CSE 373

Advanced heap implementation; ordering/Comparator

read: Weiss Ch. 6

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Int PQ ADT interface

- Let's write our own implementation of a priority queue.
  - To simplify the problem, we only store `int`s in our set for now.
  - As is (usually) done in the Java Collection Framework, we will define sets as an ADT by creating a Set interface.
  - Core operations are: add, peek (at min), remove (min).

```
public interface IntPriorityQueue {
    void add(int value);
    void clear();
    boolean isEmpty();
    int peek();          // return min element
    int remove();        // remove/return min element
    int size();
}
```

Generic PQ ADT

- Let's modify our priority queue so it can store any type of data.
  - As with past collections, we will use Java generics (a type parameter).

```
public interface PriorityQueue<E> {
    void add(E value);
    void clear();
    boolean isEmpty();
    E peek();           // return min element
    E remove();         // remove/return min element
    int size();
}
```

Generic HeapPQ class

- We can modify our heap priority class to use generics as usual...

```
public class HeapPriorityQueue<E>
        implements PriorityQueue<E> {
    private E[] elements;
    private int size;

    // constructs a new empty priority queue
    public HeapPriorityQueue() {
        elements = (E[]) new Object[10];
        size = 0;
    }

    ...
}
```

Problem: ordering elements

```
// Adds the given value to this priority queue in order.
public void add(E value) {
    ...
    int index = size + 1;
    boolean found = false;
    while (!found && hasParent(index)) {
        int parent = parent(index);
        if (elements[index] < elements[parent]) {   // error
            swap(elements, index, parent(index));
            index = parent(index);
        } else {
            found = true;  // found proper location; stop
        }
    }
}
```

- Even changing the < to a `compareTo` call does not work.
  - Java cannot be sure that type `E` has a `compareTo` method.

Comparing objects

- Heaps rely on being able to *order* their elements.
- Operators like < and > do not work with objects in Java.
  - But we do think of some types as having an ordering (e.g. `Date`s).
  - (In other languages, we can enable <, > with *operator overloading*.)

- **natural ordering**: Rules governing the relative placement of all values of a given type.
  - Implies a notion of equality (like `equals`) but also < and > .
  - **total ordering**: All elements can be arranged in A ≤ B ≤ C ≤ ... order.
  - The `Comparable` interface provides a natural ordering.

The Comparable interface

- The standard way for a Java class to define a comparison function for its objects is to implement the `Comparable` interface.

```
public interface Comparable<T> {
    public int compareTo(T other);
}
```

- A call of **A**`.compareTo(`**B**`)` should return:

  a value < 0 if **A** comes "before" **B** in the ordering,
  a value > 0 if **A** comes "after" **B** in the ordering,
  or exactly 0 if **A** and **B** are considered "equal" in the ordering.

- **Effective Java Tip #12**: Consider implementing `Comparable`.

Bounded type parameters

<**Type** extends **SuperType**>

- An upper bound; accepts the given supertype or any of its subtypes.
- Works for multiple superclass/interfaces with & :

  <**Type** extends **ClassA** & **InterfaceB** & **InterfaceC** & **...**>

<**Type** super **SuperType**>

- A lower bound; accepts the given supertype or any of its supertypes.

- Example:

```
// can be instantiated with any animal type
public class Nest<T extends Animal> {
    ...
}
...
Nest<Bluebird> nest = new Nest<Bluebird>();
```

Corrected HeapPQ class

```
public class HeapPriorityQueue<E extends Comparable<E>>
        implements PriorityQueue<E> {
    private E[] elements;
    private int size;

    // constructs a new empty priority queue
    public HeapPriorityQueue() {
        elements = (E[]) new Object[10];
        size = 0;
    }
    ...
    public void add(E value) {
        ...
        while (...) {
            if (elements[index].compareTo(
                    elements[parent]) < 0) {
                swap(...);
            }
        }
    }
}
```

Other heap operations

- Java collections support these methods in addition to the ones we listed. How would we implement them in our heap PQ?
  - (What would be each method's Big-Oh?)

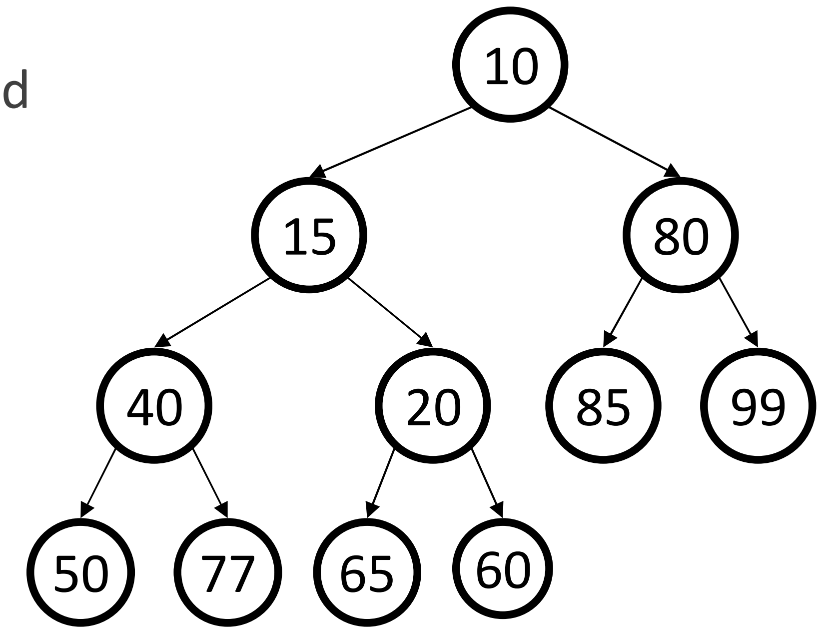
- `public boolean contains(E element)`
  - returns true if the priority queue contains the given value

- `public void remove(E element)`
  - deletes an arbitrary element in the priority queue, if it is found

- `public String toString()`
  - returns a string representation of the priority queue's elements

The contains operation

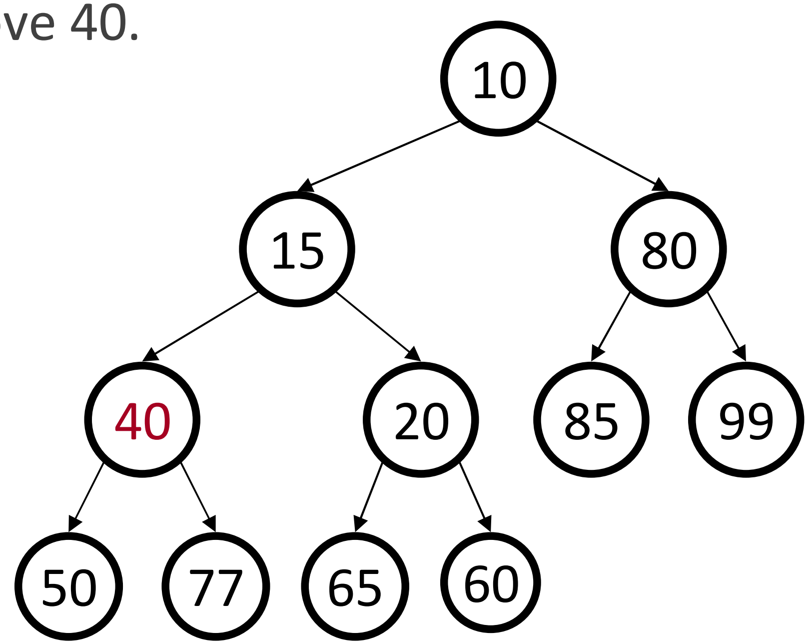
- Though there is ordering to the heap, it is not easy to take advantage of the ordering to optimize `contains`.
  - Why not?
  - What elements *must* be examined to see if the heap contains:
    - 11?
    - 19?
    - 31?
  - In practice we usually just loop over the heap array linearly.



index	0	1	2	3	4	5	6	7	8	9	0	1	2	...
value	0	10	15	80	40	20	85	99	50	77	65	60	0	...
size	11													

Removing arbitrary element

- Similar to `contains`, removing an arbitrary element from a heap is not easy to optimize because you must first *find* the value.
  - Suppose the client wants to remove 40.
  - How can we remove it safely without disturbing the heap?

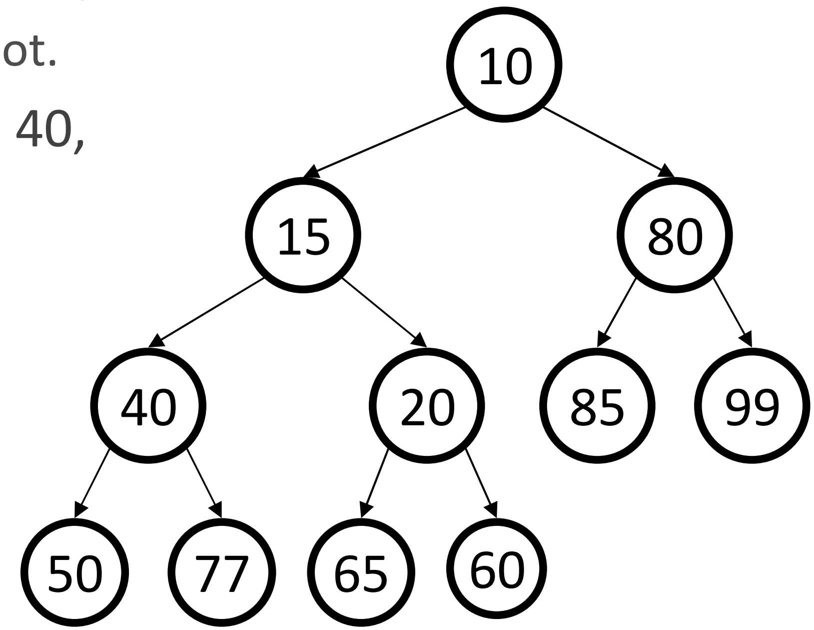


index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*...*
value	0	10	15	80	40	20	85	99	50	77	65	60	0	...
size	11													

Implementing remove

```
queue.remove(40);
```

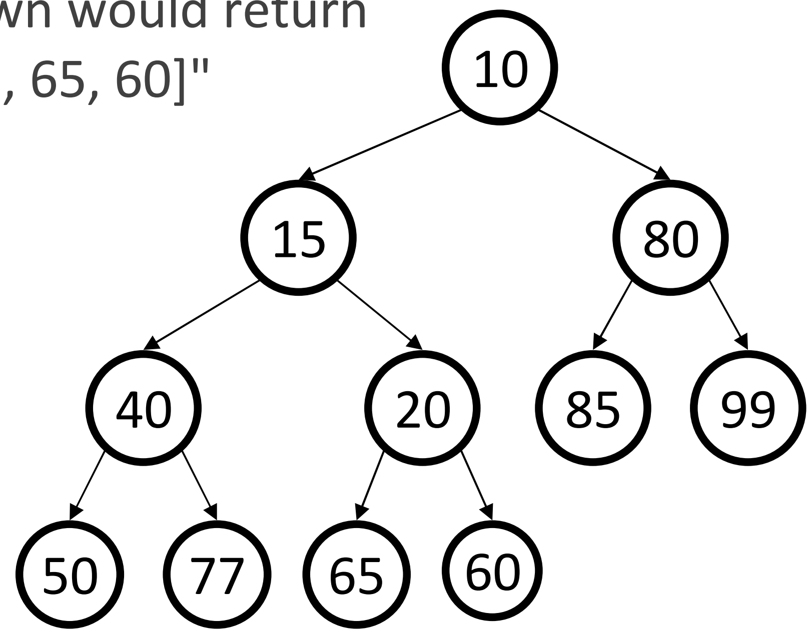
- *Step 1:* Pretend 40's value is -∞ (very small)
  - Bubble 40 all the way up to the root.
- *Step 2:* Perform a remove-min on 40, which is currently the root.
  - Do it the same as usual:
    Swap up the rightmost leaf (60),
    then bubble that leaf down.



index	0	1	2	3	4	5	6	7	8	9	0	1	2	...
value	0	10	15	80	40	20	85	99	50	77	65	60	0	...
size	11													

The toString operation

- A typical heap PQ implementation does "the simple thing" and produces a `toString` with the elements in the heap order.
  - e.g. `toString` on the heap shown would return "[10, 15, 80, 40, 20, 85, 99, 50, 77, 65, 60]"
  - Why not output the elements in their sorted order?
    - Wouldn't that make more sense to the client?



index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*...*
value	0	10	15	80	40	20	85	99	50	77	65	60	0	...
size	11													

Ordering and Comparators

What's the "natural" order?

```
public class Rectangle implements Comparable<Rectangle> {
    private int x, y, width, height;

    public int compareTo(Rectangle other) {
        // ...?
    }
}
```

- What is the "natural ordering" of rectangles?
  - By x, breaking ties by y?
  - By width, breaking ties by height?
  - By area? By perimeter?
- Do rectangles have any "natural" ordering?
  - Might we want to arrange rectangles into some order anyway?

Comparator interface

```
public interface Comparator<T> {
    public int compare(T first, T second);
}
```

- Interface `Comparator` is an external object that specifies a comparison function over some other type of objects.
  - Allows you to define multiple orderings for the same type.
  - Allows you to define a specific ordering(s) for a type even if there is no obvious "natural" ordering for that type.
  - Allows you to externally define an ordering for a class that, for whatever reason, you are not able to modify to make it Comparable:
    - a class that is part of the Java class libraries
    - a class that is final and can't be extended
    - a class from another library or author, that you don't control
    - ...

Comparator examples

```
public class RectangleAreaComparator
        implements Comparator<Rectangle> {
    // compare in ascending order by area (WxH)
    public int compare(Rectangle r1, Rectangle r2) {
        return r1.getArea() - r2.getArea();
    }
}

public class RectangleXYComparator
        implements Comparator<Rectangle> {
    // compare by ascending x, break ties by y
    public int compare(Rectangle r1, Rectangle r2) {
        if (r1.getX() != r2.getX()) {
            return r1.getX() - r2.getX();
        } else {
            return r1.getY() - r2.getY();
        }
    }
}
```

Using Comparators

- `TreeSet`, `TreeMap`, `PriorityQueue` can use `Comparator`:

```
Comparator<Rectangle> comp = new RectangleAreaComparator();
Set<Rectangle> set = new TreeSet<Rectangle>(comp);
Queue<Rectangle> pq = new PriorityQueue<Rectangle>(10,comp);
```

- Searching and sorting methods can accept `Comparator`s.

```
Arrays.binarySearch(array, value, comparator)
Arrays.sort(array, comparator)
Collections.binarySearch(list, comparator)
Collections.max(collection, comparator)
Collections.min(collection, comparator)
Collections.sort(list, comparator)
```

- Methods are provided to reverse a `Comparator`'s ordering:

```
public static Comparator Collections.reverseOrder()
public static Comparator Collections.reverseOrder(comparator)
```

PQ and Comparator

- Our heap priority queue currently relies on the `Comparable` natural ordering of its elements:

```
public class HeapPriorityQueue<E extends Comparable<E>>
        implements PriorityQueue<E> {
    ...
    public HeapPriorityQueue() {...}
}
```

- To allow other orderings, we can add a constructor that accepts a `Comparator` so clients can arrange elements in any order:

```
    ...
    public HeapPriorityQueue(Comparator<E> comp) {...}
```

PQ Comparator exercise

- Write code that stores strings in a priority queue and reads them back out in ascending order *by length*.
  - If two strings are the same length, break the tie *by ABC order*.

```
Queue<String> pq = new PriorityQueue<String>(...);
pq.add("you");
pq.add("meet");
pq.add("madam");
pq.add("sir");
pq.add("hello");
pq.add("goodbye");
while (!pq.isEmpty()) {
    System.out.print(pq.remove() + " ");
}

// sir you meet hello madam goodbye
```

PQ Comparator answer

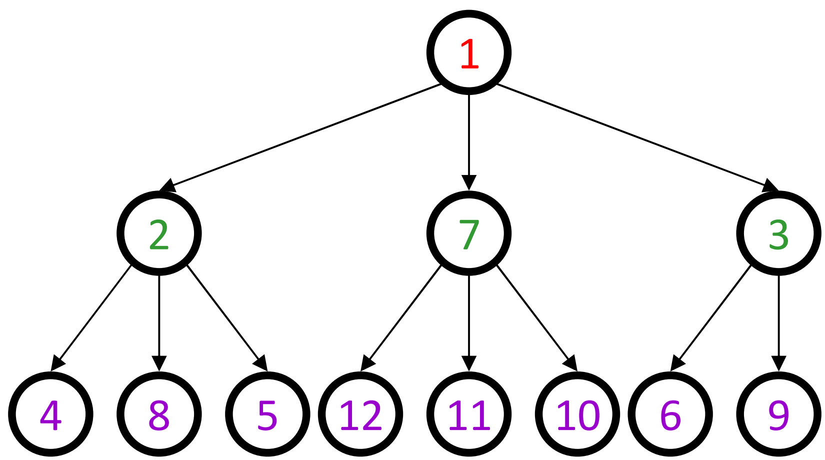
- Use the following comparator class to organize the strings:

```
public class LengthComparator
        implements Comparator<String> {
    public int compare(String s1, String s2) {
        if (s1.length() != s2.length()) {
            // if lengths are unequal, compare by length
            return s1.length() - s2.length();
        } else {
            // break ties by ABC order
            return s1.compareTo(s2);
        }
    }
}
...
Queue<String> pq = new PriorityQueue<String>(100,
                                     new LengthComparator());
```

d-heaps; heap sort

Generalization: d-Heaps

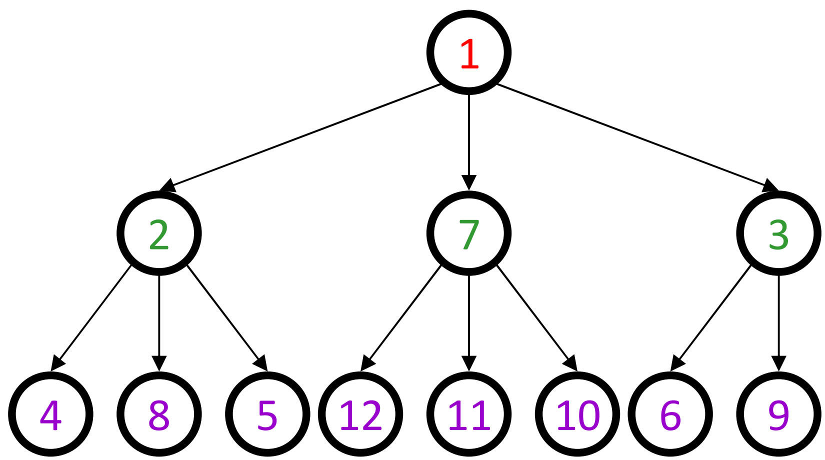
- **d-heap**: one where each node has *d* children (*d* ≥ 2)
  - Can still be represented by an array.
  - How does its height compare to that of a binary (*d* = 2) heap?
  - Example, a 3-heap:



1	2	7	3	4	8	5	12	11	10	6	9

d-heap runtime

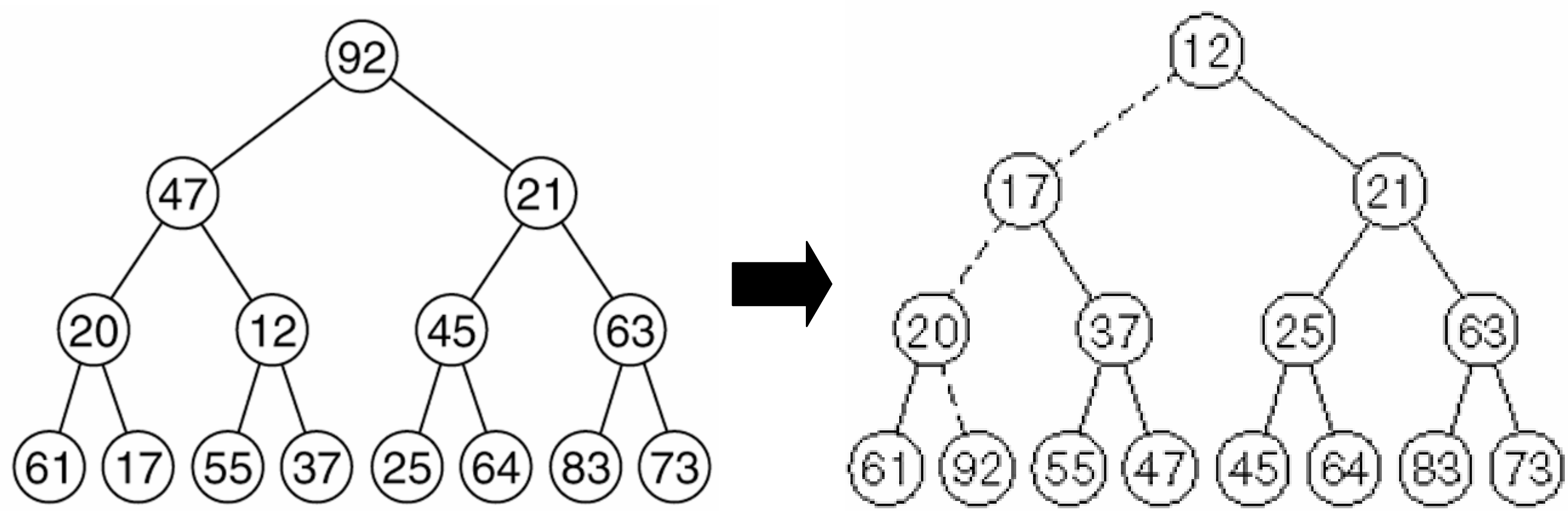
- What is the effect on runtime of using a *d*-heap?
  - `add`: $O(\log_d N)$ - fewer parents to examine; faster.
  - `peek`: $O(1)$
  - `remove`: $O(d \log_d N)$ - must look at all *d* children each time; slower.
    - Adding is slightly faster; removing is slightly slower.



1	2	7	3	4	8	5	12	11	10	6	9

Heap sort

- **heap sort**: An algorithm to sort an array of *N* elements by turning the array into a heap, then calling `remove` *N* times.
  - The elements will come out in sorted order.
  - We can put them into a new sorted array.
  - What is the runtime?



Heap sort implementation

```
public static void heapSort(int[] a) {
    PriorityQueue<Integer> pq =
            new HeapPriorityQueue<Integer>();
    for (int n : a) {
        pq.add(a);
    }
    for (int i = 0; i < a.length; i++) {
        a[i] = pq.remove();
    }
}
```

- This code is correct and runs in O(*N* log *N*) time but wastes memory.
- It makes an entire copy of the array `a` into the internal heap of the priority queue.
- Can we perform a heap sort without making a copy of `a`?

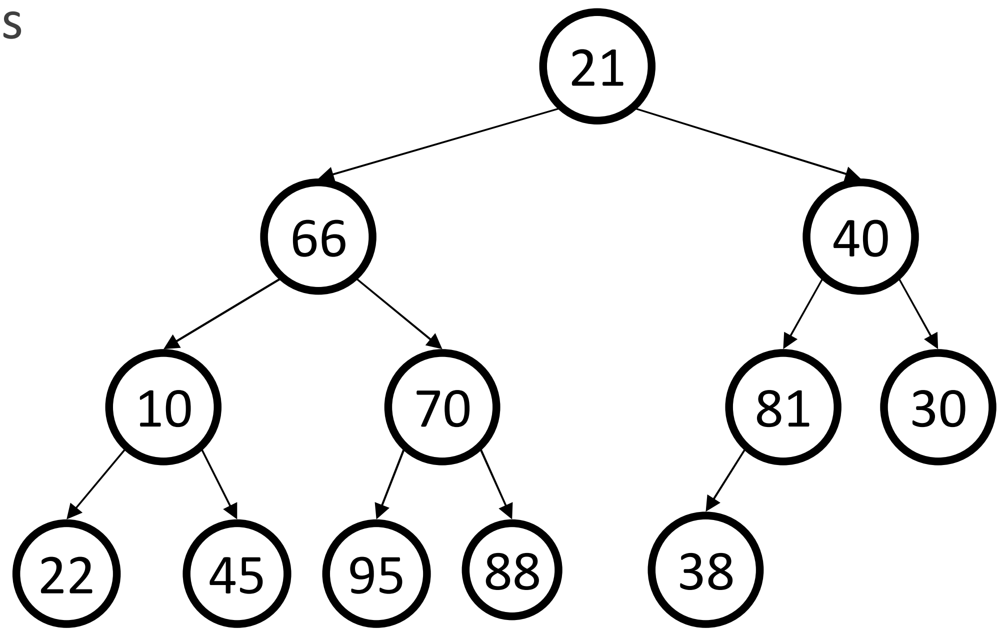
Improving the code

- *Idea:* Treat `a` itself as a max-heap, whose data starts at 0 (not 1).
  - `a` is not actually in heap order.
  - But if you repeatedly "bubble down" each *non-leaf* node, starting from the last one, you will eventually have a proper heap.

- Now that `a` is a valid max-heap:
  - Call `remove` repeatedly until the heap is empty.
  - But make it so that when an element is "removed", it is moved to the end of the array instead of completely evicted from the array.
  - When you are done, voila! The array is sorted.

Step 1: Build heap in-place

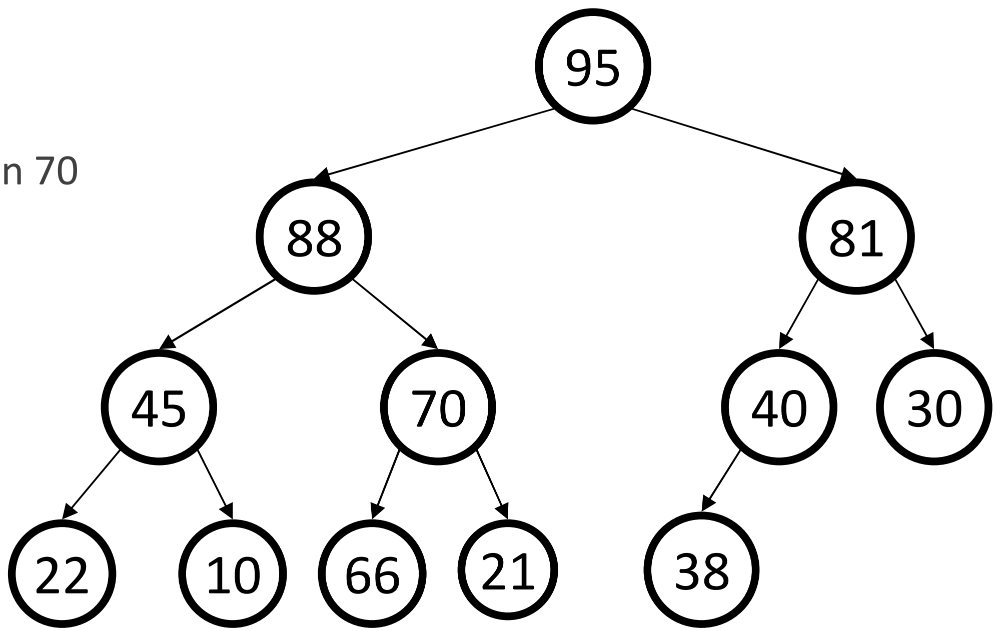
- "Bubble" down non-leaf nodes until the array is a *max*-heap:
  - `int[] a = {21, 66, 40, 10, 70, 81, 30, 22, 45, 95, 88, 38};`
  - Swap each node with its larger child as needed.



index	0	1	2	3	4	5	6	7	8	9	0	1	2	...
value	21	66	40	10	70	81	30	22	45	95	88	38	0	...
size	12													

Build heap in-place answer

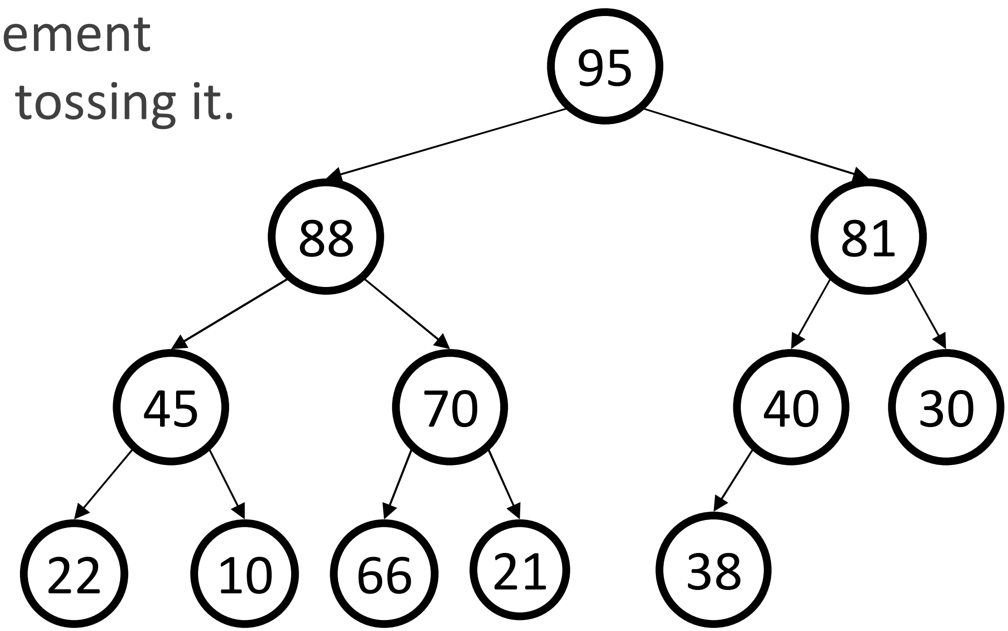
- 30: nothing to do
- 81: nothing to do
- 70: swap with 95
- 10: swap with 45
- 40: swap with 81
- 66: swap with 95, then 88
- 21: swap with 95, then 88, then 70



index	0	1	2	3	4	5	6	7	8	9	0	1	2	...
value	95	88	81	45	70	40	30	22	10	66	21	38	0	...
size	12													

Remove to sort

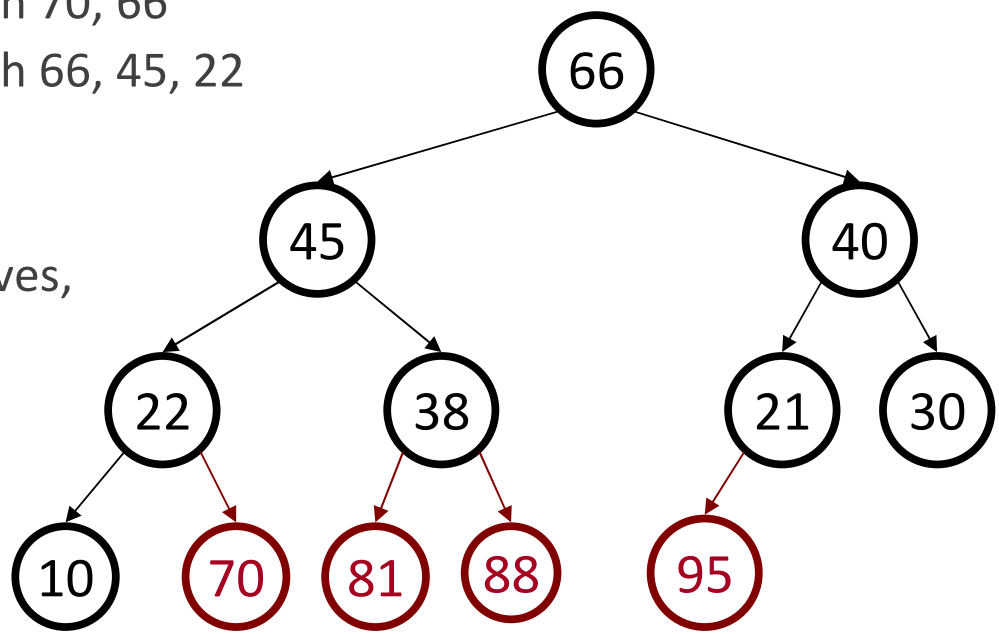
- Now that we have a max-heap, remove elements repeatedly until we have a sorted array.
  - Move each removed element to the end, rather than tossing it.



index	0	1	2	3	4	5	6	7	8	9	0	1	2	...
value	95	88	81	45	70	40	30	22	10	66	21	38	0	...
size	12													

Remove to sort answer

- 95: move 38 up, swap with 88, 70, 66
- 88: move 21 up, swap with 81, 40
- 81: move 38 up, swap with 70, 66
- 70: move 10 up, swap with 66, 45, 22
- ...
- (Notice that after 4 removes, the last 4 elements in the array are sorted. If we remove every element, the entire array will be sorted.)



index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*...*
value	66	45	40	22	38	21	30	10	70	81	88	95	0	...
size	12													