CSE373: Data Structures & Algorithms
Lecture 22: Parallel Reductions, Maps, and Algorithm Analysis

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Outline

Done:

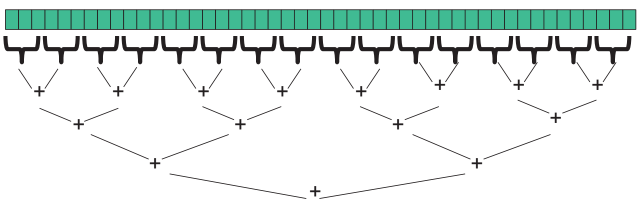
- How to write a parallel algorithm with fork and join
- Why using divide-and-conquer with lots of small tasks is best
  - Combines results in parallel
  - (Assuming library can handle "lots of small threads")

Now:

- More examples of simple parallel programs that fit the "map" or "reduce" patterns
- Teaser: Beyond maps and reductions
- Asymptotic analysis for fork-join parallelism
- Amdahl's Law

What else looks like this?

- Saw summing an array went from $O(n)$ sequential to $O(\log n)$ parallel (*assuming **a lot** of processors and very large n!*)
  - Exponential speed-up in theory (n / `log` n grows exponentially)



- Anything that can use results from two halves and merge them in $O(1)$ time has the same property…

Examples

- Maximum or minimum element
- Is there an element satisfying some property (e.g., is there a 17)?
- Left-most element satisfying some property (e.g., first 17)
  - What should the recursive tasks return?
  - How should we merge the results?
- Corners of a rectangle containing all points (a "bounding box")
- Counts, for example, number of strings that start with a vowel
  - This is just summing with a different base case
  - Many problems are!

Reductions

- Computations of this form are called reductions (or reduces?)
- Produce single answer from collection via an associative operator
  - Associative: a + (b+c) = (a+b) + c
  - Examples: max, count, leftmost, rightmost, sum, product, …
  - Non-examples: median, subtraction, exponentiation
- But some things are inherently sequential
  - How we process `arr[i]` may depend entirely on the result of processing `arr[i-1]`

Even easier: Maps (Data Parallelism)

- A map operates on each element of a collection independently to create a new collection of the same size
  - No combining results
  - For arrays, this is so trivial some hardware has direct support
- Canonical example: Vector addition

```
int[] vector_add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  result = new int[arr1.length];
  FORALL(i=0; i < arr1.length; i++) {
    result[i] = arr1[i] + arr2[i];
  }
  return result;
}
```

In Java

```
class VecAdd extends java.lang.Thread {
  int lo; int hi; int[] res; int[] arr1; int[] arr2;
  VecAdd(int l,int h,int[] r,int[] a1,int[] a2){ … }
  protected void run(){
    if(hi - lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = arr1[i] + arr2[i];
    } else {
      int mid = (hi+lo)/2;
      VecAdd left = new VecAdd(lo,mid,res,arr1,arr2);
      VecAdd right= new VecAdd(mid,hi,res,arr1,arr2);
      left.start();
      right.run();
      left.join();
    }
  }
}
int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  int[] ans = new int[arr1.length];
  (new VecAdd(0,arr.length,ans,arr1,arr2).run();
  return ans;
}
```


Maps and reductions

Maps and reductions: the "workhorses" of parallel programming

- By far the two most important and common patterns
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- Use maps and reductions to describe (parallel) algorithms
- Programming them becomes "trivial" with a little practice
  - Exactly like sequential for-loops seem second-nature

Beyond maps and reductions

- Some problems are "inherently sequential"
  "Nine women can't make a baby in one month"
- But not all parallelizable problems are maps and reductions
- If had one more lecture, would show "parallel prefix", a clever algorithm to parallelize the *problem* that this sequential *code* solves

input	6	4	16	10	16	14	2	8
output	6	10	26	36	52	66	68	76

```
int[] prefix_sum(int[] input){
  int[] output = new int[input.length];
  output[0] = input[0];
  for(int i=1; i < input.length; i++)
    output[i] = output[i-1]+input[i];
  return output;
}
```


Digression: MapReduce on clusters

- You may have heard of Google's "map/reduce"
  - Or the open-source version Hadoop
- Idea: Perform maps/reduces on data using many machines
  - The system takes care of distributing the data and managing fault tolerance
  - You just write code to map one element and reduce elements to a combined result
- Separates how to do recursive divide-and-conquer from what computation to perform
  - Separating concerns is good software engineering

Analyzing algorithms

- Like all algorithms, parallel algorithms should be:
  - Correct
  - Efficient
- For our algorithms so far, correctness is "obvious" so we'll focus on efficiency
  - Want asymptotic bounds
  - Want to analyze the algorithm without regard to a specific number of processors
  - Here: Identify the "best we can do" *if* the underlying *thread-scheduler* does its part

Work and Span

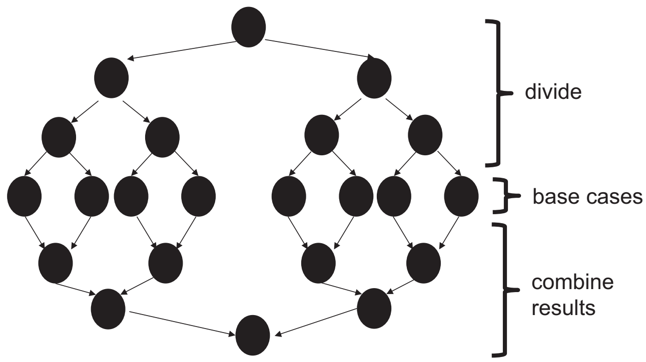
Let $\mathbf{T_P}$ be the running time if there are **P** processors available

Two key measures of run-time:

- Work: How long it would take 1 processor = $\mathbf{T_1}$
  - Just "sequentialize" the recursive forking
- Span: How long it would take infinity processors = $\mathbf{T_\infty}$
  - The longest dependence-chain
  - Example: $O(\texttt{log}\ n)$ for summing an array
    - Notice having > $n/2$ processors is no additional help

Our simple examples

- Picture showing all the "stuff that happens" during a reduction or a map: it's a (conceptual!) DAG



Connecting to performance

- Recall: $\mathbf{T_P}$ = running time if there are **P** processors available
- Work = $\mathbf{T_1}$ = sum of run-time of all nodes in the DAG
  - That lonely processor does everything
  - Any topological sort is a legal execution
  - $O(n)$ for maps and reductions
- Span = $\mathbf{T_\infty}$ = sum of run-time of all nodes on the most-expensive path in the DAG
  - Note: costs are on the nodes not the edges
  - Our infinite army can do everything that is ready to be done, but still has to wait for earlier results
  - $O(\texttt{log}\ n)$ for simple maps and reductions

Speed-up

Parallel algorithms is about decreasing span without increasing work too much

- Speed-up on **P** processors: $\mathbf{T_1 / T_P}$
- Parallelism is the maximum possible speed-up: $\mathbf{T_1 / T_\infty}$
  - At some point, adding processors won't help
  - What that point is depends on the span
- In practice we have **P** processors. How well can we do?
  - We cannot do better than $O(\mathbf{T_\infty})$ ("must obey the span")
  - We cannot do better than $O(\mathbf{T_1 / P})$ ("must do all the work")
  - Not shown: With a "good thread scheduler", can do this well (within a constant factor of optimal!)

Examples

$$\mathbf{T_P} = O(\max((\mathbf{T_1 / P}), \mathbf{T_\infty}))$$

- In the algorithms seen so far (e.g., sum an array):
  - $\mathbf{T_1} = O(n)$
  - $\mathbf{T_\infty} = O(\texttt{log}\ n)$
  - So expect (ignoring overheads): $\mathbf{T_P} = O(\max(n/\mathbf{P}, \texttt{log}\ n))$
- Suppose instead:
  - $\mathbf{T_1} = O(n^2)$
  - $\mathbf{T_\infty} = O(n)$
  - So expect (ignoring overheads): $\mathbf{T_P} = O(\max(n^2/\mathbf{P}, n))$

Amdahl's Law (mostly bad news)

- So far: analyze parallel programs in terms of work and span
- In practice, typically have parts of programs that parallelize well…
  - Such as maps/reductions over arrays

  …and parts that don't parallelize at all

  - Such as reading a linked list, getting input, doing computations where each needs the previous step, etc.

Amdahl's Law (mostly bad news)

Let the ***work*** (time to run on 1 processor) be 1 unit time

Let **S** be the portion of the execution that can't be parallelized

Then: $\mathbf{T_1 = S + (1\text{-}S) = 1}$

Suppose *parallel portion parallelizes perfectly (generous assumption)*

Then: $\mathbf{T_P = S + (1\text{-}S)/P}$

So the overall speedup with **P** processors is (Amdahl's Law):

$$\mathbf{T_1 / T_P = 1 / (S + (1\text{-}S)/P)}$$

And the parallelism (infinite processors) is:

$$\mathbf{T_1 / T_\infty = 1 / S}$$

Why such bad news

$T_1 / T_P = 1 / (S + (1-S)/P)$ $T_1 / T_\infty = 1 / S$

- Suppose 33% of a program's execution is sequential
  - Then a billion processors won't give a speedup over 3
- Suppose you miss the good old days (1980-2005) where 12ish years was long enough to get 100x speedup
  - Now suppose in 12 years, clock speed is the same but you get 256 processors instead of 1
  - For 256 processors to get at least 100x speedup, we need

    $100 \leq 1 / (\mathbf{S} + (1-\mathbf{S})/256)$

    Which means $\mathbf{S} \leq .0061$ (i.e., 99.4% perfectly parallelizable)

All is not lost

Amdahl's Law is a bummer!
- Unparallelized parts become a bottleneck very quickly
- But it doesn't mean additional processors are worthless

- We can find new parallel algorithms
  - Some things that seem sequential are actually parallelizable
- We can change the problem or do new things
  - Example: Video games use tons of parallel processors
    - They are not rendering 10-year-old graphics faster
    - They are rendering more beautiful(?) monsters

Moore and Amdahl

- Moore's "Law" is an observation about the progress of the semiconductor industry
  - Transistor density doubles roughly every 18 months
- Amdahl's Law is a mathematical theorem
  - Diminishing returns of adding more processors
- Both are incredibly important in designing computer systems