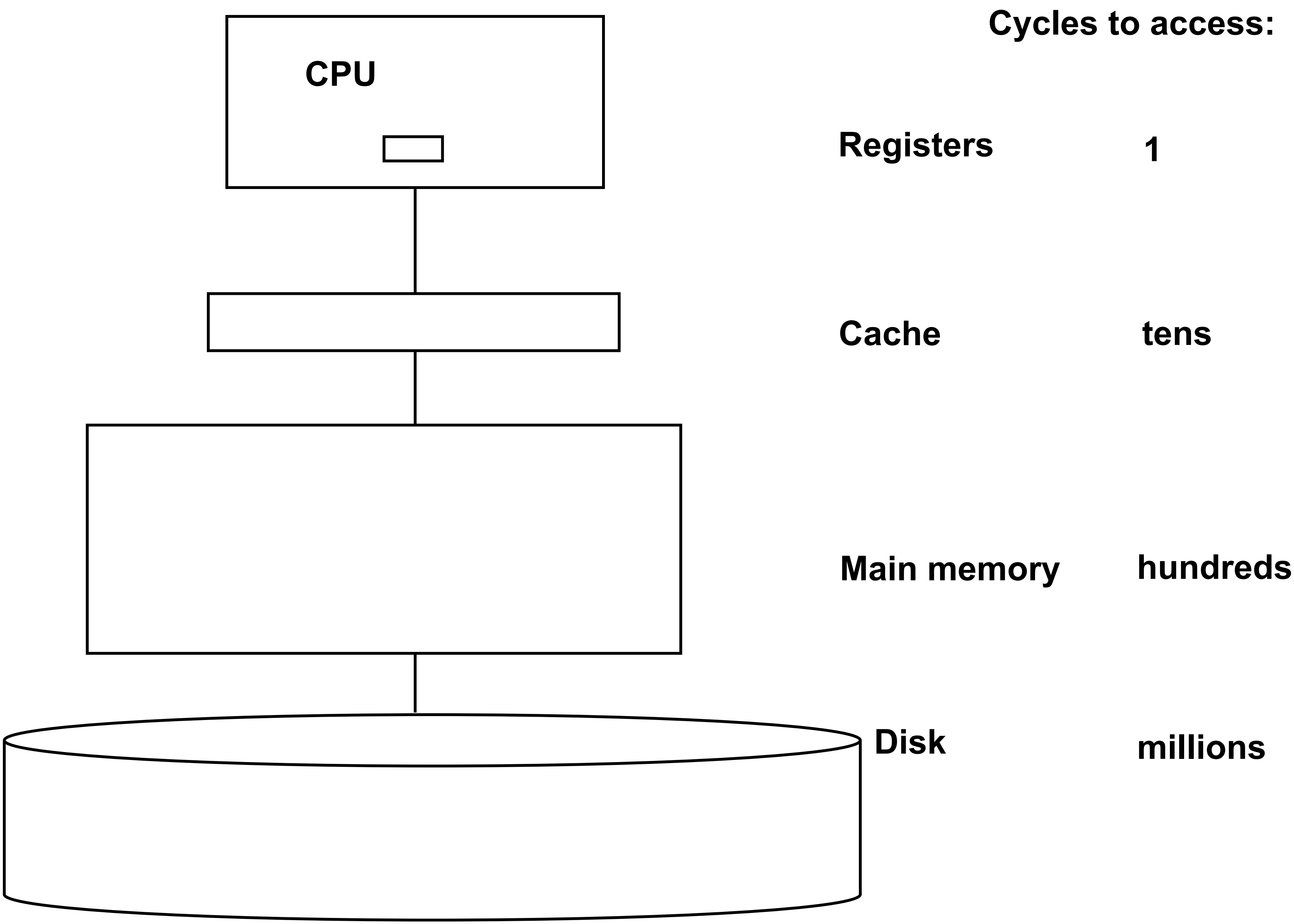
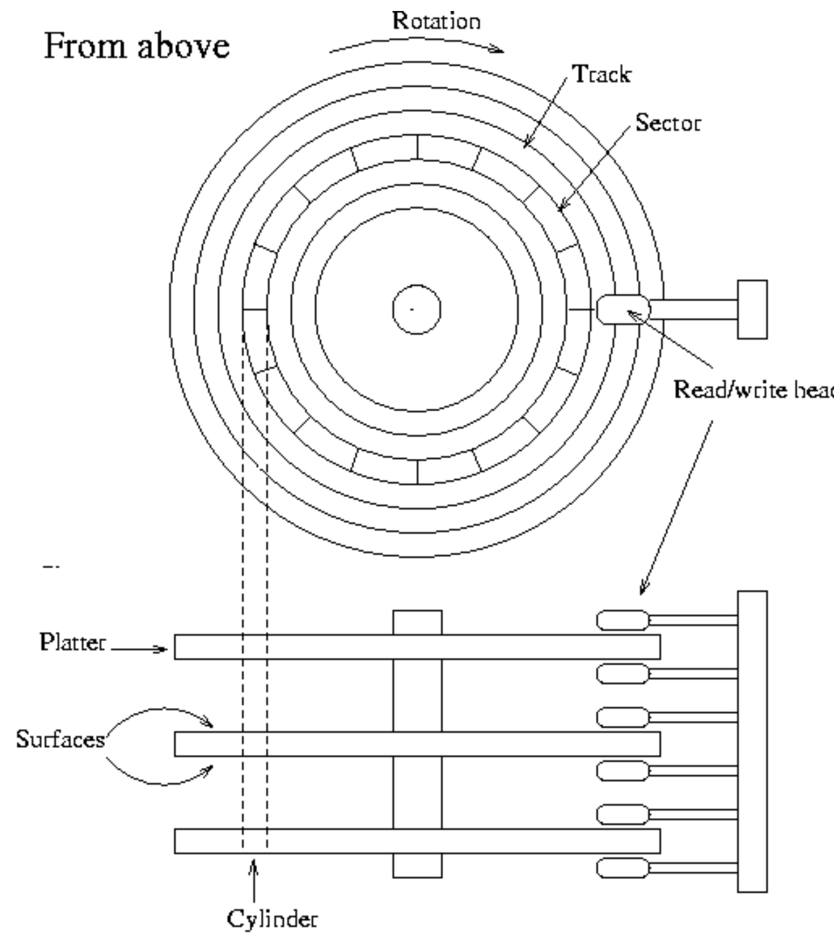
CSE 373: Data Structures and Algorithms

Lecture 25: B-Trees


Cycles to access:
CPU
Registers
1
Cache
tens
Main memory
hundreds
Disk
millions

Hard Disks

- Large amount of storage but slow access
- Identifying a page takes a long time
  - Pays to read or write data in pages (i.e. blocks) of 0.5 – 8 KB in size



Algorithm Analysis

- Running time of disk-based data structures measured in terms of
  - computing time (CPU)
  - number of disk accesses
    - sequential reads
    - random reads

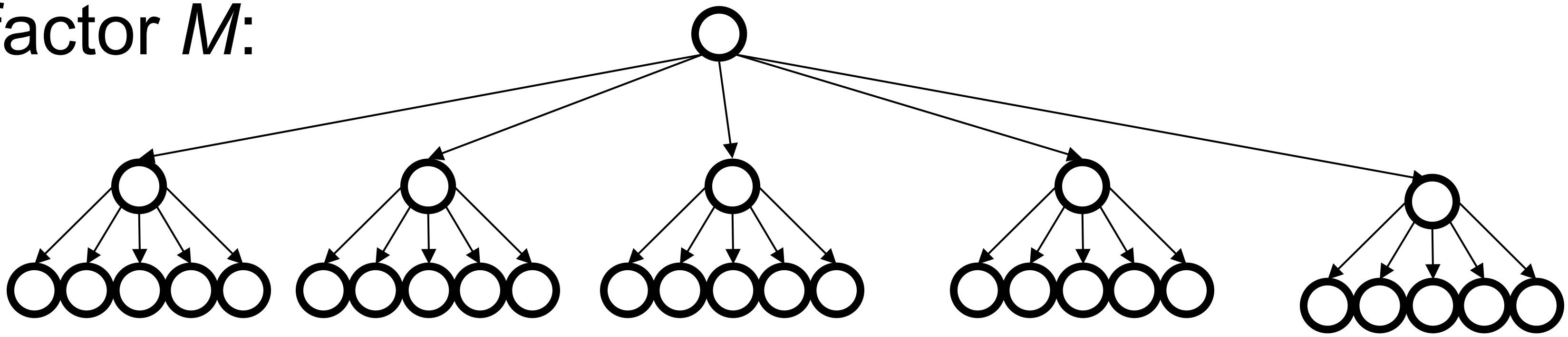
- Regular main-memory algorithms that work one data element at a time can not be "ported" to secondary storage in a straight forward way

Principles

- Almost all of our data structure is on disk.
- Every time we access a node in the tree it amounts to a random disk access.
- How can we address this problem?

M-ary Search Tree

- Suppose we devised a search tree with branching factor *M*:



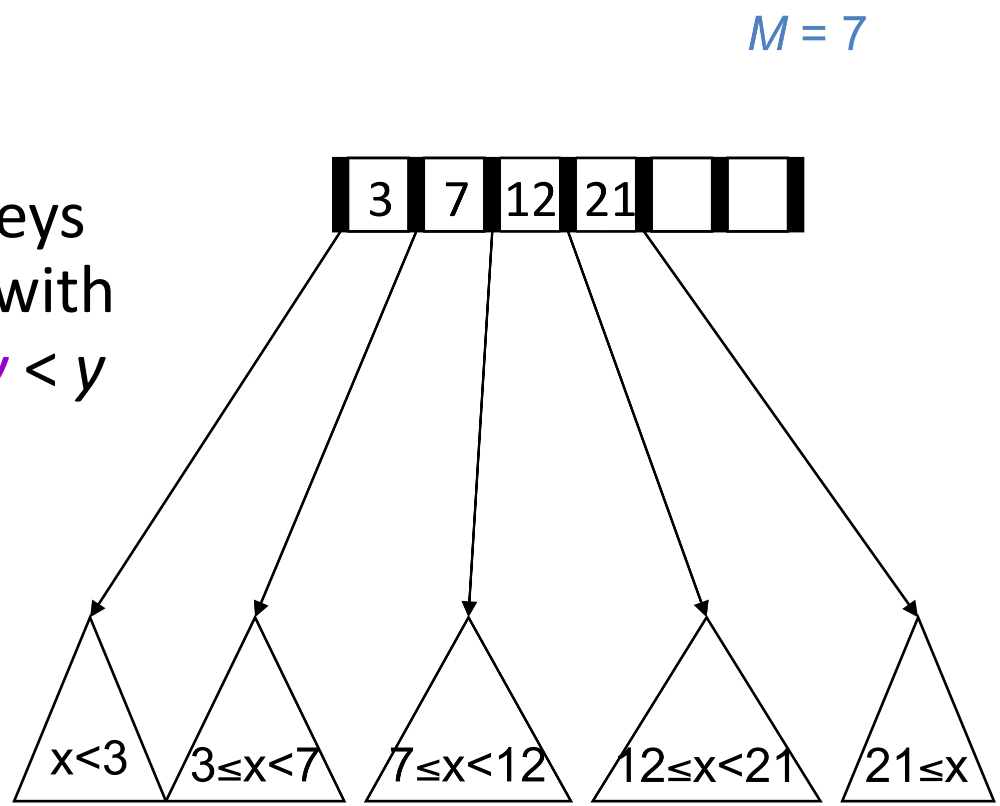
- M – 1 keys needed to decide branch to take
- Complete tree has height: $\Theta(\log_M n)$
- # Nodes accessed for *search*: $\Theta(\log_M n)$

B-Trees

- Internal nodes store (up to) M - 1 keys

- Order property:
  - subtree between two keys x and y contain leaves with *values* v such that $x \leq v < y$
  - Note the "**≤**"

- Leaf nodes contain up to L sorted values/ records.



Disk Friendliness

What makes B-trees disk-friendly?

1. Many keys stored in a node
   - Each node is one disk page/block.
   - All brought to memory/cache in one disk access.

2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - Much of tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

What is limiting you from increasing the number of keys stored in each node?

Exercise: If disk block is 4000 bytes, key size is 20 bytes, pointer size is 4 bytes, and data/value size is 200 bytes, what should M and L be for our B-Tree?

B-Tree Structure Properties

- Root (special case)
  - has between 2 and *M* children (or could be a leaf)
- Internal nodes

  Nodes are at least ½ full
  - store up to M-1 keys
  - have between floor(M/2) and M children
- Leaf nodes

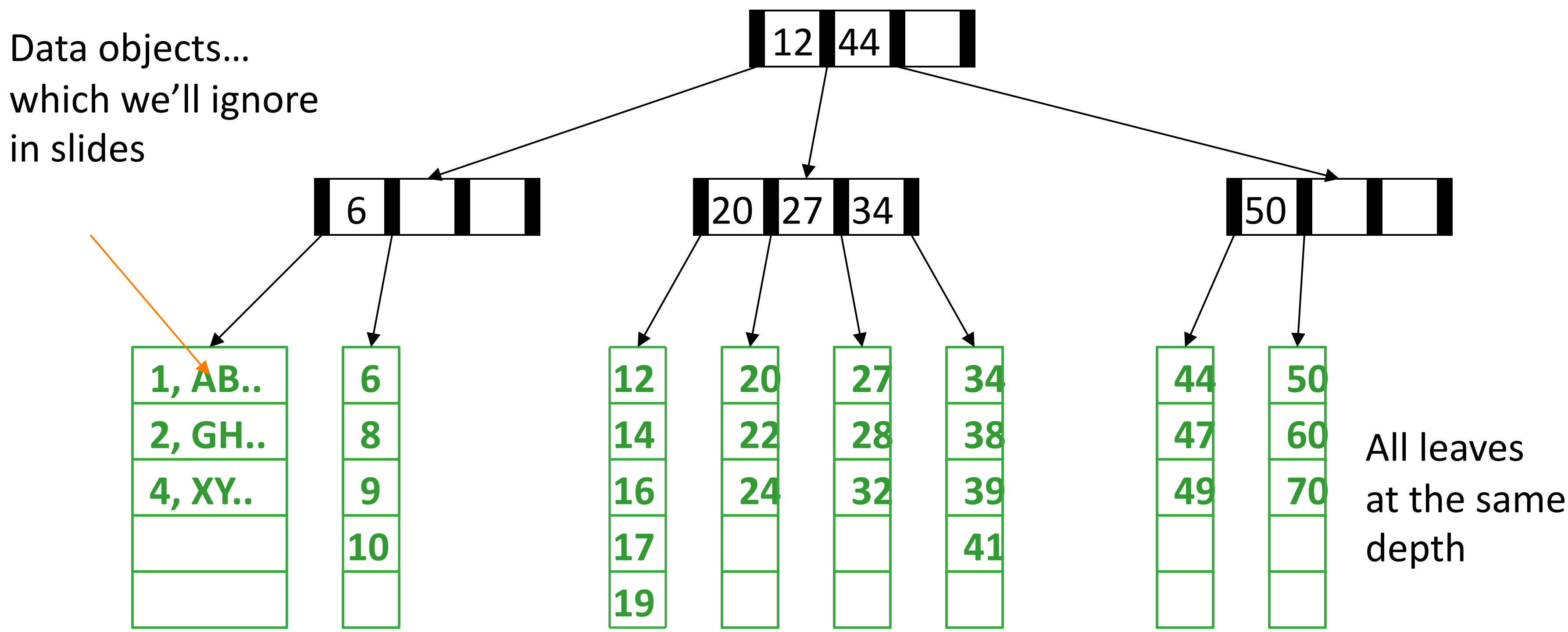
  Leaves are at least ½ full
  - where data is stored
  - contain between floor(*L*/2) and *L* data items

The tree is ***perfectly balanced*** !

B-Tree: Example

B-Tree with *M* = 4 (# pointers in internal node)

and *L* = 5 (# data items in leaf)



Definition for later: "neighbor" is the next sibling to the left or right.

B-trees vs. AVL trees

Suppose we have $n = 10^9$ data items:

- Depth of AVL Tree: $\log_2 10^9 = 30$
- Depth of B-Tree with $M = 256$, $L = 256$: $\log_{128} 10^9 = 4.3$

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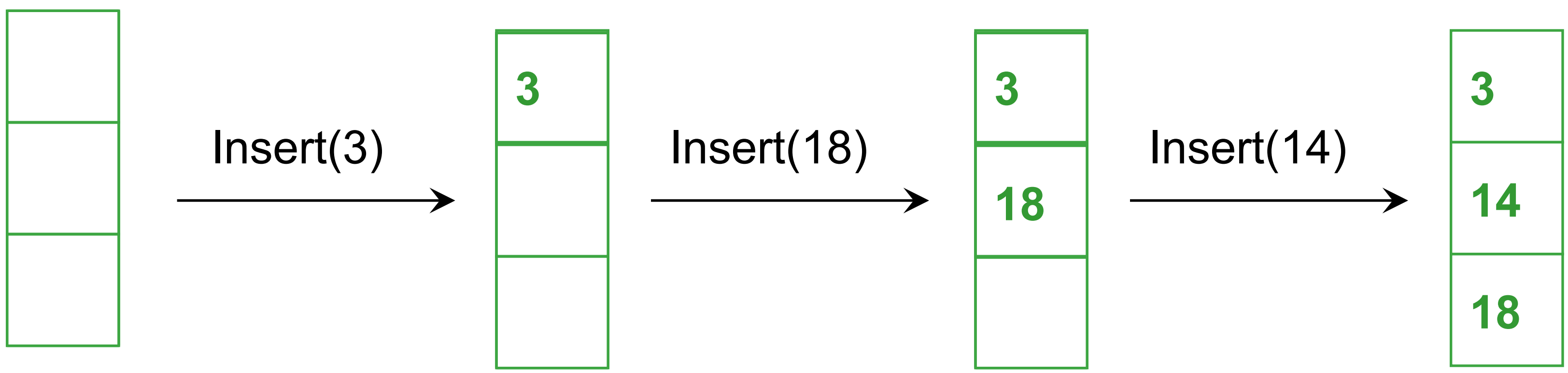
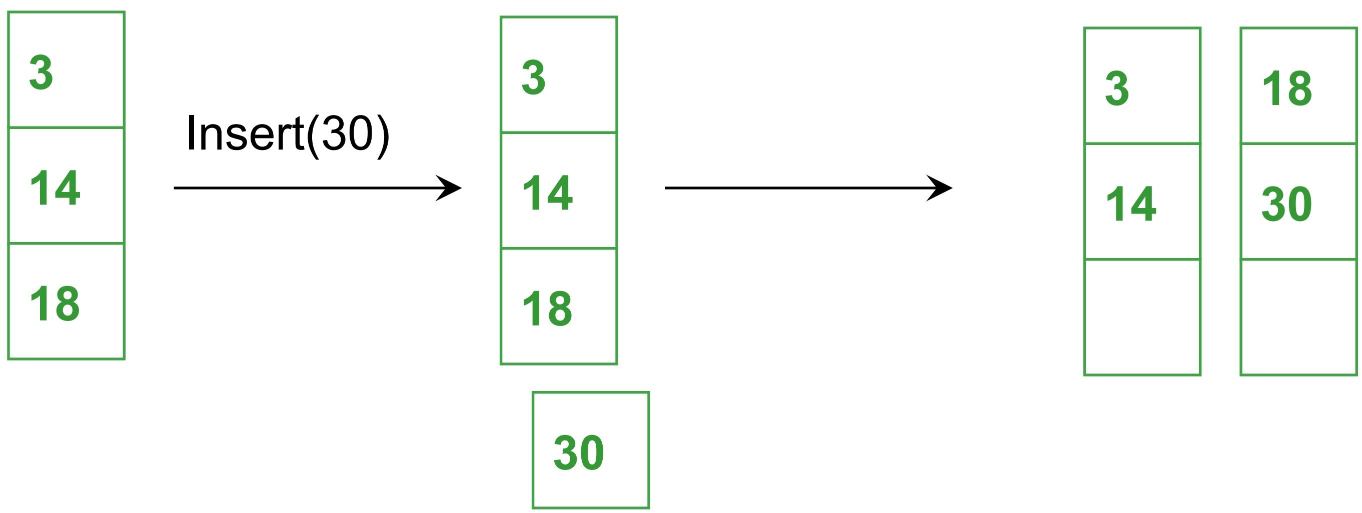
Building a B-Tree with Insertions

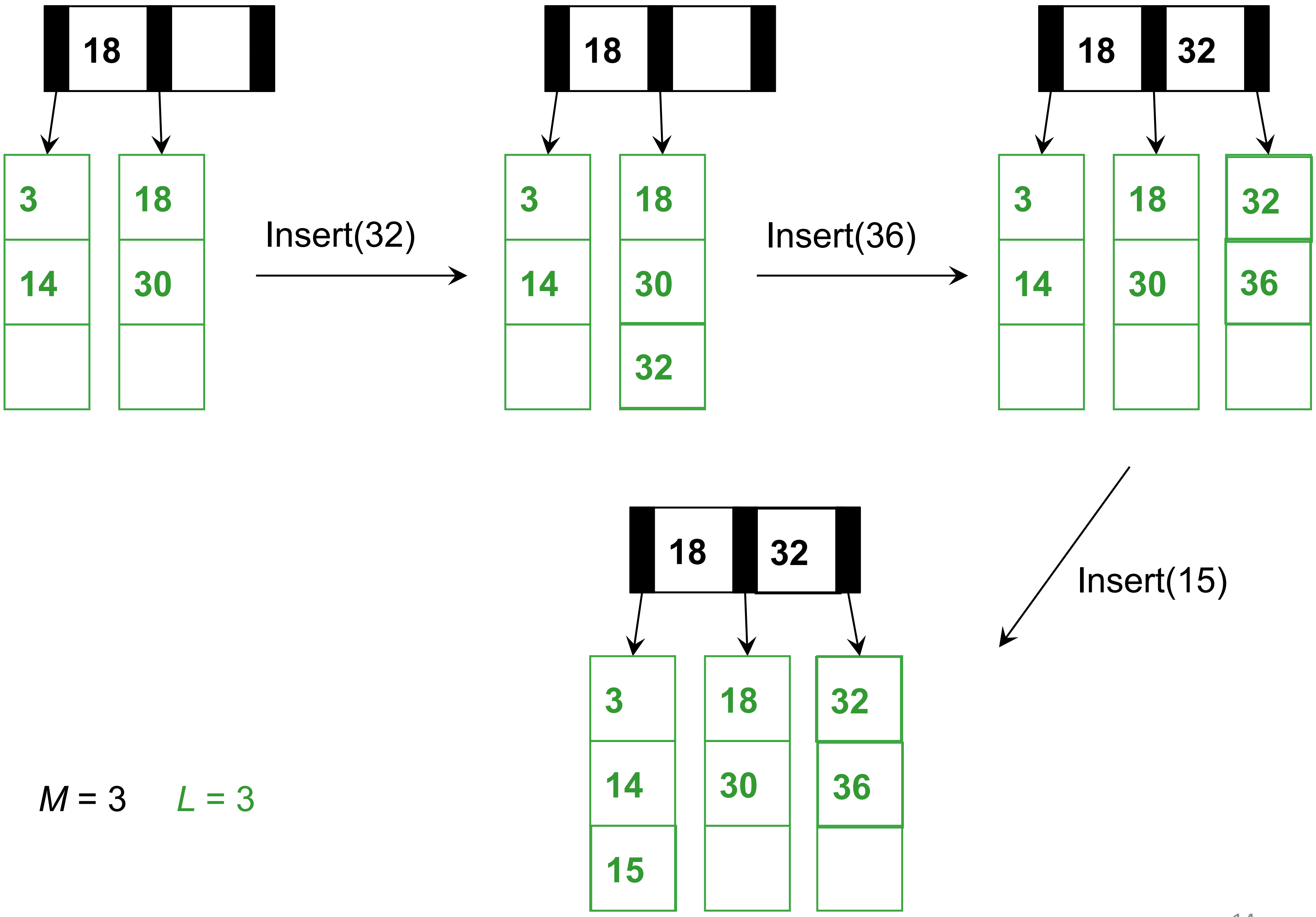


The empty B-Tree

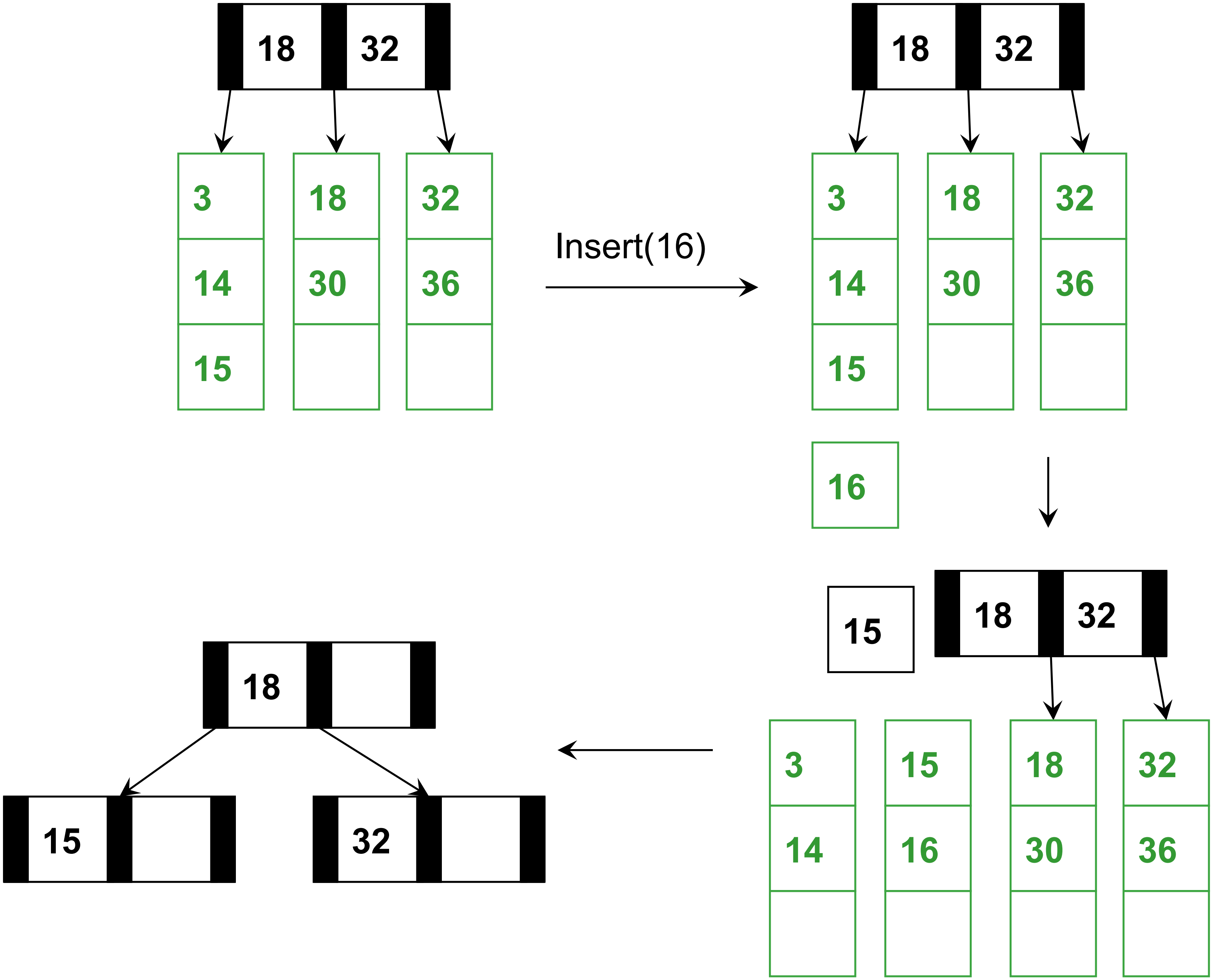
$M = 3$ $L = 3$



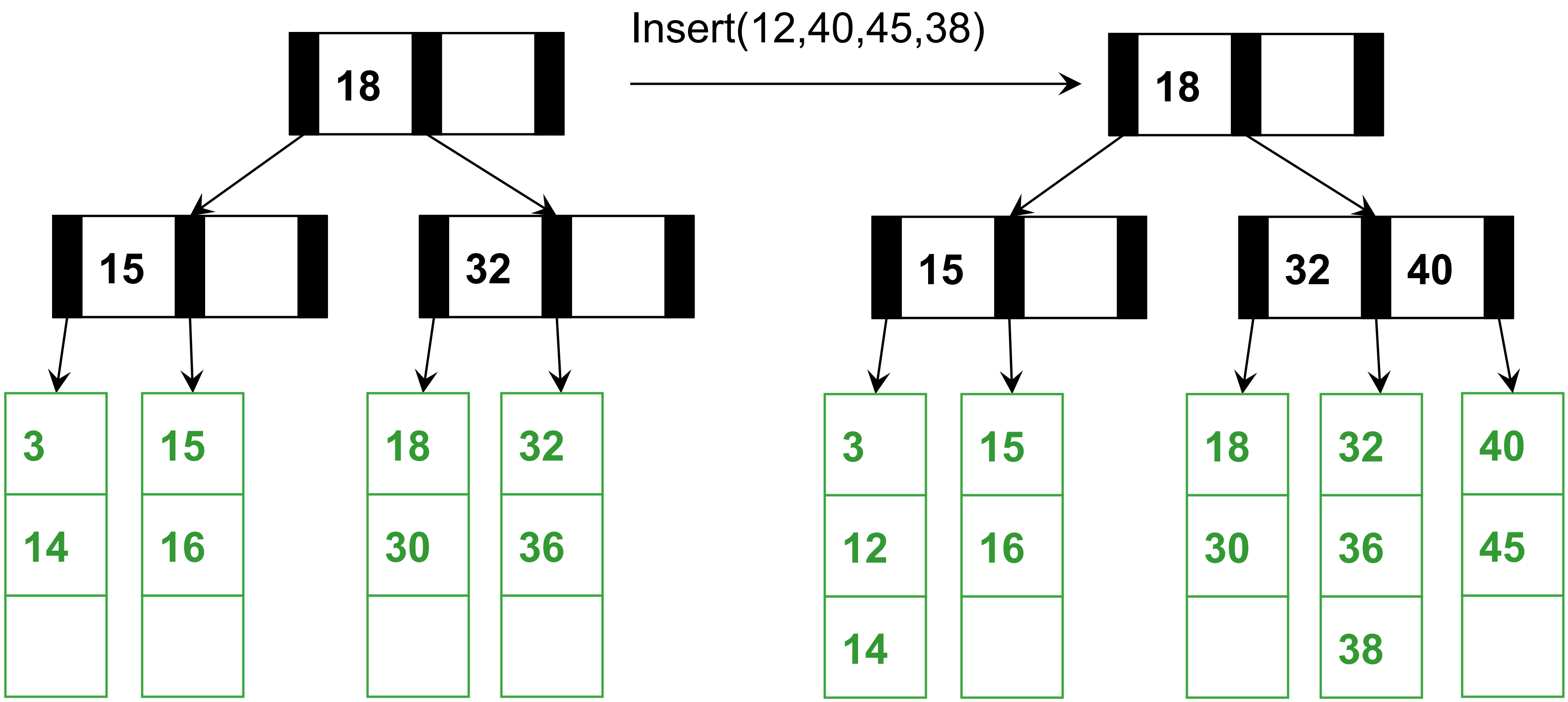
$M = 3$ $L = 3$


18
3
14
18
30
Insert(32)
18
3
14
18
30
32
Insert(36)
18
32
3
14
18
30
32
36
Insert(15)
18
32
3
14
15
18
30
32
36

M = 3 *L* = 3

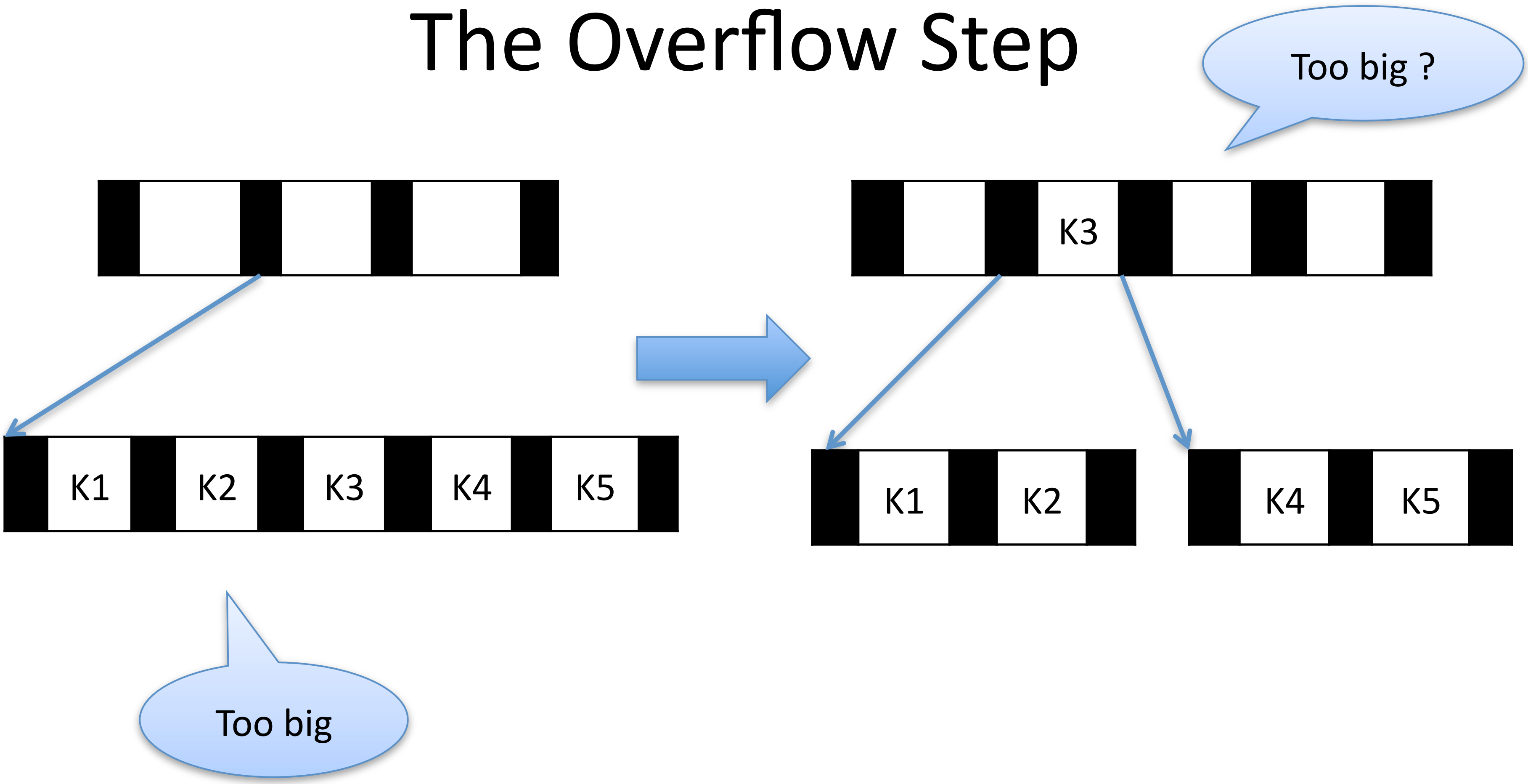

18
32
3
14
15
18
30
32
36
Insert(16)
18
32
3
14
15
18
30
32
36
16
15
18
32
3
14
15
16
18
30
32
36
18
15
32

$M = 3$ $L = 3$



Insert(12,40,45,38)
18
15
32
3
14
15
16
18
30
32
36
18
15
32
40
3
12
14
15
16
18
30
32
36
38
40
45

Insertion Algorithm: The Overflow Step



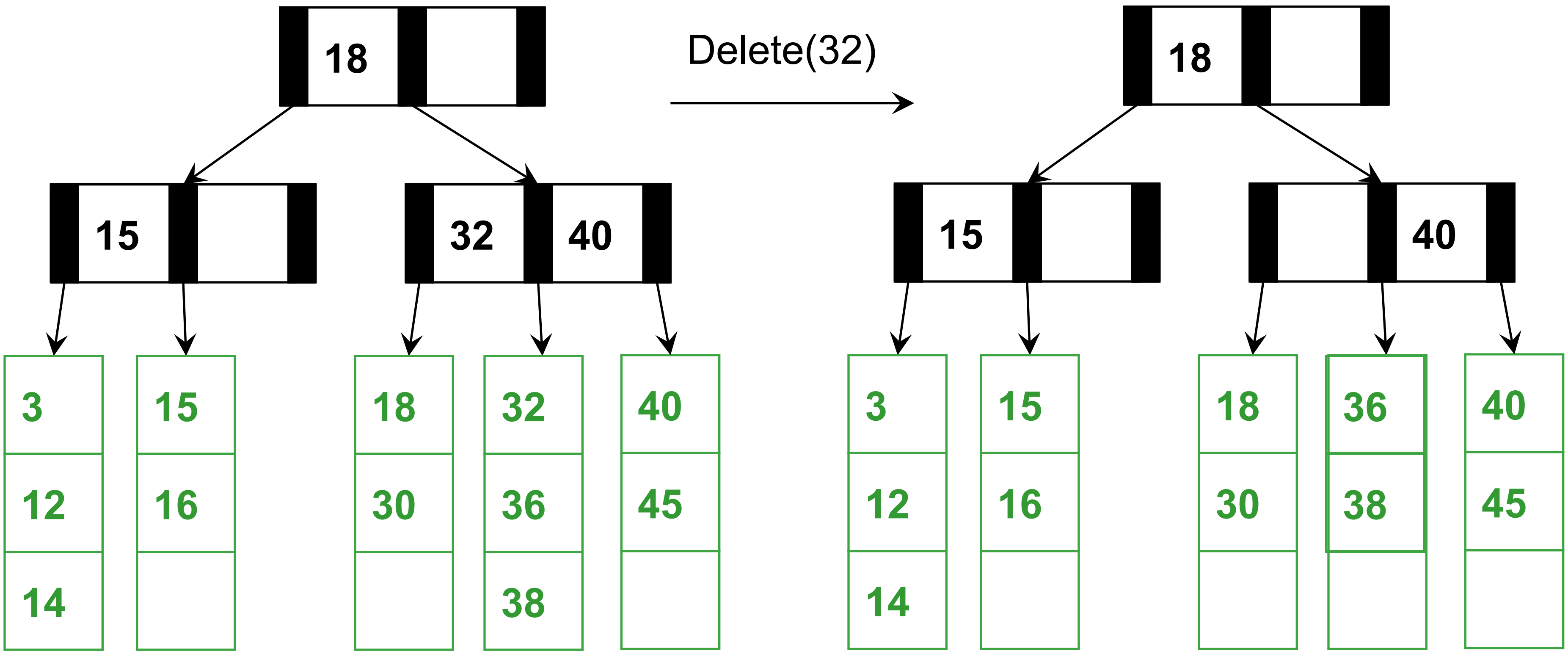
$M = 5$

Insertion Algorithm

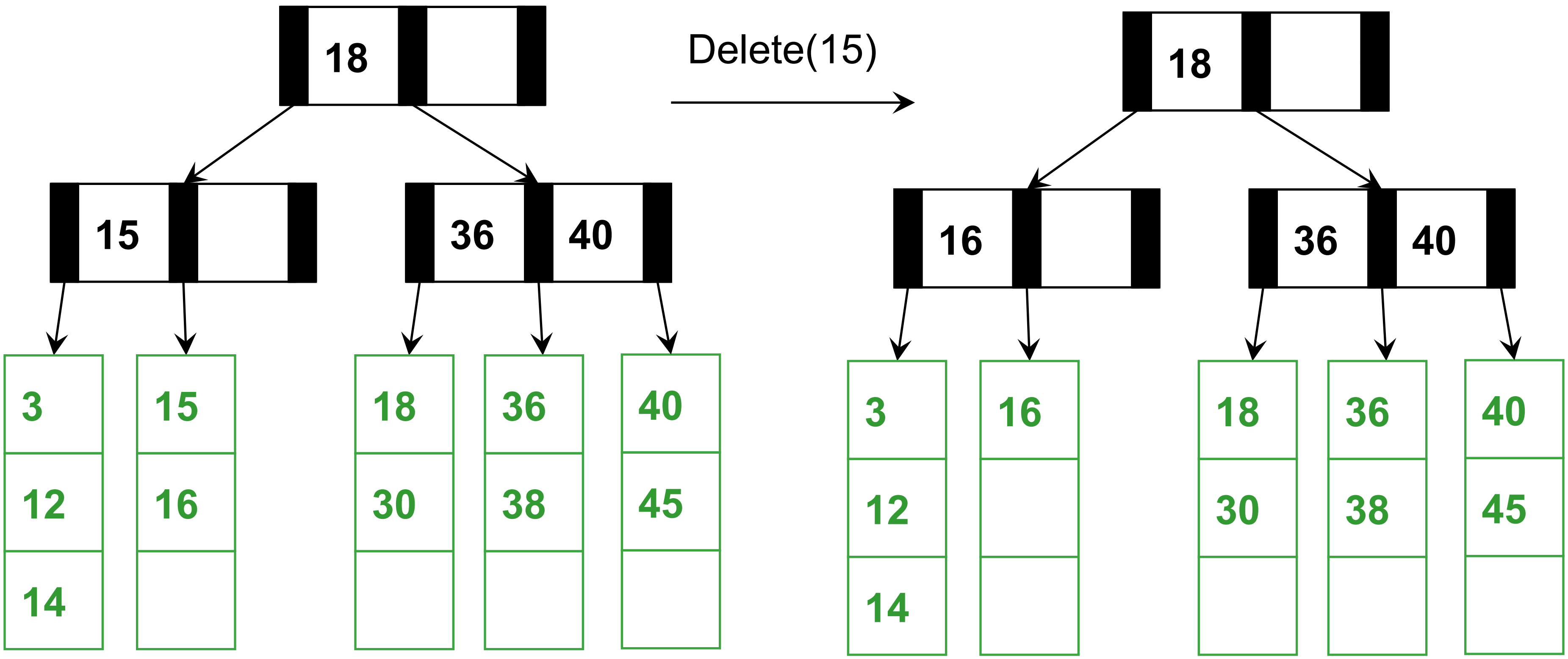
1. Insert the key in its leaf in sorted order
2. If the leaf ends up with ***L+1*** items, **overflow**!
   - Split the leaf into two nodes:
     $\lceil$**(*L*+1)/2**$\rceil$ smaller keys
     $\lfloor$**(*L*+1)/2**$\rfloor$ larger keys
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** children, **overflow**!
3. If an internal node ends up with ***M+1*** children, **overflow**!
   - Split the node into two nodes:
     $\lceil$**(*M*+1)/2**$\rceil$ children with smaller keys
     $\lfloor$**(*M*+1)/2**$\rfloor$ children with larger keys
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
4. If the root ends up with ***M+1*** children, split it in two, and create new root with two children

This makes the tree deeper!

And Now for Deletion…



M = 3 *L* = 3

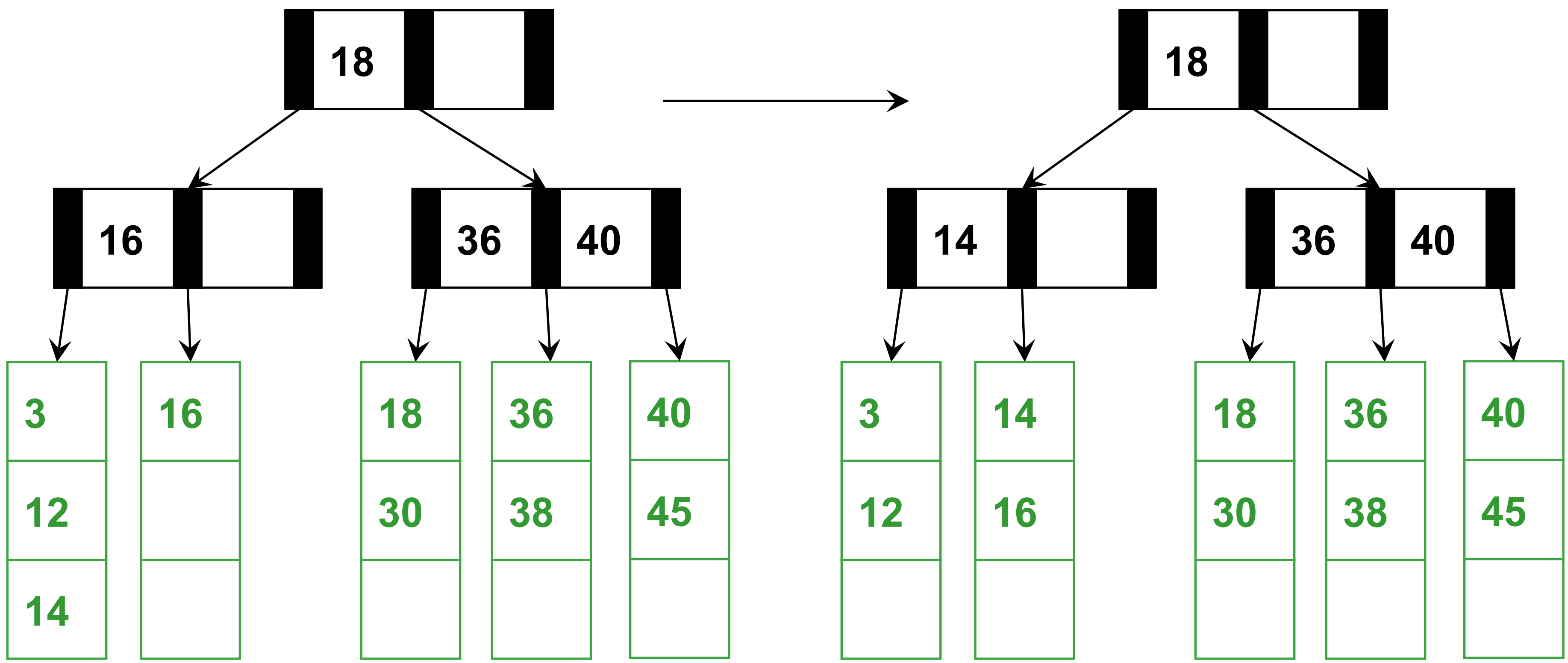


Are we okay?

M = 3 L = 3

Dang, not half full

Are you using that 14?
Can I borrow it?


18
16
36 40
3
12
14
16
18
30
36
38
40
45
18
14
36 40
3
12
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16
18
30
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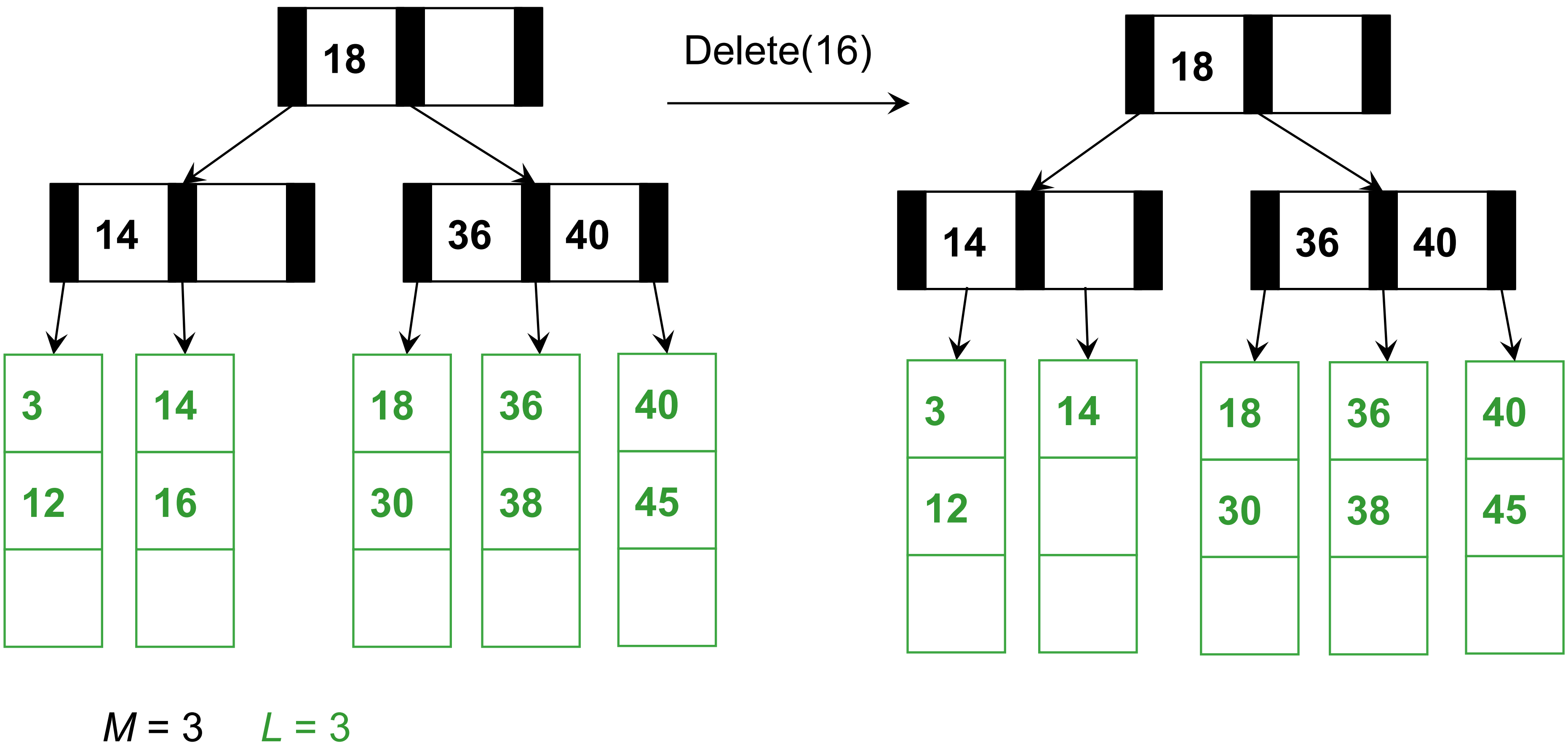
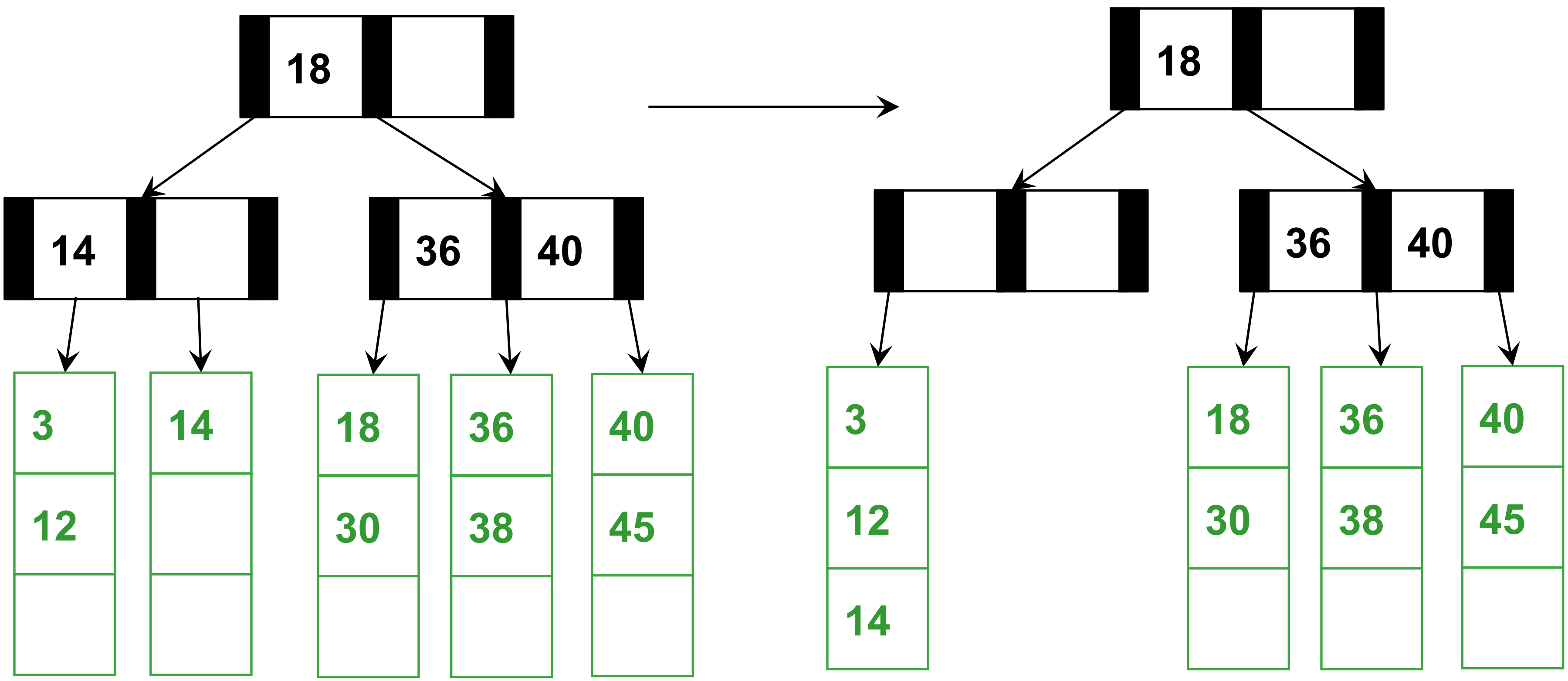
38
40
45

M = 3 *L* = 3

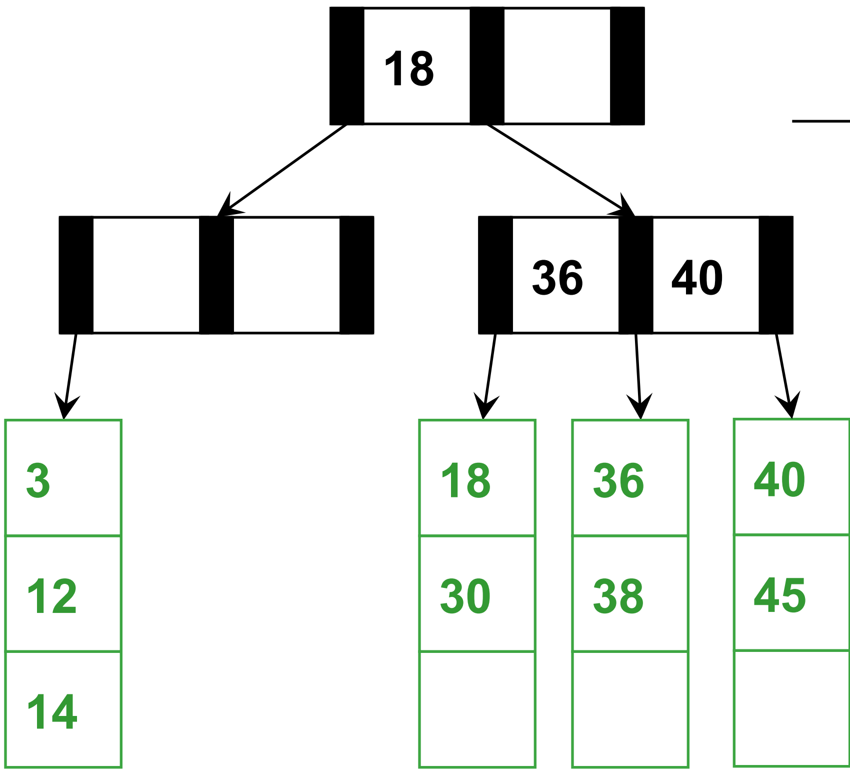


Are you using that 12?

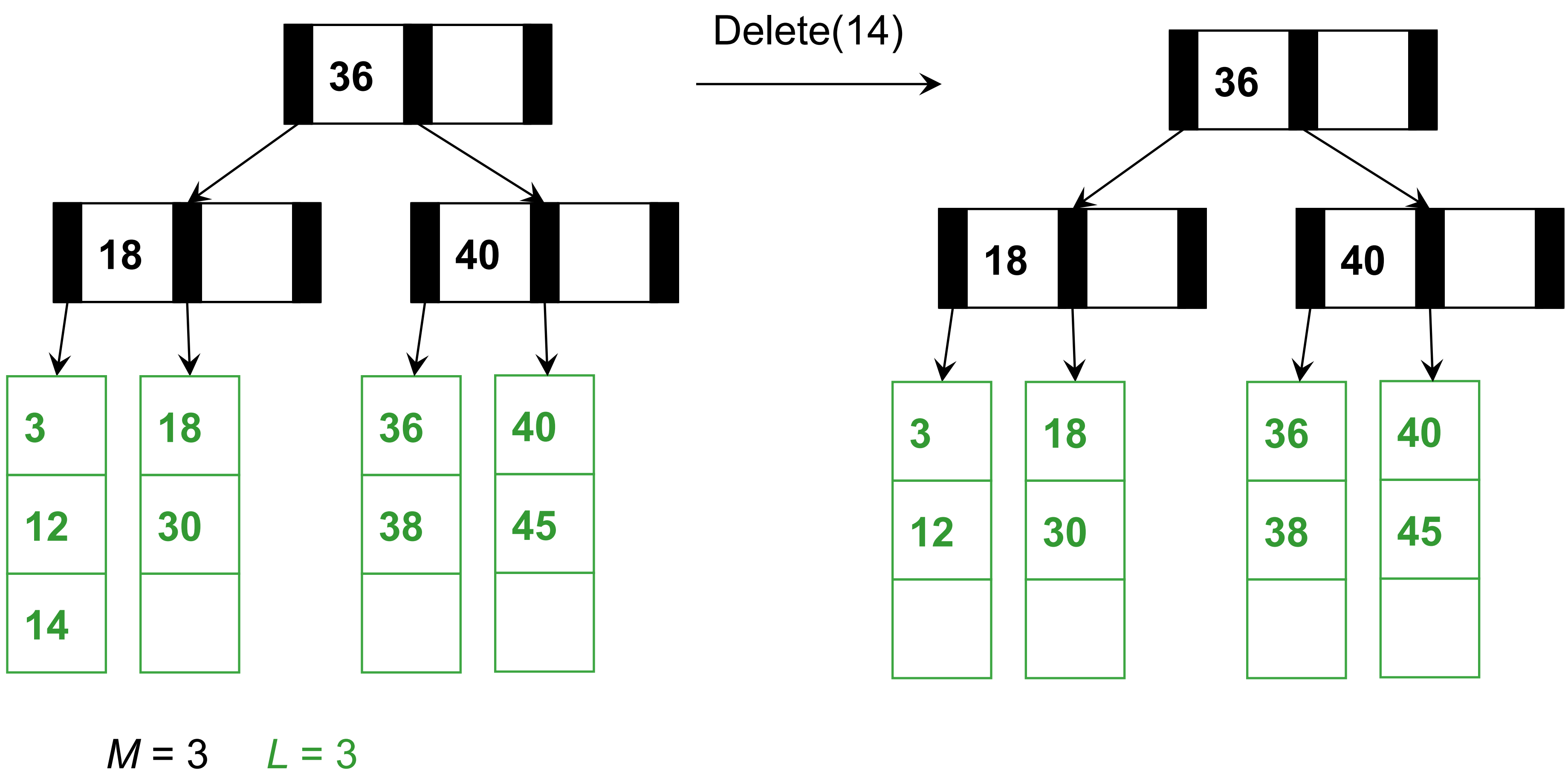


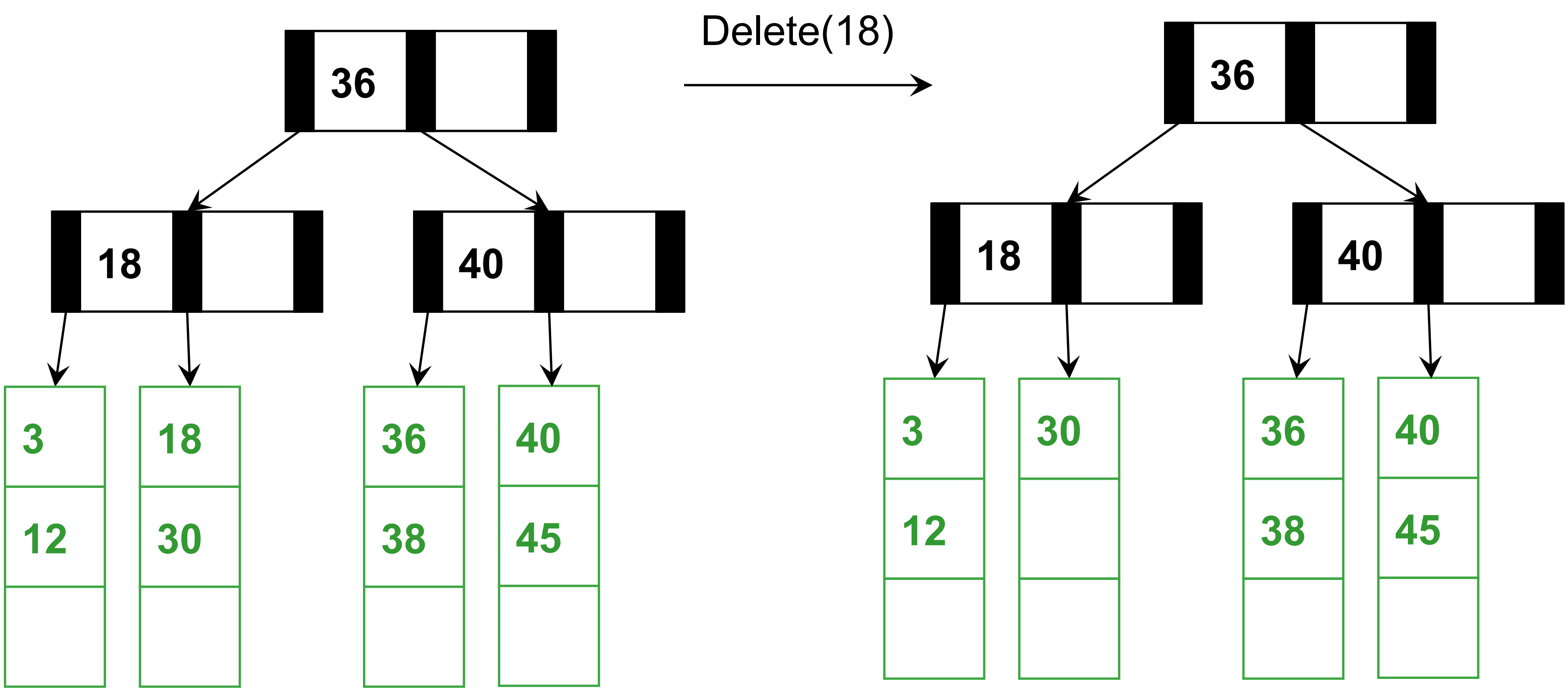
M = 3 *L* = 3

Are you using the node18/30?

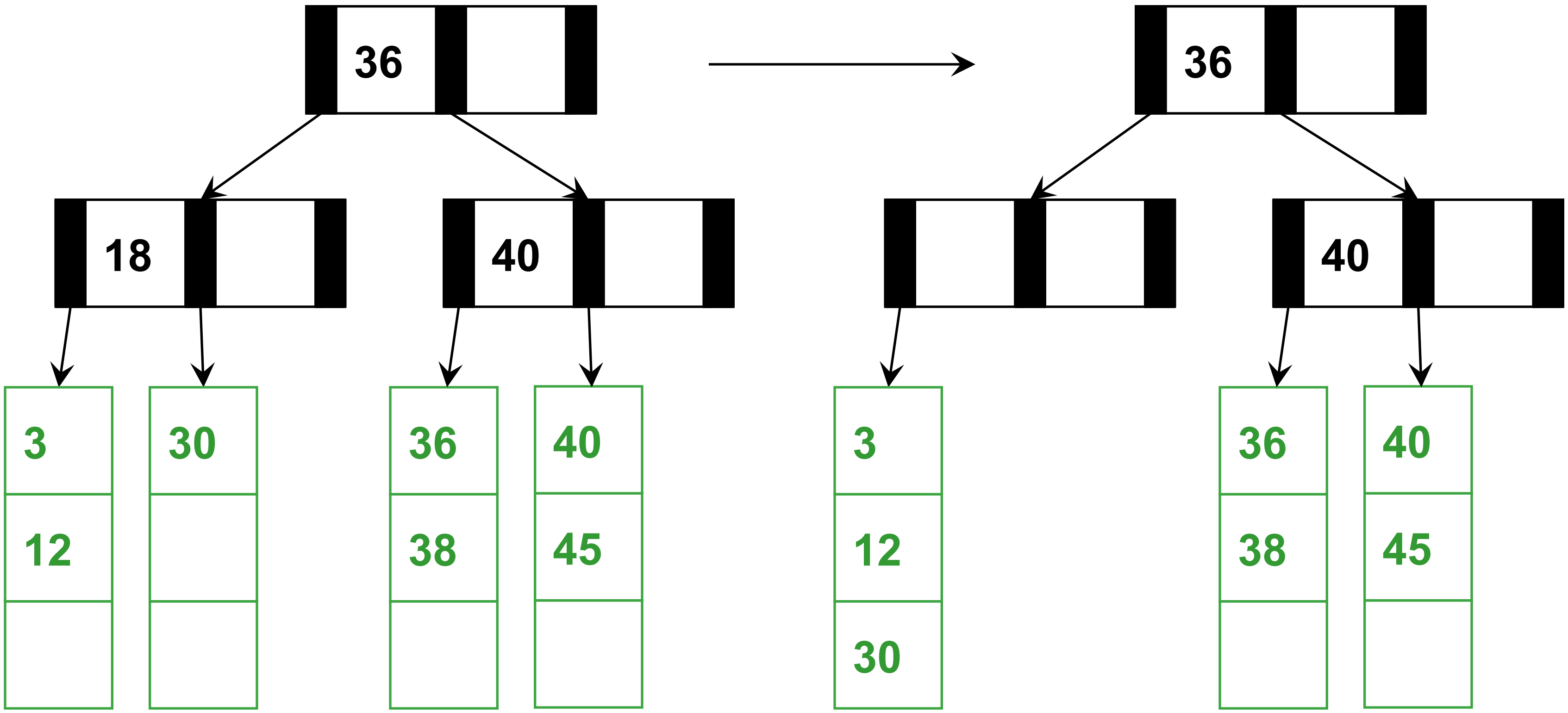


$M = 3$ $L = 3$

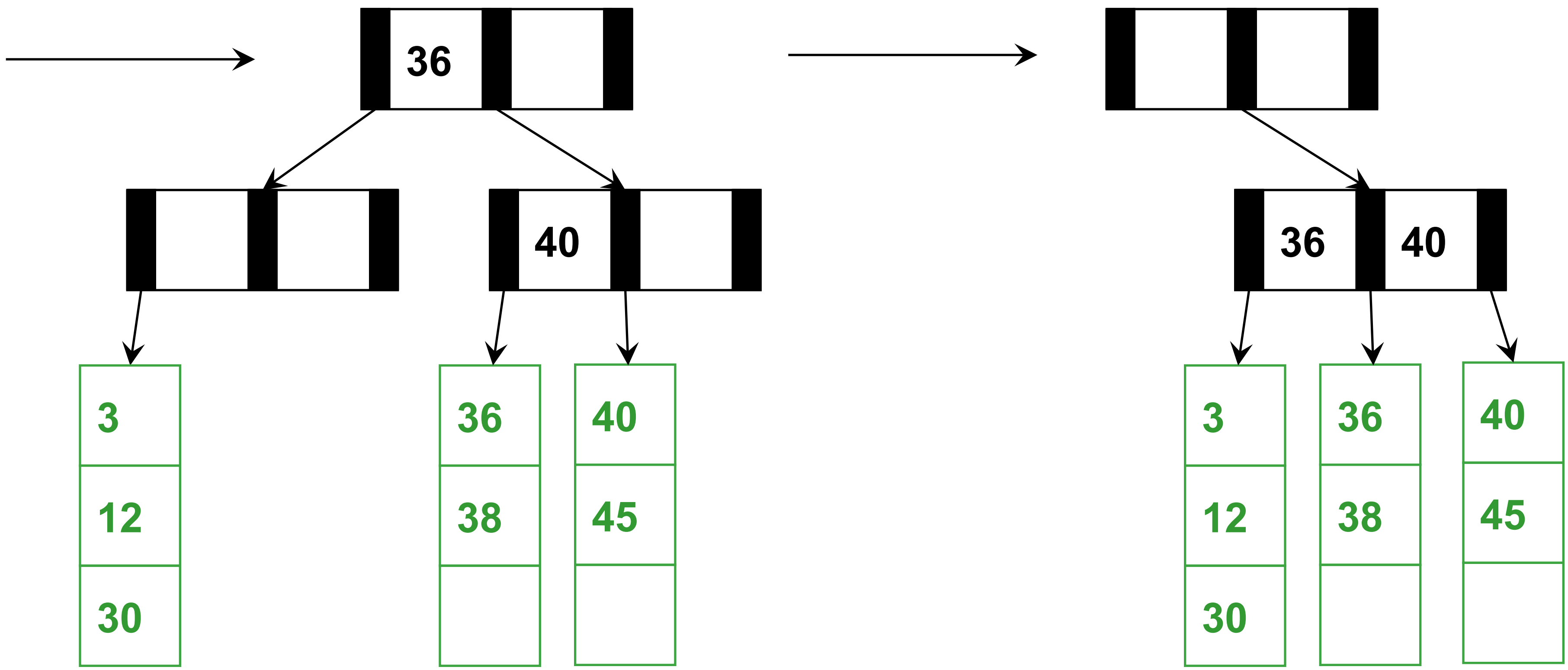

36
18
40
3
12
14
18
30
36
38
40
45
Delete(14)
36
18
40
3
12
18
30
36
38
40
45
M = 3
L = 3



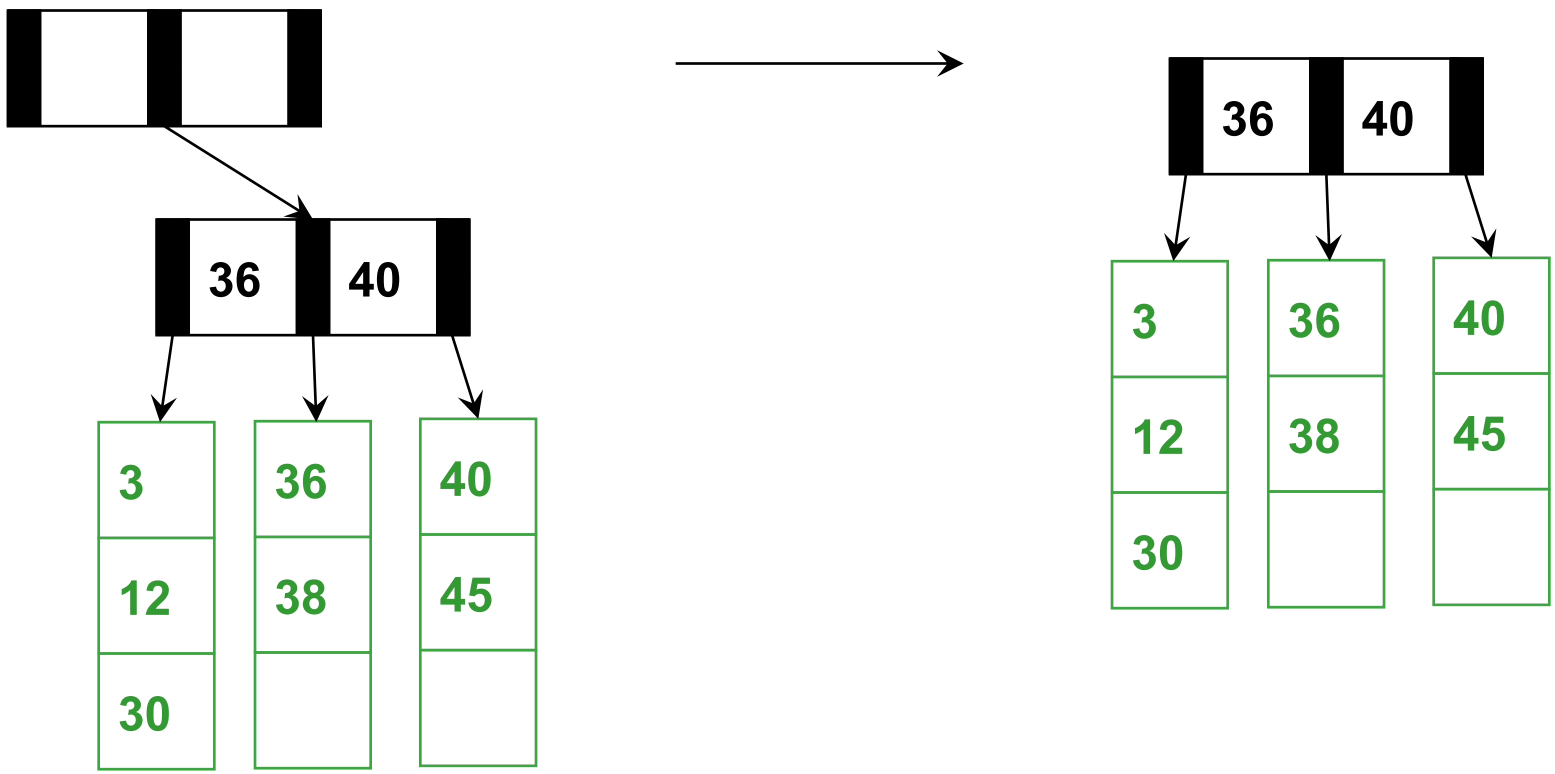
M = 3 L = 3



$M = 3$ $L = 3$

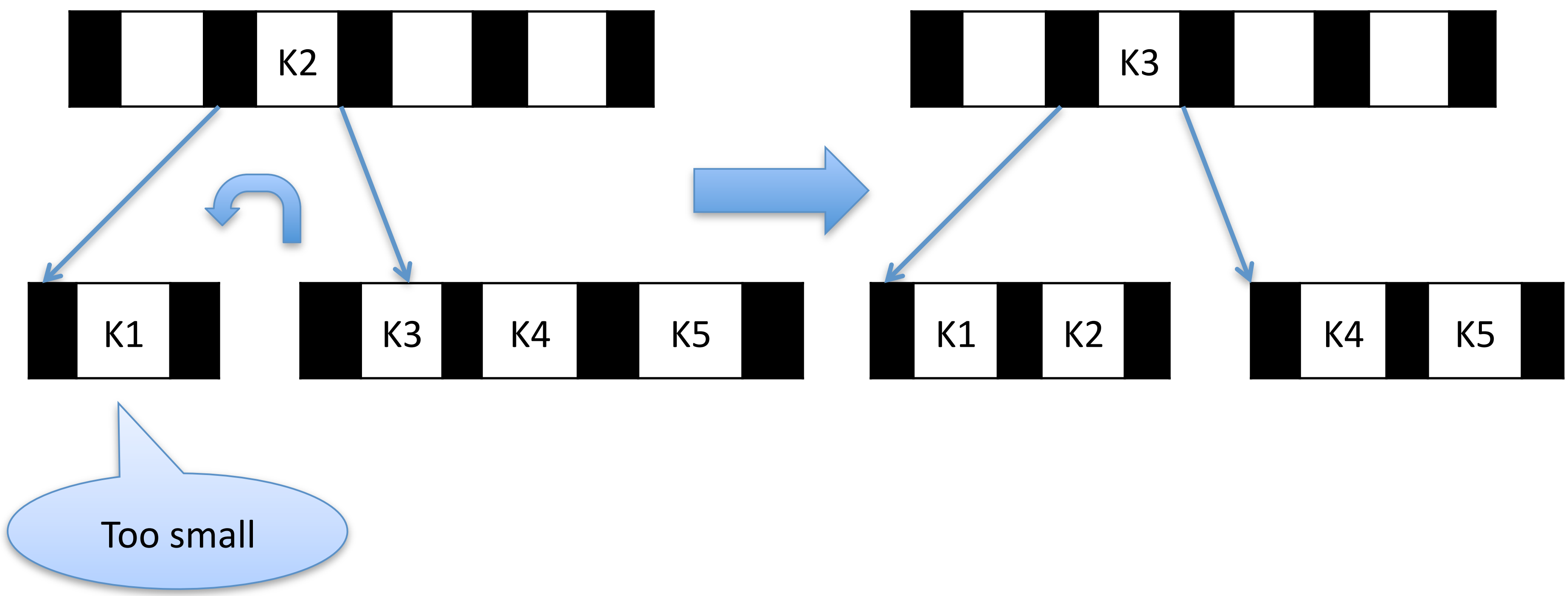


$M = 3$ $\quad L = 3$



$M = 3$ $L = 3$

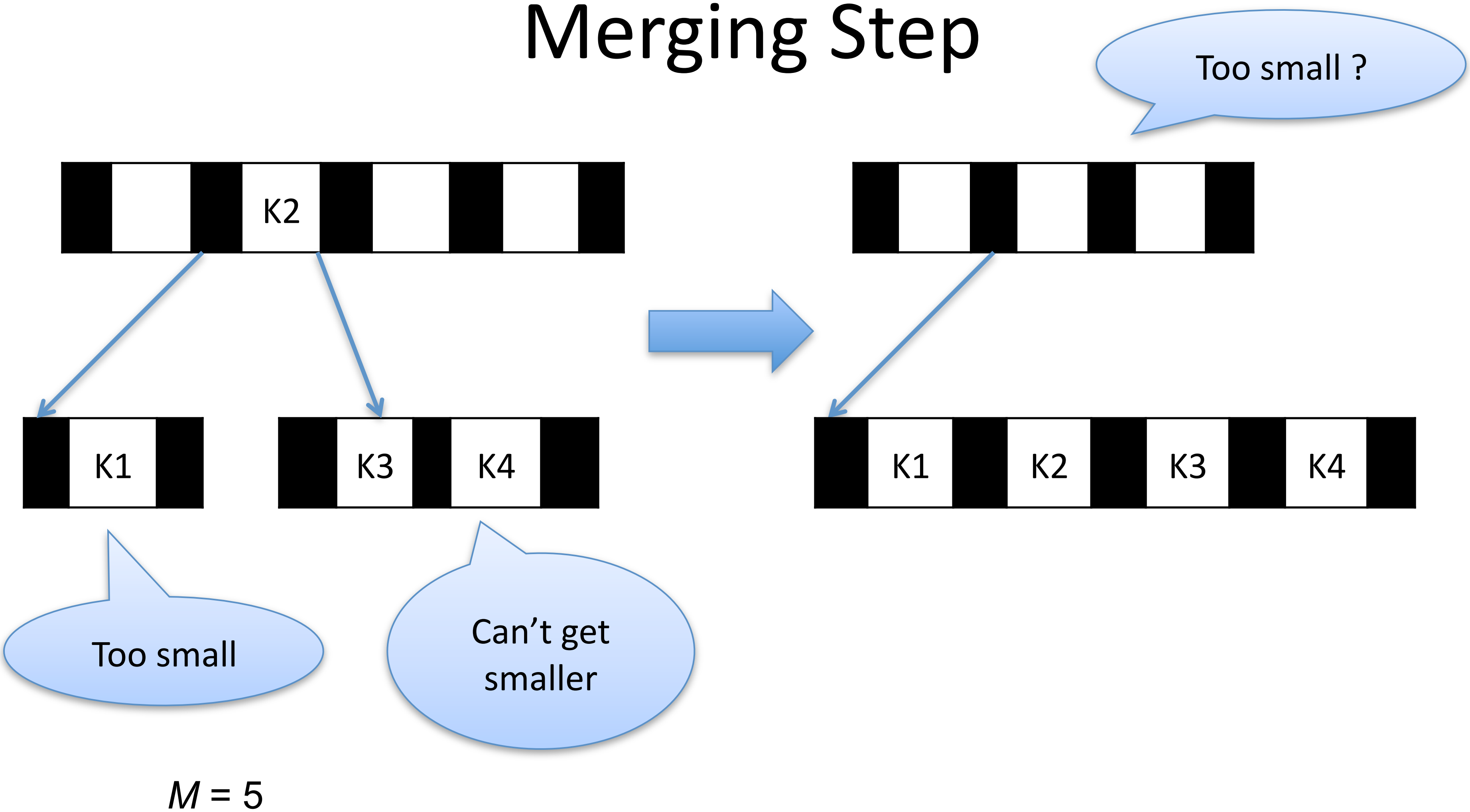
Deletion Algorithm: Rotation Step



$M = 5$

This is *left* rotation. Similarly, *right* rotation

Deletion Algorithm: Merging Step



Deletion Algorithm

1. Remove the key from its leaf

2. If the leaf ends up with fewer than ⌈***L*/2**⌉ items, **underflow**!
   - Try a left rotation
   - If not, try a right rotation
   - If not, merge, then check the parent node for underflow

Deletion Slide Two

3. If an internal node ends up with fewer than ⌈***M*/2**⌉ children, **underflow**!
   – Try a left rotation
   – If not, try a right rotation
   – If not, merge, then check the parent node for underflow

4. If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!