CSE 373

## Data Structures and Algorithms

Lecture 19: Graphs

## What are graphs?

- Yes, this is a graph....

- But we are interested in a different kind of "graph"


## Airline Routes



Nodes = cities
Edges = direct flights

## Computer Networks



Nodes = computers
Edges = transmission rates

## CSE Course Prerequisites at UW



## Graphs

b graph: a data structure containing
b a set of vertices $V$
p a set of edges $E$, where an edge represents a connection between 2 vertices
b edge is a pair $(v, w)$ where $v, w$ in $V$

- Denote graph as $G=(V, E)$
- Example:
$G=(V, E)$ where

$V=\{a, b, c\}$ and $E=\{(a, b),(b, c),(c, a)\}$


## Paths

- path: a path from vertex $A$ to $B$ is a sequence of edges that can be followed starting from $A$ to reach $B$
- Can be represented as vertices visited or edges taken
- Example: path from $V$ to $Z:\{b, h\}$ or $\{(v, x),(x, z)\}$ or $\{V, X, Z\}$
- reachability: $v_{2}$ is reachable from $v_{1}$ if a path exists from $v_{1}$ to $v_{2}$
| connected graph: one in which it is possible to reach any node from any other - Is this graph connected?



## Cycles

- cycle: path from one node back to itself
- Example: $\{\mathrm{V}, \mathrm{X}, \mathrm{Y}, \mathrm{W}, \mathrm{U}, \mathrm{V}\}$
- loop: edge directly from node to itself
- Many graphs don't allow loops



## More terminology

- degree: number of edges touching a vertex
- Example:W has degree 4
- What is the degree of $X$ ? of $Z$ ?
p adjacent vertices: vertices connected directly by an edge



## Weighted graphs

- weight: (optional) cost associated with a given edge
- Example: graph of airline flights
- If we were programming this graph, what information would we have to store for each vertex / edge?



## Directed graphs

- directed graph (digraph): edges are one-way connections between vertices
- If graph is directed, a vertex has a separate in/out degree



## Trees as Graphs

- Every tree is a graph with some restrictions:
- The tree is directed
- There is exactly one directed path from the root to every node



## Graph questions

- Are the following graphs directed or undirected?
- Buddy graphs of instant messaging programs?
(vertices = users, edges = user being on another's buddy list)
- bus line graph depicting all of Seattle's bus stations and routes
- graph of movies in which actors have appeared together
- Are these graphs potentially cyclic? Why or why not?


## Graph exercise

- Consider a graph of instant messenger buddies.
, What do the vertices represent? What does an edge represent?
- Is this graph directed or undirected? Weighted or unweighted?
- What does a vertex's degree mean? In degree? Out degree?
- Can the graph contain loops? Cycles?


## Graph exercise

- Consider this graph data:
- Jessica's buddy list: Meghan, Alan, Martin.
- Meghan's buddy list:Alan, Lori.
- Toni's buddy list: Lori, Meghan.
- Martin's buddy list: Lori, Meghan.
, Alan's buddy list: Martin, Jessica.
- Lori's buddy list: Meghan.
- Compute the in/out degree of each vertex. Is the graph connected?
- Who is the most popular? Least? Who is the most antisocial?
- If we're having a party and want to distribute the message the most quickly, who should we tell first?


## Graph exercise

- Consider a graph of Facebook friends.
, What do the vertices represent? What does an edge represent?
- Is this graph directed or undirected? Weighted or unweighted?
- What does a vertex's degree mean? In degree? Out degree?
- Can the graph contain loops? Cycles?

