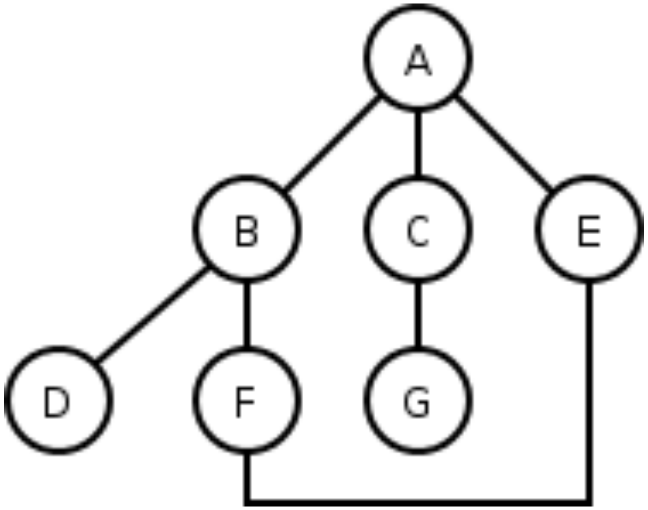
CSE 373: Data Structures and Algorithms

Lecture 19: Graphs III

Depth-first search

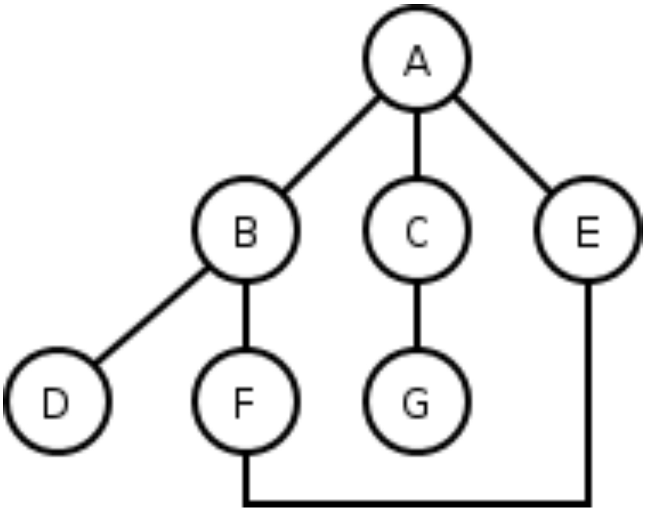
- **depth-first search (DFS)**: finds a path between two vertices by exploring each possible path as many steps as possible before backtracking
  - – often implemented recursively



DFS template

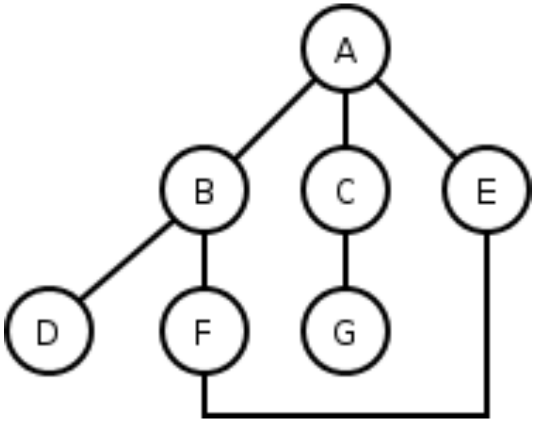
- Pseudo-code for depth-first template:

```
dfs(Vertex v):
    mark v as visited
    for each unvisited neighbor $v_i$ of v
   where there is an edge from v to $v_i$:
       if( !$v_i$.visited )
           dfs($v_i$)
```



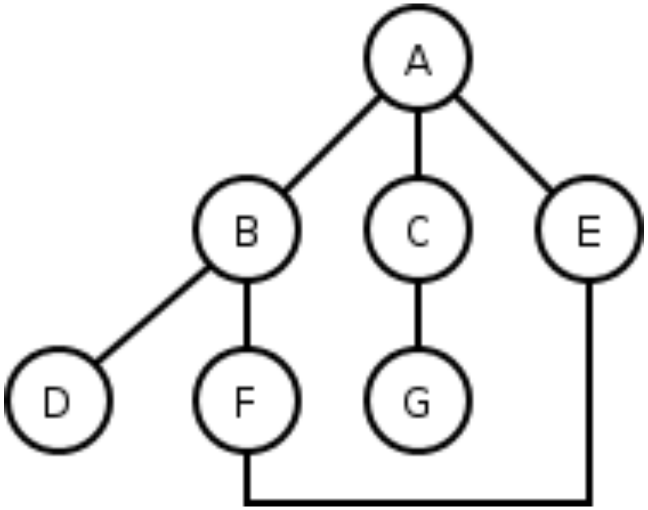
Breadth-first search

- **breadth-first search (BFS)**: finds a path between two nodes by taking one step down all paths and then immediately backtracking
  - often implemented by maintaining a list or queue of vertices to visit
  - BFS always returns the path with the fewest edges between the start and the goal vertices



BFS example

- All BFS paths from A to others (assumes ABC edge order)
  - A
  - A -> B
  - A -> C
  - A -> E
  - A -> B -> D
  - A -> B -> F
  - A -> C -> G



- What are the paths that BFS did not find?

BFS pseudocode

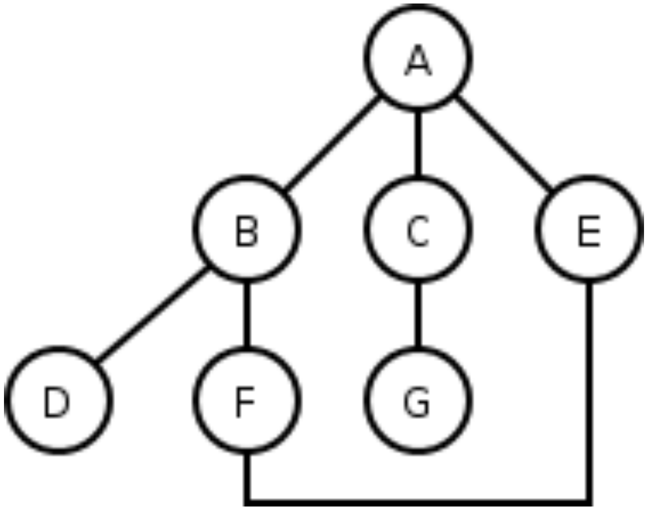
- Pseudo-code for breadth-first search:

```
bfs(v1, v2):
    List := {v1}.
    mark v1 as visited.

    while List not empty:
        v := List.removeFirst().
        if v is v2:
            path is found.

        for each unvisited neighbor $v_i$ of v
        where there is an edge from v to $v_i$:
            mark $v_i$ as visited.
            List.addLast($v_i$).

    path is not found.
```

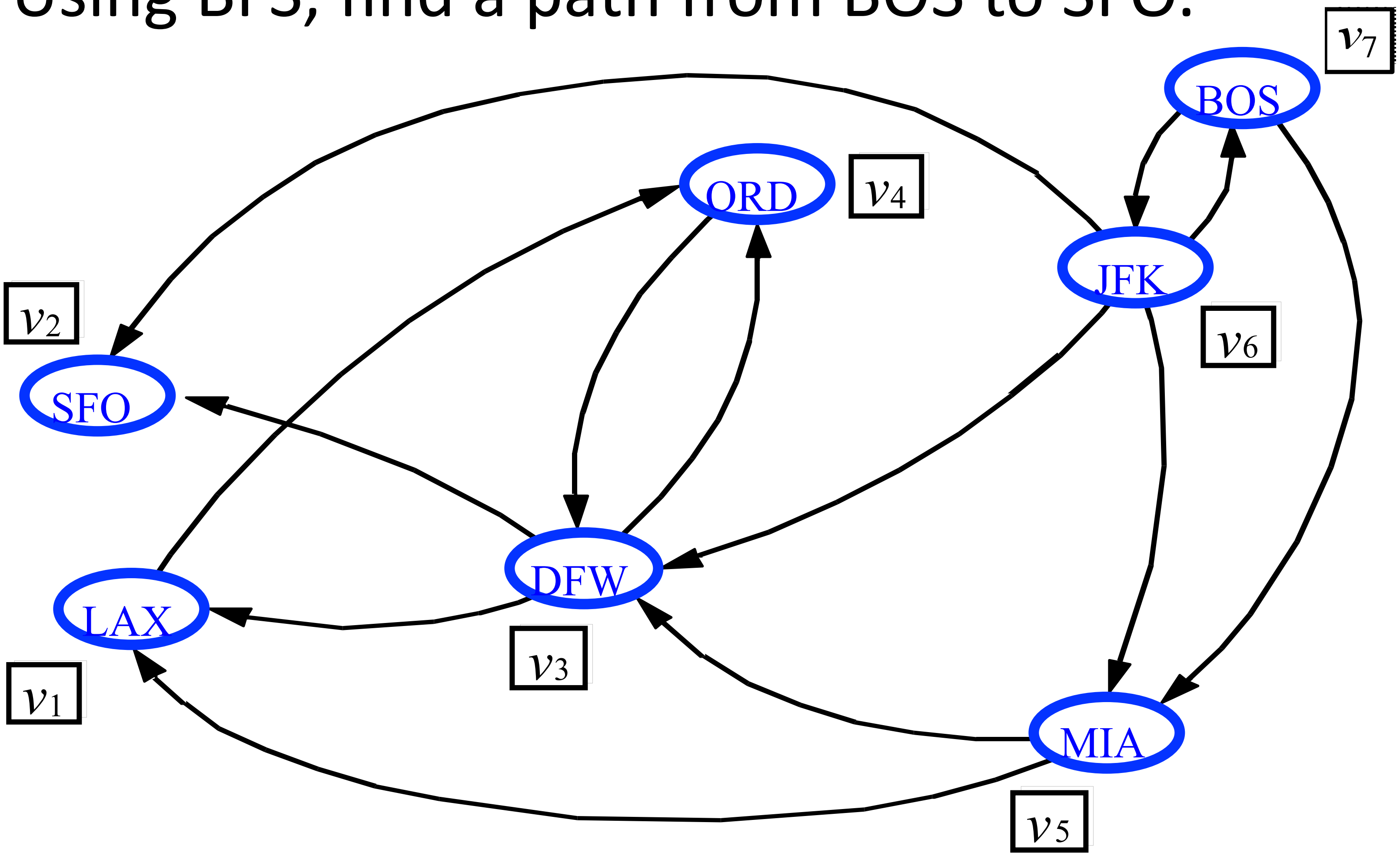


BFS observations

- *optimality*:
  - in unweighted graphs, optimal. (fewest edges = best)
  - In weighted graphs, not optimal.
    (path with fewest edges might not have the lowest weight)
- *disadvantage*: harder to reconstruct what the actual path is once you find it
  - conceptually, BFS is exploring many possible paths in parallel, so it's not easy to store a Path array/list in progress
- observation: any particular vertex is only part of one partial path at a time
  - We can keep track of the path by storing *predecessors* for each vertex (references to the previous vertex in that path)

Another BFS example

- Using BFS, find a path from BOS to SFO.



DFS, BFS runtime

- What is the expected runtime of DFS, in terms of the number of vertices V and the number of edges E ?

- What is the expected runtime of BFS, in terms of the number of vertices V and the number of edges E ?

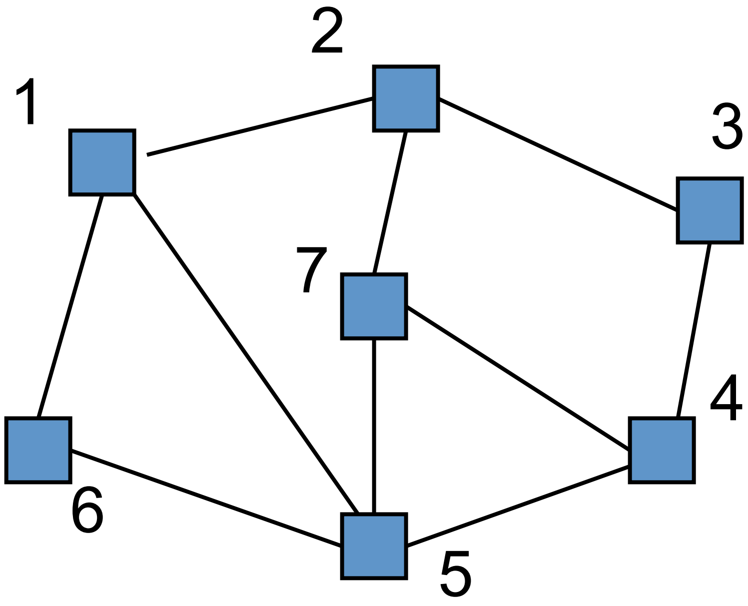
- Answer: O($|V| + |E|$)
  - each algorithm must potentially visit every node and/or examine every edge once.
  - why not O($|V| * |E|$) ?

- What is the space complexity of each algorithm?

Implementing graphs

Implementing a graph

- If we wanted to program an actual data structure to represent a graph, what information would we need to store?
  - for each vertex?
  - for each edge?



- What kinds of questions would we want to be able to answer quickly:
  - about a vertex?
  - about its edges / neighbors?
  - about paths?
  - about what edges exist in the graph?

- We'll explore three common graph implementation strategies:
  - edge list, adjacency list, adjacency matrix

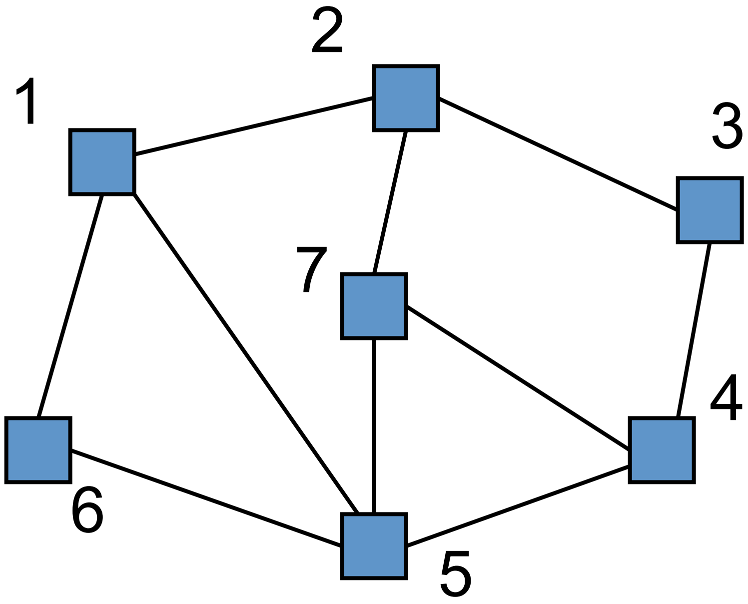
Edge list

- **edge list**: an unordered list of all edges in the graph

- *advantages*
  - easy to loop/iterate over all edges

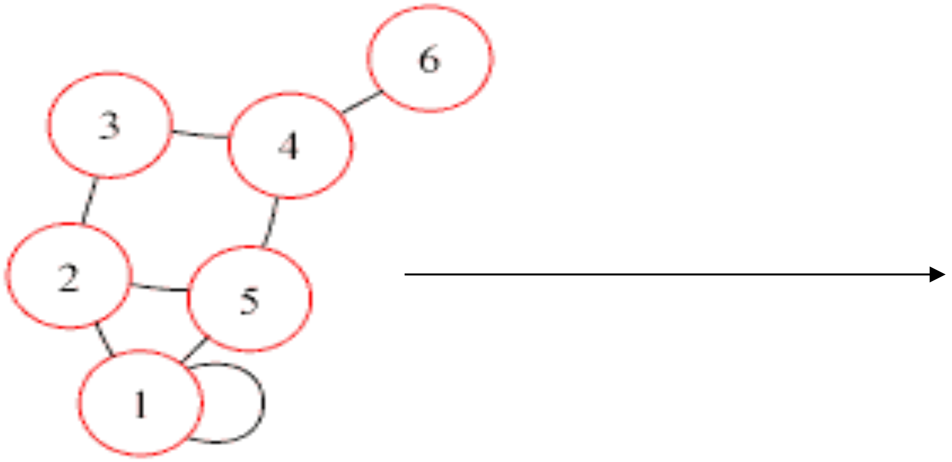
- *disadvantages*
  - hard to tell if an edge exists from A to B
  - hard to tell how many edges a vertex touches (its degree)

1	1	1	2	2	3	5	5	5	7
2	5	6	7	3	4	6	7	4	4



Adjacency matrix

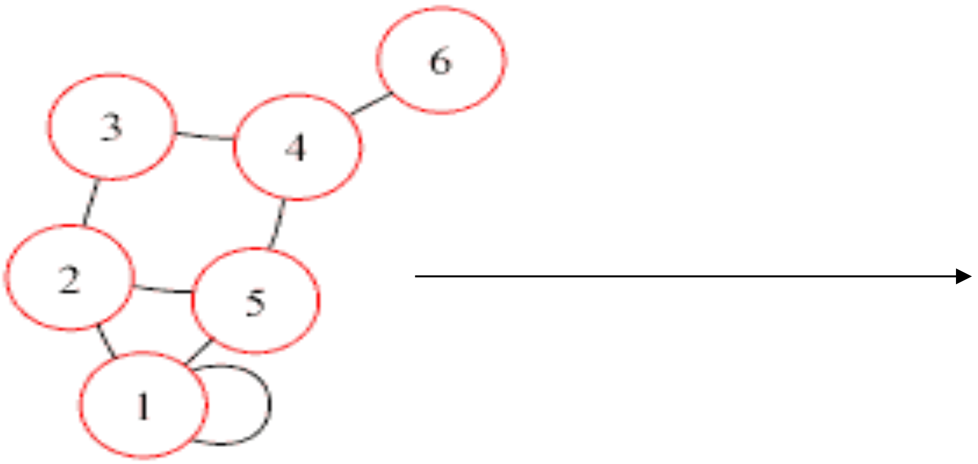
- **adjacency matrix**: an $n \times n$ matrix where:
  - the nondiagonal entry a_{ij} is the number of edges joining vertex *i* and vertex *j* (or the weight of the edge joining vertex *i* and vertex *j*)
  - the diagonal entry a_{ii} corresponds to the number of loops (self-connecting edges) at vertex *i*



$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Pros/cons of Adj. matrix

- *advantage*: fast to tell whether edge exists between any two vertices *i* and *j* (and to get its weight)
- *disadvantage*: consumes a lot of memory on sparse graphs (ones with few edges)

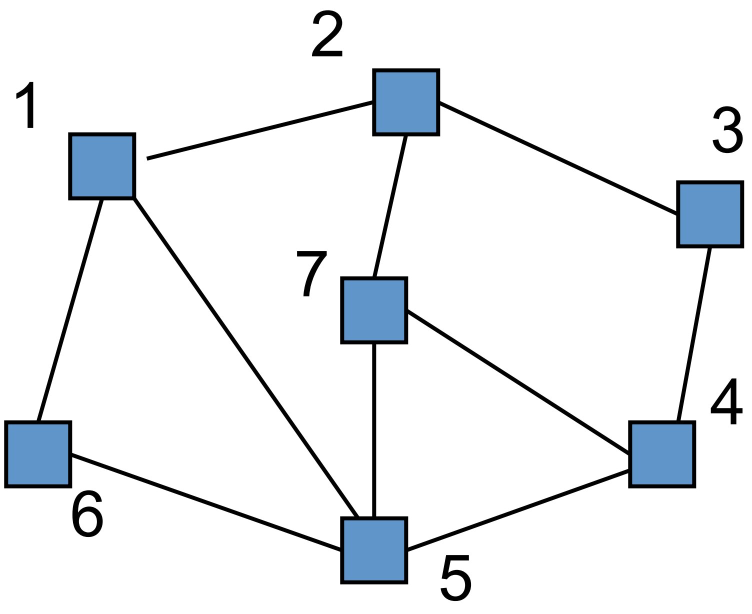


$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Adjacency matrix example

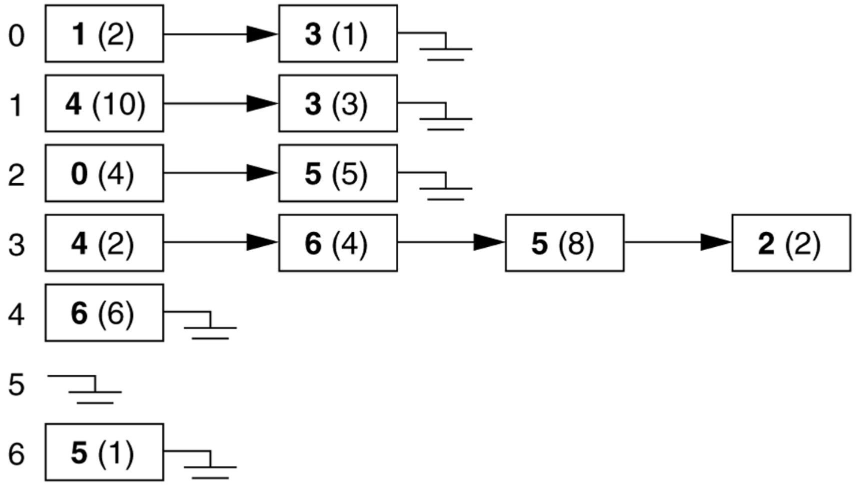
- The graph at right has the following adjacency matrix:
  - How do we figure out the degree of a given vertex?
  - How do we find out whether an edge exists from A to B?
  - How could we look for loops in the graph?

	1	2	3	4	5	6	7
1	0	1	0	0	1	1	0
2	1	0	1	0	0	0	1
3	0	1	0	1	0	0	0
4	0	0	1	0	1	0	1
5	1	0	0	1	0	1	1
6	1	0	0	0	1	0	0
7	0	1	0	1	1	0	0



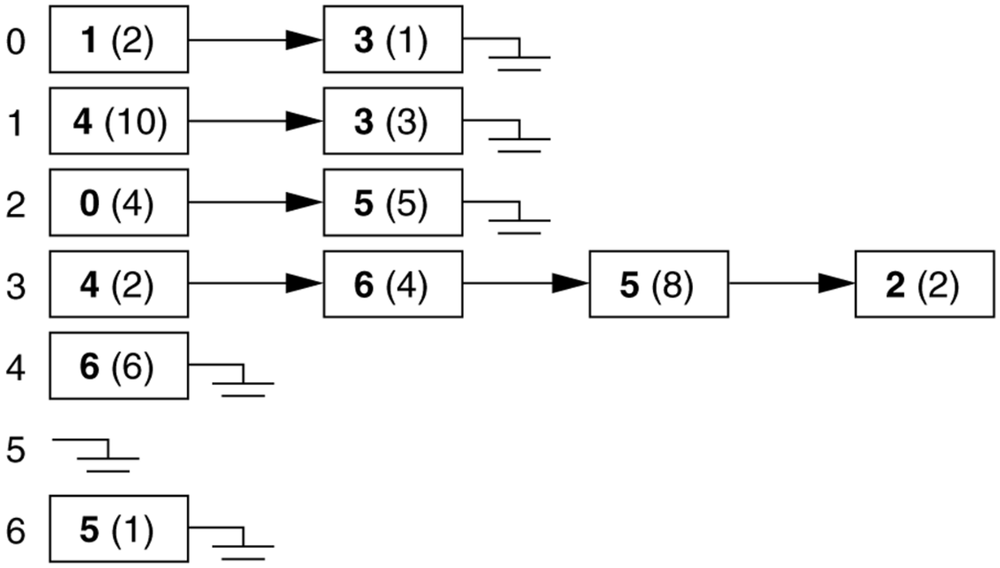
Adjacency lists

- **adjacency list**: stores edges as individual linked lists of references to each vertex's neighbors
  - generally, no information needs to be stored in the edges, only in nodes, these arrays can simply be pointers to other nodes and thus represent edges with little memory requirement



Pros/cons of adjacency list

- *advantage*: new nodes can be added to the graph easily, and they can be connected with existing nodes simply by adding elements to the appropriate arrays; "who are my neighbors" easily answered
- *disadvantage*: determining whether an edge exists between two nodes requires O(*n*) time, where *n* is the average number of incident edges per node



Adjacency list example

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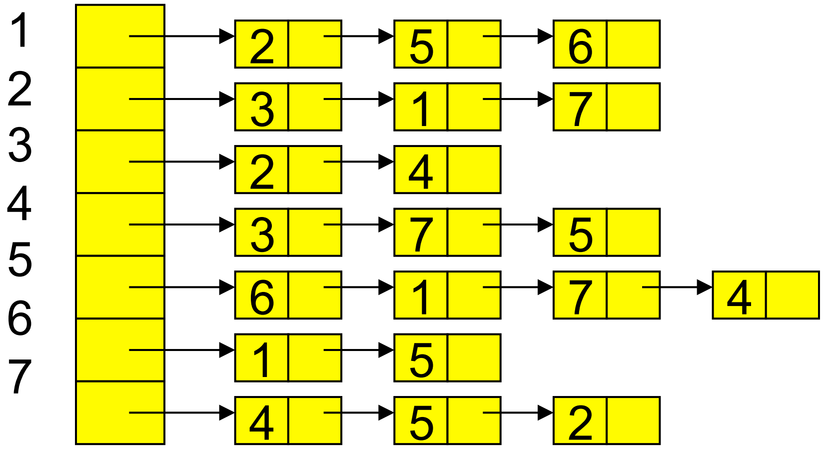

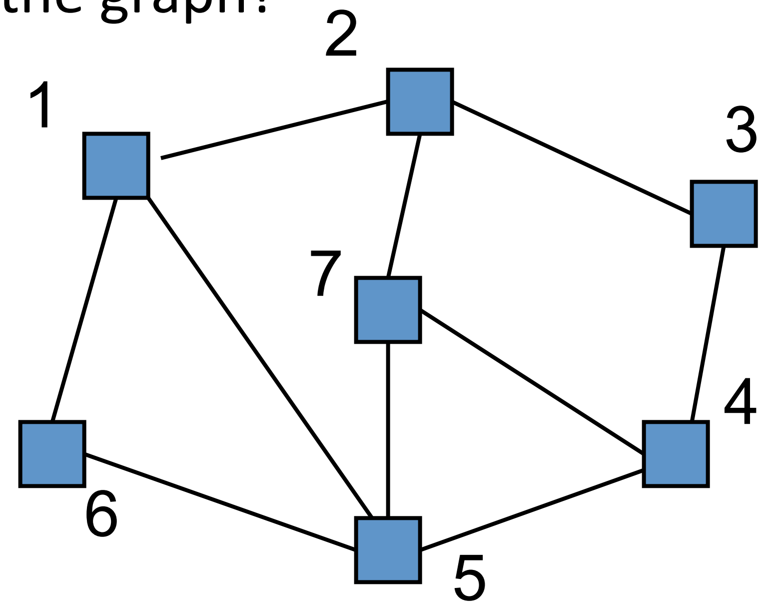

Runtime table

- ***n*** vertices, ***m*** edges - no parallel edges - no self-loops	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
Finding all adjacent vertices to ***v***	m	deg(v)	n
Determining if ***v*** is adjacent to ***w***	m	min(deg(v), deg (w))	1
inserting a vertex	1	1	n^2
inserting an edge	1	1	1
removing vertex ***v***	m	deg(v)	n^2
removing an edge	m	deg(v)	1