CSE 373 Data Structures and Algorithms

Lecture 19: Graphs

What are graphs?

Yes, this is a graph....



But we are interested in a different kind of "graph"

Airline Routes



Edges = direct flights

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Computer Networks



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CSE Course Prerequisites at UW



Graphs

• graph: a data structure containing

- a set of vertices V
- a set of edges E, where an edge represents a connection between 2 vertices
 - edge is a pair (v, w) where v, w in V
- Denote graph as G = (V, E)
- Example:
 - G = (V, E) where
 - $V = \{a, b, c\}$ and $E = \{(a, b), (b, c), (c, a)\}$



Paths

- path: a path from vertex A to B is a sequence of edges that can be followed starting from A to reach B
 - Can be represented as vertices visited or edges taken
 - Example: path from V to Z: {b, h} or {(v,x), (x,z)} or {V, X, Z}



Cycles

cycle: path from one node back to itself

- Example: {V, X, Y, W, U, V}
- Ioop: edge directly from node to itself
 - Many graphs don't allow loops



More terminology

- degree: number of edges touching a vertex
 - Example: W has degree 4
 - What is the degree of X? of Z?
- adjacent vertices: vertices connected directly by an edge



Weighted graphs

• weight: (optional) cost associated with a given edge

- Example: graph of airline flights
 - If we were programming this graph, what information would we have to store for each vertex / edge?



Directed graphs

- directed graph (digraph): edges are one-way connections between vertices
 - If graph is directed, a vertex has a separate in/out degree



Trees as Graphs

• Every tree is a graph with some restrictions:

- The tree is directed
- There is exactly one directed path from the root to every node



Graph questions

Are the following graphs directed or undirected?

- Buddy graphs of instant messaging programs?
 (vertices = users, edges = user being on another's buddy list)
- bus line graph depicting all of Seattle's bus stations and routes
- graph of movies in which actors have appeared together
- Are these graphs potentially cyclic? Why or why not?

Graph exercise

• Consider a graph of instant messenger buddies.

- What do the vertices represent? What does an edge represent?
- Is this graph directed or undirected? Weighted or unweighted?
- What does a vertex's degree mean? In degree? Out degree?
- Can the graph contain loops? Cycles?

Graph exercise

Consider this graph data:

- Jessica's buddy list: Meghan, Alan, Martin.
- Meghan's buddy list: Alan, Lori.
- Toni's buddy list: Lori, Meghan.
- Martin's buddy list: Lori, Meghan.
- Alan's buddy list: Martin, Jessica.
- Lori's buddy list: Meghan.
- Compute the in/out degree of each vertex. Is the graph connected?
- Who is the most popular? Least? Who is the most antisocial?
- If we're having a party and want to distribute the message the most quickly, who should we tell first?

Graph exercise

Consider a graph of Facebook friends.

- What do the vertices represent? What does an edge represent?
- Is this graph directed or undirected? Weighted or unweighted?
- > What does a vertex's degree mean? In degree? Out degree?
- Can the graph contain loops? Cycles?