

CSE 373 Optional Section

Runtime Analysis

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Interview Question / Runtime Analysis

- How do we find the integer square root of a number?

$$\sqrt{x}$$

Integer Square Root Solution 1 $\lfloor \sqrt{x} \rfloor$

- Increment through the list of numbers from 1 to x . Call this value i .
- What cases do we have?
 - If $i \times i < x$ increment i
 - If $i \times i == x$ return i
 - If $i \times i > x$ return $i - 1$

Integer Square Root Solution 1 $\lfloor \sqrt{x} \rfloor$

- Implementation:

```
int i = 1
while(i * i < x)
    i++
return i - 1
```

- Runtime
 - $O(n^{0.5})$
- Implement it in Java together & add timing

Integer Square Root Solution 2 $\lfloor \sqrt{x} \rfloor$

- Binary search
- Imagine all of the numbers are in an array (indexed by itself) and search for the integer root
- What number should we start with first?
 - 0, 1, $x / 2$, x

Integer Square Root Solution 2 $\left[\sqrt{x} \right]$

- Starting with i , $jump = x$; $search(x, i, jump)$
- Proposition A: $i \times i \leq x$
- Proposition B: $(i + 1) \times (i + 1) > x$
- What do we do in each case?
 - A & B return i
 - A only jump /= 2; search(x, i + jump, jump)
 - B only jump /= 2; search(x, i - jump, jump)
 - Neither throw *WhatJustHappenedException()*

Integer Square Root Solution 2 $\lfloor \sqrt{x} \rfloor$

- Implement

```
intSqrt(x):
    return search(x, x / 2, x / 2)
Search(x, i, jump):
    if(i * i <= x)
        if((i+1)(i+1) > x)
            return i
        else
            jump /= 2
            return search(x, i + jump, jump)
    else
        jump /= 2
        return search(x, i - jump, jump)
```

Integer Square Root Solution 2 $\lfloor \sqrt{x} \rfloor$

- Runtime
 - $T(1) = 1$
 - $T(N) = 1 + T(N / 2)$

 - $O(\log(n))$

Stuck on Proof by Induction?

- Khan Academy!
 - https://www.khanacademy.org/math/trigonometry/seq_induction/proof_by_induction/v/proof-by-induction