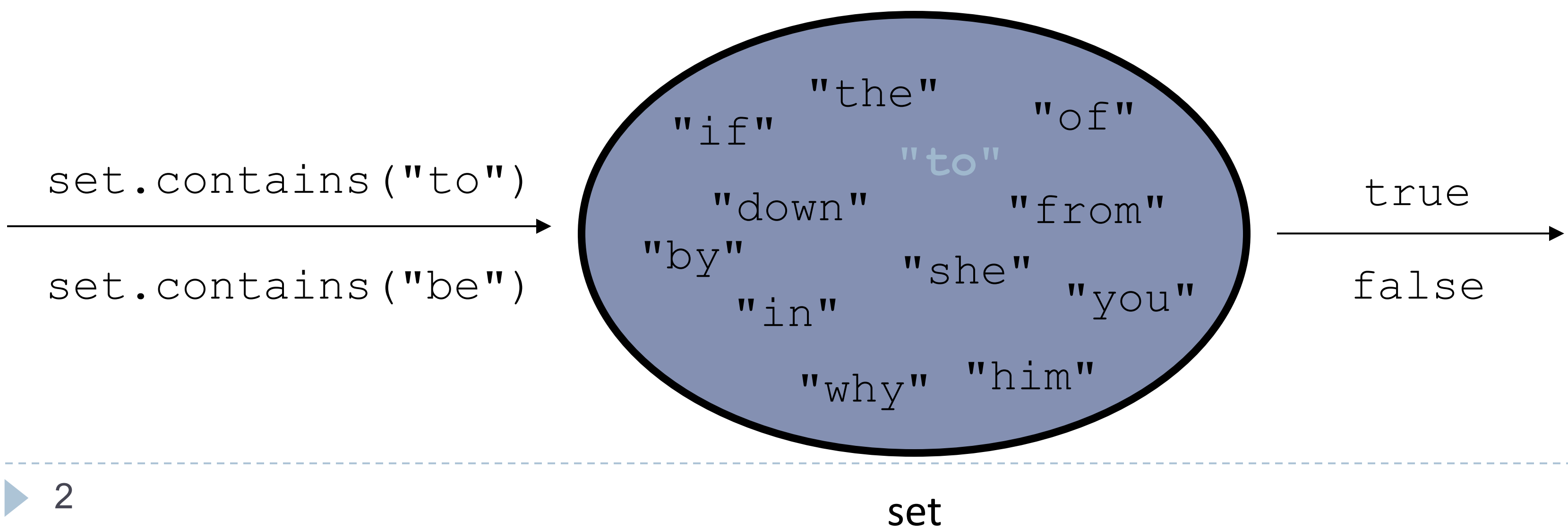
CSE 373
Data Structures and Algorithms

Lecture 16: Hashing

Set ADT

- **set**: A collection that does not allow duplicates
  - We don't think of a set as having indices or any order

- Basic set operations:
  - **insert**: Add an element to the set (order doesn't matter).
  - **remove**: Remove an element from the set.
  - **search**: Efficiently determine if an element is a member of the set.



Implementing Set ADT (Revisited)

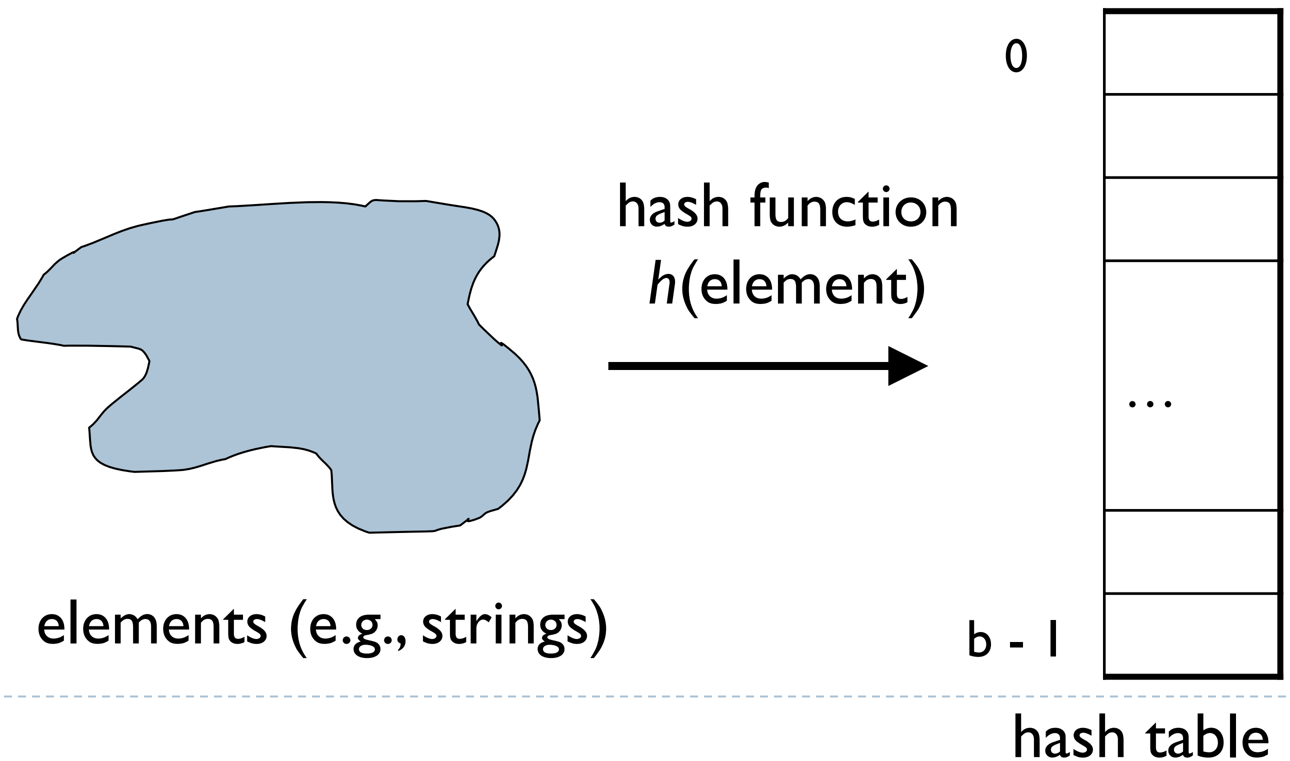
	Insert	Remove	Search
Unsorted array	O(1)	O(n)	O(n)
Sorted array	O(log n + n)	O(log n + n)	O(log n)
Linked list	O(1)	O(n)	O(n)
BST (if balanced)	O(log n)	O(log n)	O(log n)

A different tactic

- How do you check to see if a word is in the dictionary?
  - linear search?
  - binary search?
  - A – Z tabs?

Hash tables

- table maintains b different "buckets" (numbered 0 to b-1)
- **hash function** maps elements to value in 0 to b – 1
- use hash to determine which bucket an element belongs in and only searches/modifies this one bucket



Hashing, hash functions

- The idea: We somehow map every element into some index in the array ("hash" it); this is its one and only place that it should go
  - Lookup becomes <u>constant-time</u>: simply look at that one slot again later to see if the element is there
  - insert, remove, search all become O(1) !

- For now, let's look at storing integers
  - Assume the following "hash function" *h*: Store int *i* at index *i* (a direct mapping)
    - if `i >= array.length`, store `i` at index (`i % array.length`)
  - *h(i) = i % array.length*

Simple Integer Hash Functions

- elements = integers
- *TableSize* = 10
- $h(i) = i \% 10$
- **Insert**: 7, 18, 41, 34

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

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Hash function example

- Desirable properties of a hash function
  - efficient computation
  - deterministic/stable result
  - uniformly distributes values over range
- *h(i) = i % 10*
  - Does this function have the properties above?
- Drawbacks?
  - Lose all ordering information:
    - getMin, getMax, removeMin, removeMax
    - Ordered traversals; printing items in sorted order

0	
1	41
2	
3	
4	34
5	
6	
7	7
8	18
9	

Hash collisions

- Example: add 7, 18, 41, 34, then 21
  - 21 hashes into the same slot as 41!
  - Should 21 replace 41?
    - No!

- **collision**: the event that two hash table elements map into the same slot in the array

- **collision resolution**: means for fixing collisions in a hash table

0	
1	41
2	
3	
4	34
5	
6	
7	7
8	18
9	

Hash function for strings

- elements = Strings
- How do we map a string into an integer index? (i.e., how do we "hash" it?)

- Let's view a string by its letters:
  - String s : $s_0, s_1, s_2, \ldots, s_{n-1}$

- One possible hash function:
  - Treat first character as an int, and hash on that
    - $h(s) = s_0 \,\%\, TableSize$
    - Is this a good hash function? When will strings "collide"?
    - What about $h(s) = s.length \,\%\, TableSize$?

Better string hash functions

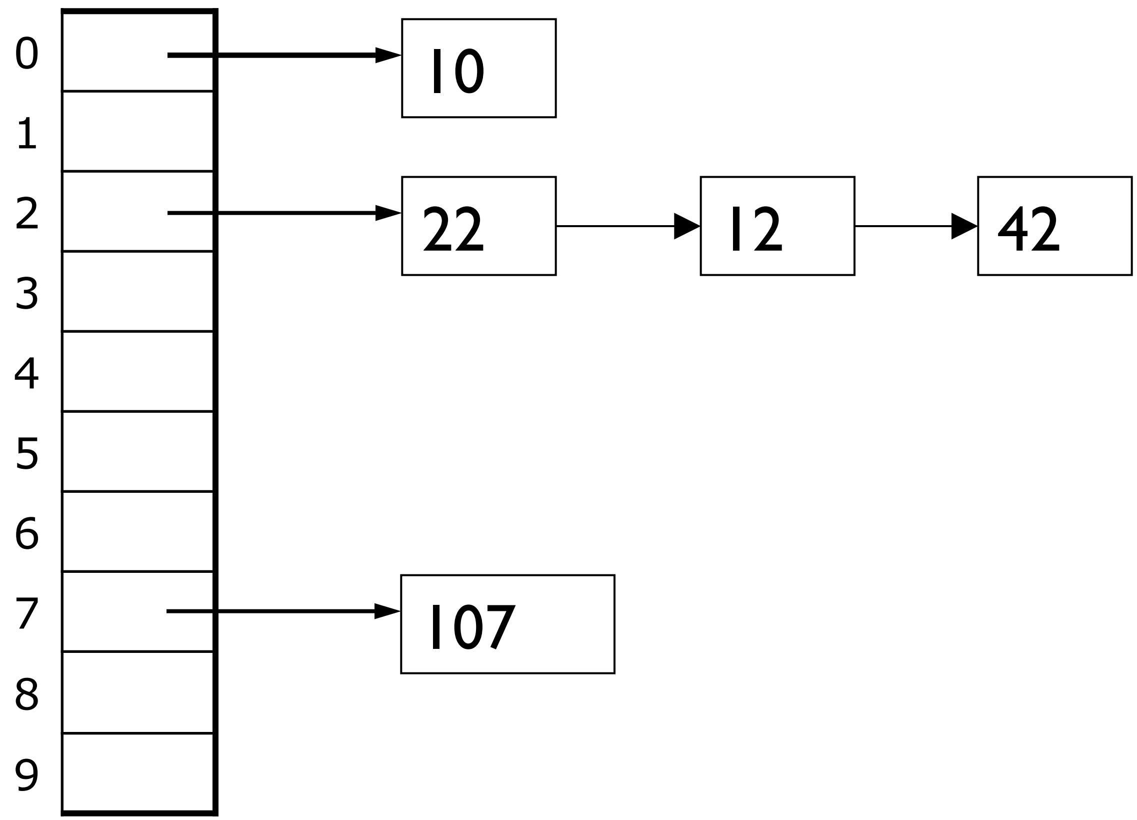
- Another possible hash function:
  - Treat each character as an int, sum them, and hash on that
    $h(s) = \left(\sum_{i=0}^{n-1} s_i \right)$ % *TableSize*
    - What's wrong with this hash function? When will strings collide?

- A third option (polynomial accumulation)
  - Perform a *weighted sum* of the letters, and hash on that
    $h(s) = \left(\sum_{i=0}^{k-1} s_i \cdot 37^i \right)$ % *TableSize*

- Coming up with a great hash function is hard.

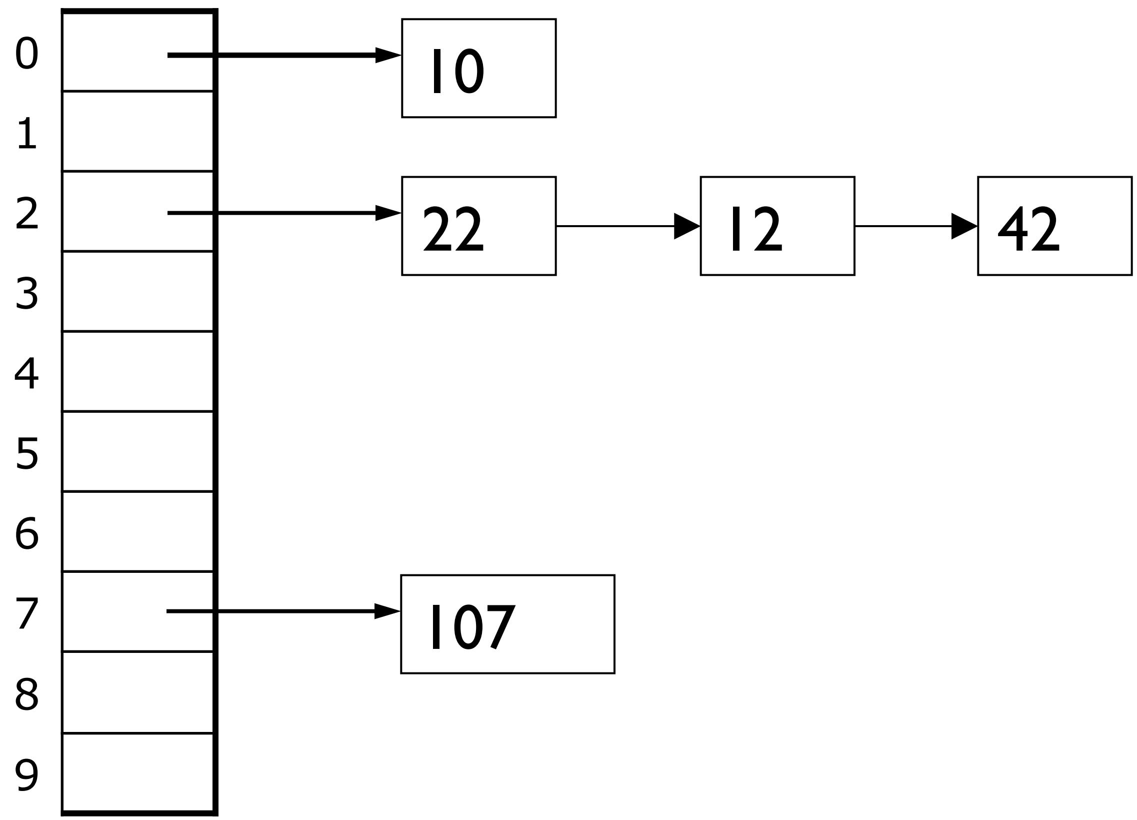
Chaining

- **chaining:** All keys that map to the same hash value are kept in a linked list



Load factor

- **load factor (λ)**: ratio of elements to capacity
  - load factor = size / capacity = 5 / 10 = 0.5



Analysis of hash table search

- Analysis of search, with chaining:
  - Unsuccessful: λ
    - The average length of a list at hash(*i*)
  - Successful: $1 + (\lambda/2)$
    - One node, plus half the average length of a list (not including the item)

Implementing Set with Hash Table

- Each Set entry adds an element to the table
  - Hash function will tell us where to put the element in the hash table

- Runtime
  - insert: O(1)
  - remove: O(1)
  - search: O(1)

Implementing Set with Hash Table

```
public interface StringSet {
    public boolean add(String value);

    public boolean contains(String value);

    public void print();

    public boolean remove(String value);

    public int size();
}
```

StringHashEntry

```
public class StringHashEntry {
    public String data;              // data stored at this node
    public StringHashEntry next;     // reference to the next entry

    // Constructs a single hash entry.
    public StringHashEntry(String data) {
        this(data, null);
    }

    public StringHashEntry(String data, StringHashEntry next) {
        this.data = data;
        this.next = next;
    }
}
```

`StringHashSet` class

```
public class StringHashSet implements StringSet {
    private static final int DEFAULT_SIZE = 11;
    private StringHashEntry[] table;
    private int size;

    ...
}
```

- Client code talks to the `StringHashSet`, not to the entry objects stored in it
- The array (table) is of `StringHashEntry`
  - Each element in the array is a linked list of elements that have the same hash

Set implementation: search

```
public boolean contains(String value) {
    // figure out where value should be...
    int valuePosition = hash(value);

    // check to see if the value is in the set
    StringHashEntry temp = table[valuePosition];
    while (temp != null) {
        if (temp.data.equals(value)) {
            return true;
        }
        temp = temp.next;
    }

    // otherwise, the value was not found
    return false;
}
```

Set implementation: insert

- Similar structure to `contains`
  - Calculate hash of new element
  - Check if the element is already in the set
- Add the element to the front of the list that is at `table[hash(value)]`

Set implementation: insert

```
public boolean add(String value) {
    int valuePosition = hash(value);

    // check to see if the value is already in the set
    StringHashEntry temp = table[valuePosition];
    while (temp != null) {
        if (temp.data.equals(value)) {
            return false;
        }
        temp = temp.next;
    }

    // add the value to the set
    StringHashEntry newEntry = new StringHashEntry(value, table[valuePosition]);
    table[valuePosition] = newEntry;
    size++;
    return true;
}
```