CSE 373
Data Structures and Algorithms

Lecture 7: Sorting

Why Sorting?

- Practical application
  - People by last name
  - Countries by population
  - Search engine results by relevance
- Fundamental to other algorithms
- Different algorithms have different asymptotic and constant-factor trade-offs
  - No single 'best' sort for all scenarios
  - Knowing one way to sort just isn't enough
- Many to approaches to sorting which can be used for other problems

Problem statement

- There are *n* comparable elements in an array and we want to rearrange them to be in increasing order

- Pre:
  - An array **`A`** of data records
  - A value in each data record
  - A comparison function
    - `<`, `=`, `>`, `compareTo`

- Post:
  - For each distinct position **`i`** and **`j`** of **`A`**, if **`i<j`** then **`A[i] ≤ A[j]`**
  - **`A`** has all the same data it started with

Sorting Classification

In memory sorting			External sorting
Comparison sorting $\Omega(N \log N)$		Specialized Sorting	
$O(N^2)$	$O(N \log N)$	$O(N)$	# of tape accesses
• Bubble Sort • Selection Sort • Insertion Sort • Shell Sort	• Merge Sort • Quick Sort • Heap Sort	• Bucket Sort • Radix Sort	• Simple External Merge Sort • Variations

Comparison Sorting

Determine order through comparisons on the input data

Bogo sort

- bogo sort: orders a list of values by repetitively shuffling them and checking if they are sorted

- more specifically:
  - scan the list, seeing if it is sorted
  - if not, shuffle the values in the list and repeat

- This sorting algorithm has terrible performance!
  - Can we deduce its runtime?
  - What about best case?

Bogo sort code

```
public static void bogoSort(int[] a) {
    while (!isSorted(a)) {
        shuffle(a);
    }
}

// Returns true if array a's elements
// are in sorted order.
public static boolean isSorted(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        if (a[i] > a[i+1]) {
            return false;
        }
    }

    return true;
}
```

Bogo sort code, helpers

```
// Shuffles an array of ints by randomly swapping each
// element with an element ahead of it in the array.
public static void shuffle(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // pick random number in [i+1, a.length-1] inclusive
        int range = (a.length - 1) - (i + 1) + 1;
        int j = (int)(Math.random() * range + (i + 1));
        swap(a, i, j);
    }
}

// Swaps a[i] with a[j].
private static void swap(int[] a, int i, int j) {
    if (i == j)
        return;

    int temp = a[i];
    a[i] = a[j];
    a[j] = temp;
}
```

$O(n^2)$ Comparison Sorting

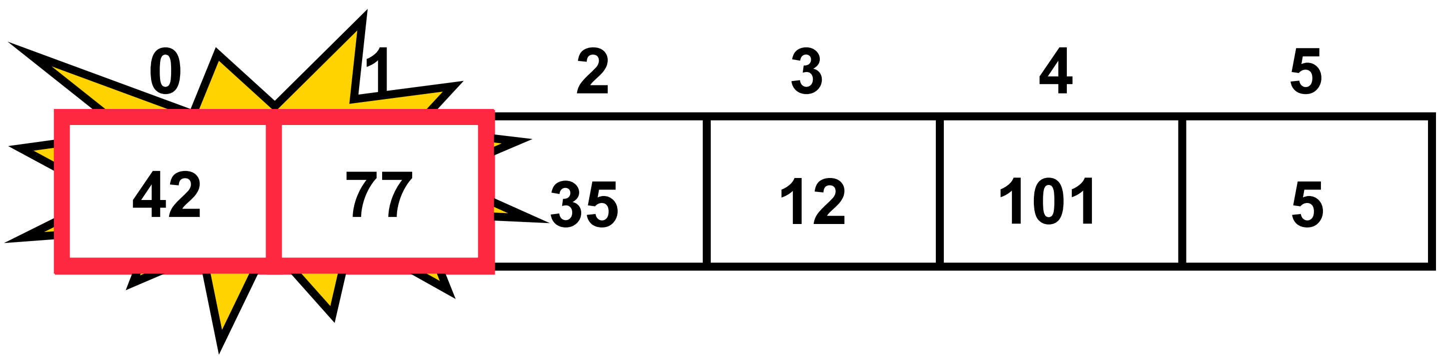
Bubble sort

- **bubble sort**: orders a list of values by repetitively comparing neighboring elements and swapping their positions if necessary

- more specifically:
  - scan the list, exchanging adjacent elements if they are not in relative order; this bubbles the highest value to the top
  - scan the list again, bubbling up the second highest value
  - repeat until all elements have been placed in their proper order

"Bubbling" largest element

- Traverse a collection of elements
  - Move from the front to the end
  - "Bubble" the largest value to the end using pair-wise comparisons and swapping



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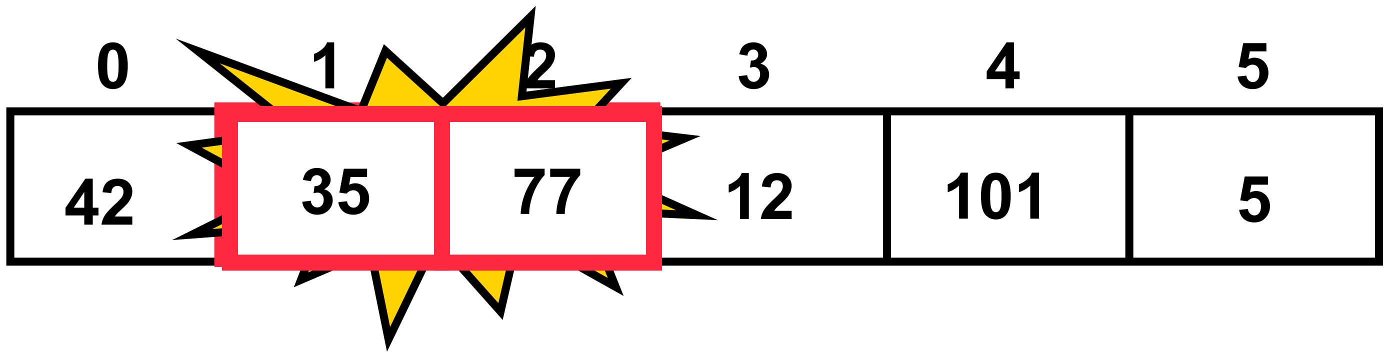

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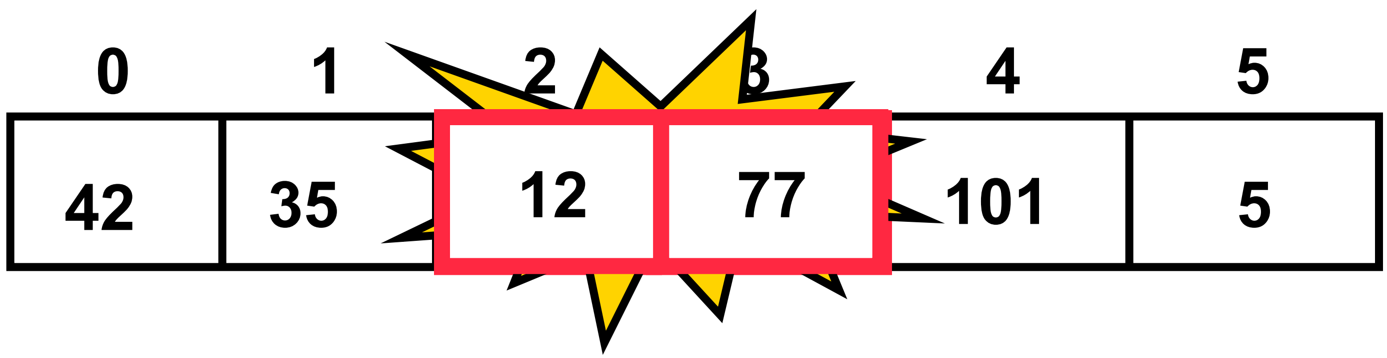

"Bubbling" largest element

- Traverse a collection of elements
  - Move from the front to the end
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0	1	2	3	4	5
42	35	12	77	101	5

No need to swap

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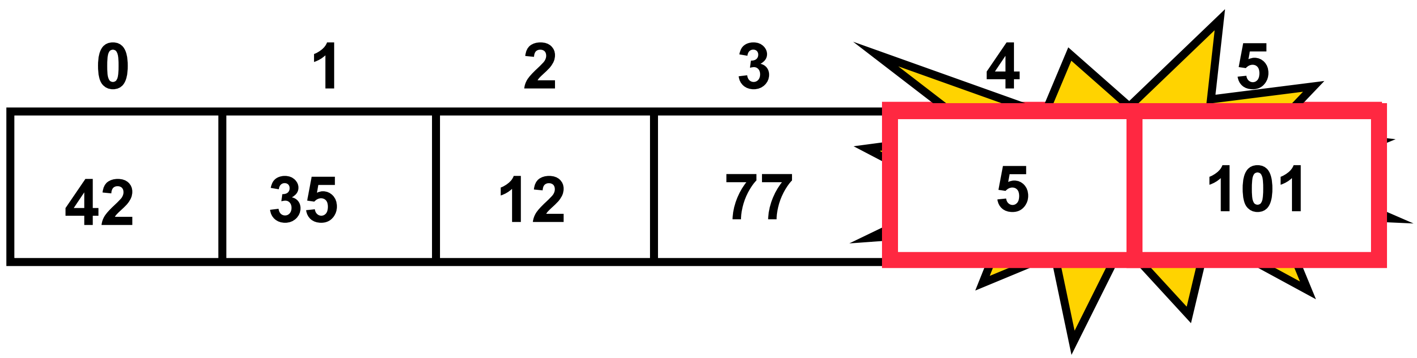

"Bubbling" largest element

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42	35	12	77	5	101

Largest value correctly placed

Bubble sort code

```
public static void bubbleSort(int[] a) {
    for (int i = 0; i < a.length; i++) {
        for (int j = 1; j < a.length - i; j++) {
            // swap adjacent out-of-order elements
            if (a[j-1] > a[j]) {
                swap(a, j-1, j);
            }
        }
    }
}
```

Bubble sort runtime

- Running time (# comparisons) for input size *n*:

$$\sum_{i=0}^{n-1} \sum_{j=1}^{n-1-i} 1 = \sum_{i=0}^{n-1} (n-1-i)$$

$$= n\sum_{i=0}^{n-1} 1 - \sum_{i=0}^{n-1} 1 - \sum_{i=0}^{n-1} i$$

$$= n^2 - n - \frac{(n-1)n}{2}$$

$$= \Theta(n^2)$$

  - number of actual swaps performed depends on the data; out-of-order data performs many swaps

Selection sort

- **selection sort**: orders a list of values by repetitively putting a particular value into its final position

- more specifically:
  - find the smallest value in the list
  - switch it with the value in the first position
  - find the next smallest value in the list
  - switch it with the value in the second position
  - repeat until all values are in their proper places

Selection sort example

	3	9	6	1	2
Scan right starting with 3. 1 is the smallest. Exchange 1 and 3.					
	1	9	6	3	2
Scan right starting with 9. 2 is the smallest. Exchange 9 and 2.					
	1	2	6	3	9
Scan right starting with 6. 3 is the smallest. Exchange 6 and 3.					
	1	2	3	6	9
Scan right starting with 6. 6 is the smallest. Exchange 6 and 6.					
	1	2	3	6	9

Selection sort example 2

Index	0	1	2	3	4	5	6	7
Value	27	63	1	72	64	58	14	9
1st pass	**1**	63	27	72	64	58	14	9
2nd pass	1	**9**	27	72	64	58	14	63
3rd pass	1	9	**14**	72	64	58	27	63
…								

Selection sort code

```
public static void selectionSort(int[] a) {
    for (int i = 0; i < a.length; i++) {
        // find index of smallest element
        int minIndex = i;
        for (int j = i + 1; j < a.length; j++) {
            if (a[j] < a[minIndex]) {
                minIndex = j;
            }
        }

        // swap smallest element with a[i]
        swap(a, i, minIndex);
    }
}
```

Selection sort runtime

- Running time for input size *n*:
  - In practice, a bit faster than bubble sort. Why?

$$\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-1} (n - 1 - (i + 1) + 1)$$

$$= \sum_{i=0}^{n-1} (n - i - 1)$$

$$= n \sum_{i=0}^{n-1} 1 - \sum_{i=0}^{n-1} i - \sum_{i=0}^{n-1} 1$$

$$= n^2 - \frac{(n-1)n}{2} - n$$

$$= \Theta(n^2)$$

Insertion sort

- **insertion sort**: orders a list of values by repetitively inserting a particular value into a sorted subset of the list

- more specifically:
  - consider the first item to be a sorted sublist of length 1
  - insert the second item into the sorted sublist, shifting the first item if needed
  - insert the third item into the sorted sublist, shifting the other items as needed
  - repeat until all values have been inserted into their proper positions

Insertion sort

- Simple sorting algorithm.
  - n-1 passes over the array
  - At the end of pass *i*, the elements that occupied A[0]…A[*i*] originally are still in those spots and in sorted order.

	2	15	8	1	17	10	12	5
	0	1	2	3	4	5	6	7
after pass 2	2	8	15	1	17	10	12	5
	0	1	2	3	4	5	6	7
after pass 3	1	2	8	15	17	10	12	5
	0	1	2	3	4	5	6	7

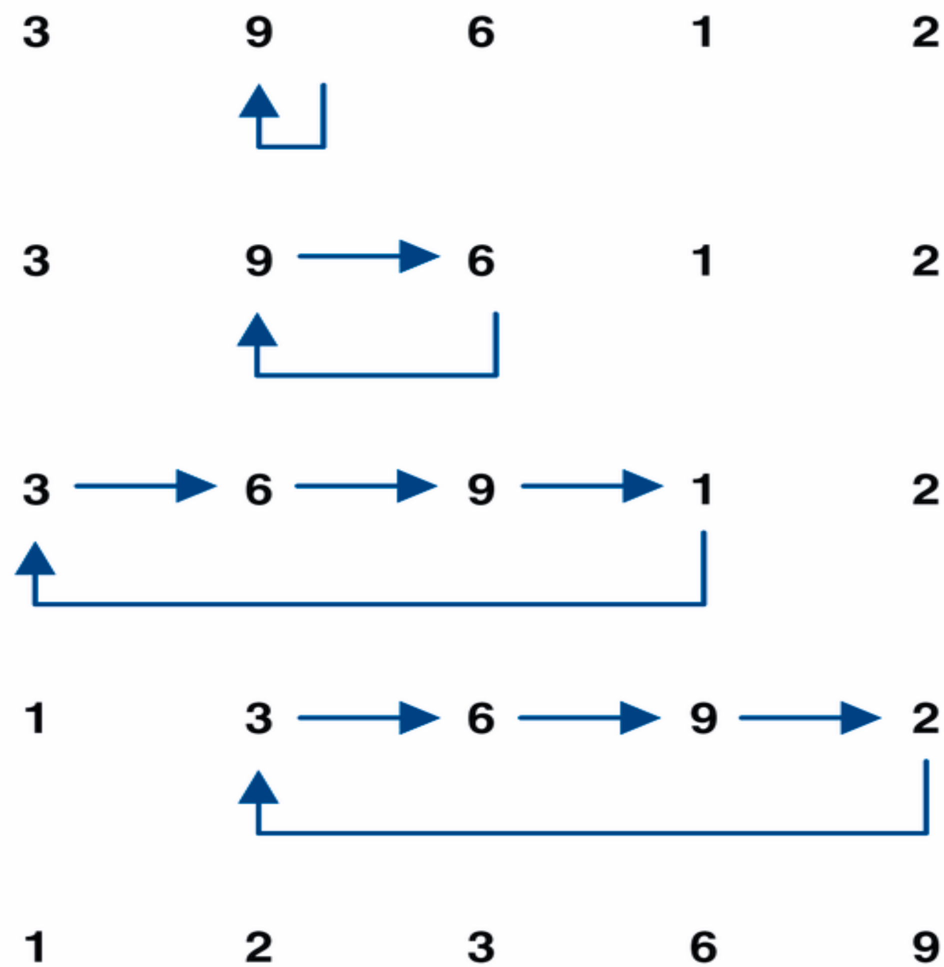
Insertion sort example

3 is sorted.
Shift nothing. Insert 9.

3 and 9 are sorted.
Shift 9 to the right. Insert 6.

3, 6, and 9 are sorted.
Shift 9, 6, and 3 to the right. Insert 1.

1, 3, 6, and 9 are sorted.
Shift 9, 6, and 3 to the right. Insert 2.



Insertion sort code

```
public static void insertionSort(int[] a) {
    for (int i = 1; i < a.length; i++) {
        int temp = a[i];

        // slide elements down to make room for a[i]
        int j = i;
        while (j > 0 && a[j - 1] > temp) {
            a[j] = a[j - 1];
            j--;
        }

        a[j] = temp;
    }
}
```

Insertion sort runtime

- worst case: reverse-ordered elements in array.

$$\sum_{i=1}^{n-1} i = 1 + 2 + 3 + \ldots + (n-1) = \frac{(n-1)n}{2}$$

$$= \Theta(n^2)$$

- best case: array is in sorted ascending order.

$$\sum_{i=1}^{n-1} 1 = n - 1 = \Theta(n)$$

- average case: each element is about halfway in order.

$$\sum_{i=1}^{n-1} \frac{i}{2} = \frac{1}{2}(1 + 2 + 3 \ldots + (n-1)) = \frac{(n-1)n}{4}$$

$$= \Theta(n^2)$$

Comparing sorts

- We've seen "simple" sorting algorithms so far, such as selection sort and insertion sort.
- They all use nested loops and perform approximately n^2 comparisons
- They are relatively inefficient

Sorting practice problem

- Consider the following array of int values.
  - [22, 11, 34, -5, 3, 40, 9, 16, 6]
  - (a) Write the contents of the array after 3 passes of the outermost loop of bubble sort.
  - (b) Write the contents of the array after 5 passes of the outermost loop of insertion sort.
  - (c) Write the contents of the array after 4 passes of the outermost loop of selection sort.