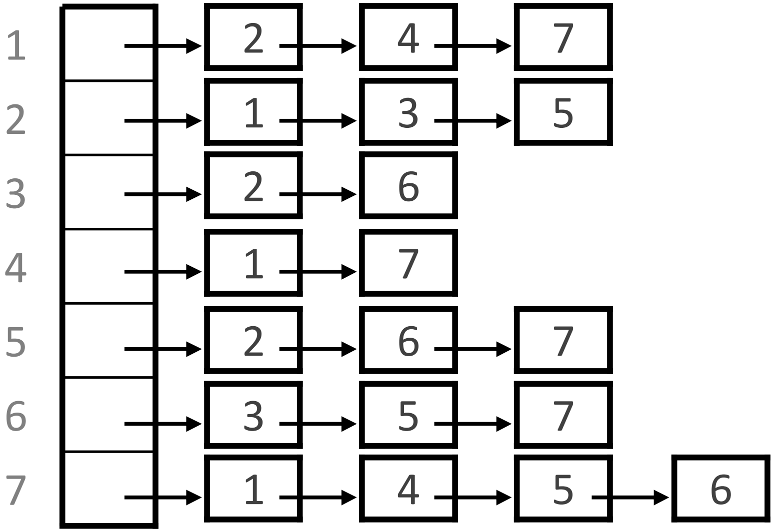
- *advantages*:
  - fast to tell whether an edge exists between any two vertices *i* and *j* (and to get its weight)
- *disadvantage*:
  - consumes a lot of memory on sparse graphs (ones with few edges)

	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0

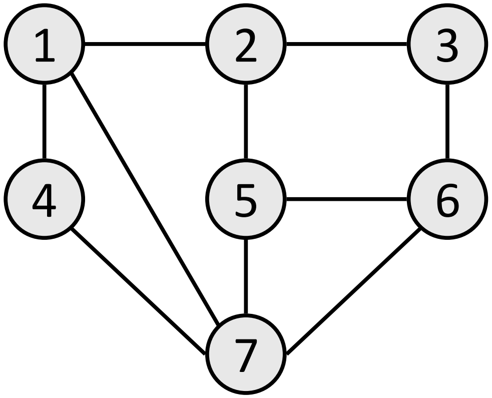


Adjacency list

- **adjacency list**: Stores edges as individual linked lists of references to each vertex's neighbors.
  - in unweighted graphs, the lists can simply be references to other vertices and thus use little memory
  - in undirected graphs, edge (*i*, *j*) is stored in both *i*'s and *j*'s lists

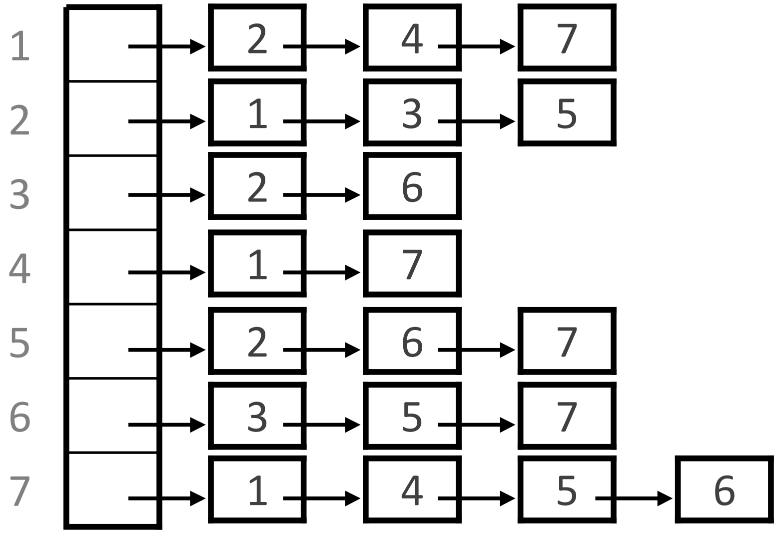


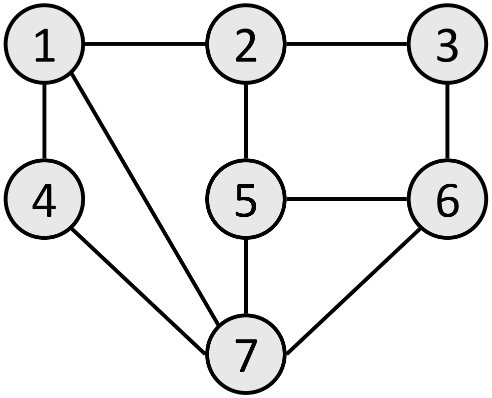
1 → 2 → 4 → 7
2 → 1 → 3 → 5
3 → 2 → 6
4 → 1 → 7
5 → 2 → 6 → 7
6 → 3 → 5 → 7
7 → 1 → 4 → 5 → 6



Graph operations

- Using an *adjacency list*, how would you find:
  - all neighbors of a given vertex?
  - the degree of a given vertex?
  - whether there is an edge from *A* to *B*?
  - whether there are any loops (self-edges)?
    - What is the Big-Oh of each operation?





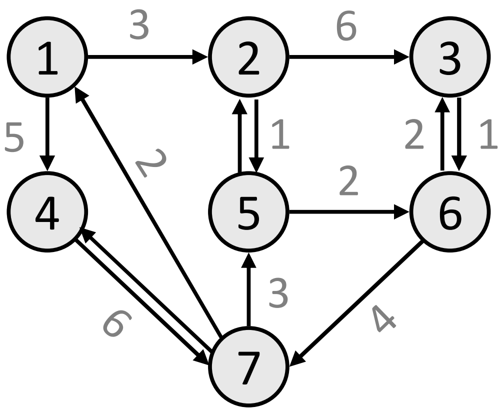
Adj list pros / cons

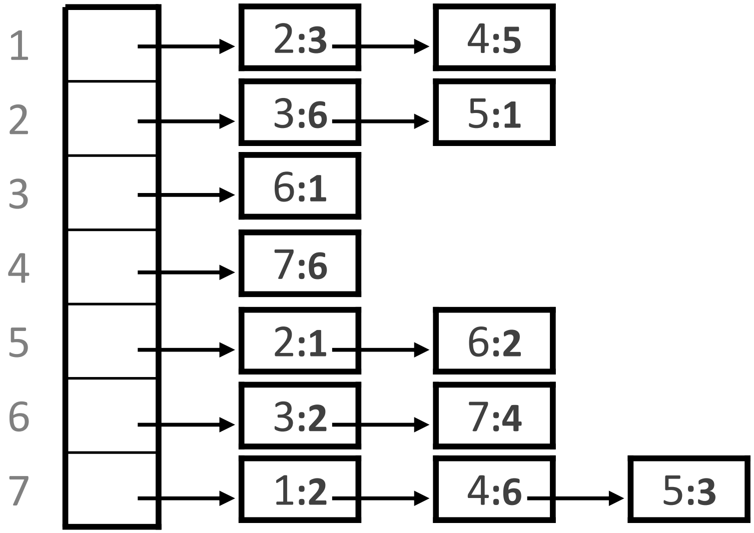
- *advantages*:
  - new vertices can be added to the graph easily, and they can be connected with existing nodes simply by adding elements to the appropriate arrays;
  - easy to find all neighbors of a given vertex (and its degree)

- *disadvantages*:
  - determining whether an edge exists between two vertices requires O(*N*) time, where *N* is the average number of edges per node

Weighted/directed graphs

- *weighted*:
  - *adj. list*: store weight in each edge node
  - *adj. matrix*: store weight in each matrix box
- directed:
  - *adj. list*: edges appear only in start vertex's list
  - *adj. matrix*: no longer diagonally symmetric





1	2:3	4:5	
2	3:6	5:1	
3	6:1		
4	7:6		
5	2:1	6:2	
6	3:2	7:4	
7	1:2	4:6	5:3

	1	2	3	4	5	6	7
1	0	**3**	0	**5**	0	0	0
2	0	0	**6**	0	**1**	0	0
3	0	0	0	0	0	**1**	0
4	0	0	0	0	0	0	**6**
5	0	**1**	0	0	0	**2**	0
6	0	0	**2**	0	0	0	**4**
7	**2**	0	0	**6**	**3**	0	0

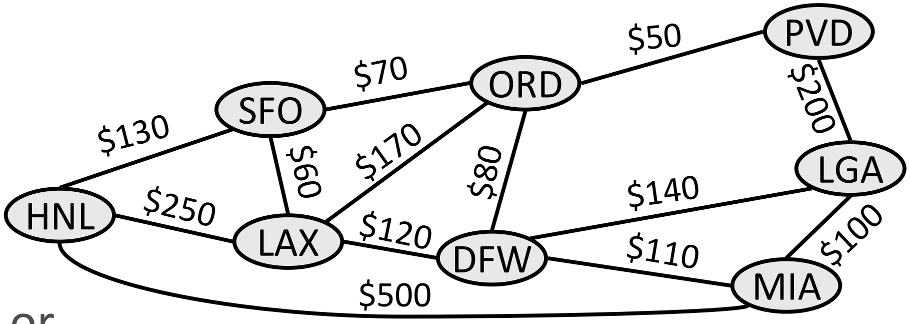
Runtime comparison

- *|V| vertices, |E| edges*
- *no parallel edges*
- *no self-loops*

	Edge List	Adjacency List	Adjacency Matrix
Memory usage	$\lvert V \rvert + \lvert E \rvert$	$\lvert V \rvert + \lvert E \rvert$	$\lvert V \rvert^2$
Find all neighbors of ***v***	$\lvert E \rvert$	degree(***v***)	$\lvert V \rvert$
Is ***v*** a neighbor of ***w***?	$\lvert E \rvert$	degree(***v***)	1
add a vertex	1	1	$\lvert V \rvert^2$
add an edge	1	1	1
remove a vertex	$\lvert E \rvert$	1	$\lvert V \rvert^2$
remove an edge	$\lvert E \rvert$	deg(***v***)	1

Representing vertices

- Not all graphs have vertices/edges that are easily "numbered".
  - How do we represent lists or matrices of vertex/edge relationships?
  - How do we quickly look up edges or vertices near a given vertex?



  - edge list:
    - `List<Edge>`
  - adjacency list:
    - `Map<Vertex, List<Edge>>` or
    - `Multimap<Vertex, Edge>`
  - adjacency matrix:
    - `Map<Vertex, Map<Vertex, Edge>>` or
    - `Table<Vertex, Vertex, Edge>`

A graph ADT

- As with other ADTs, we can create a Graph ADT interface:

```
public interface Graph<V, E> {
    void addEdge(V v1, V v2, E e, int weight);
    void addVertex(V v);
    void clear();
    boolean containsEdge(E e);
    boolean containsEdge(V v1, V v2);
    boolean containsVertex(V v);
    int cost(List<V> path);
    int degree(V v);
    E edge(V v1, V v2);
    int edgeCount();
    Set<E> edges();
    int edgeWeight(V v1, V v2);
```

A graph ADT, cont'd.

```
// public interface Graph<V, E> {
    ...
    boolean isDirected();
    boolean isEmpty();
    boolean isReachable(V v1, V v2);    // DFS
    boolean isWeighted();
    List<V> minimumWeightPath(V v);     // Dijkstra's
    Set<V> neighbors(V v);
    int outDegree(V v);
    void removeEdge(V v1, V v2);
    void removeVertex(V v);
    List<V> shortestPath(V v1, V v2);  // BFS
    String toString();
    int vertexCount();
    Set<V> vertices();
}
```

Info about vertices

- Information stored in each vertex (for internal use):
  - can store various flags and fields for use by path search algorithms

```
public class Vertex<V> {
    public int cost() {...}
    public int number() {...}
    public V previous() {...}
    public boolean visited() {...}

    public void setCost(int cost) {...}
    public void setNumber(int number) {...}
    public void setPrevious(V previous) {...}
    public void setVisited(boolean visited) {...}
    public void clear() {...}    // reset dist,prev,visited
}
```

Info about edges

- Information stored in each edge (for internal use):

```
public class Edge<V, E> {
    public boolean contains(V vertex) {...}
    public E edge() {...}
    public V end() {...}
    public V start() {...}
    public int weight() {...}    // 1 if unweighted
}
```