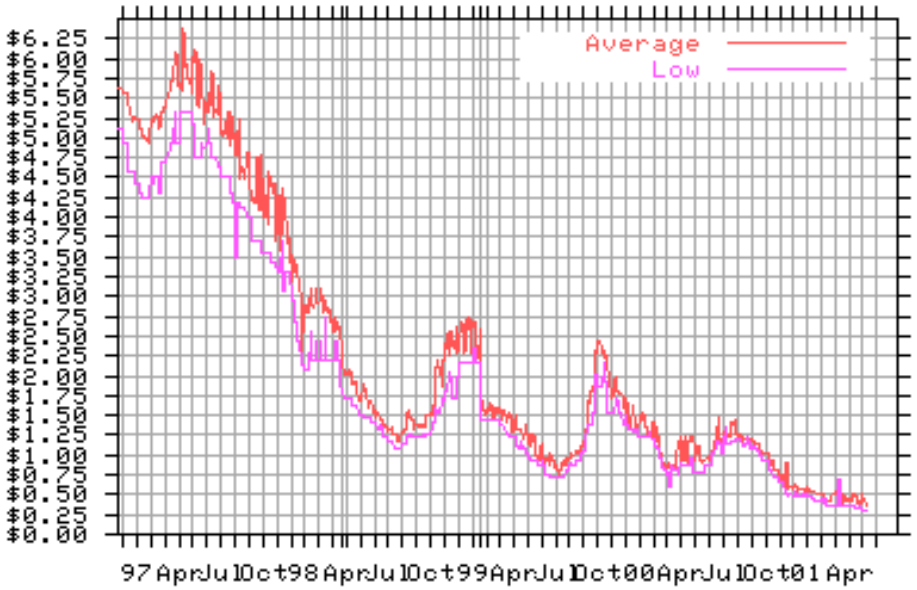
CSE 373: Data Structures and Algorithms

Lecture 17: Graphs

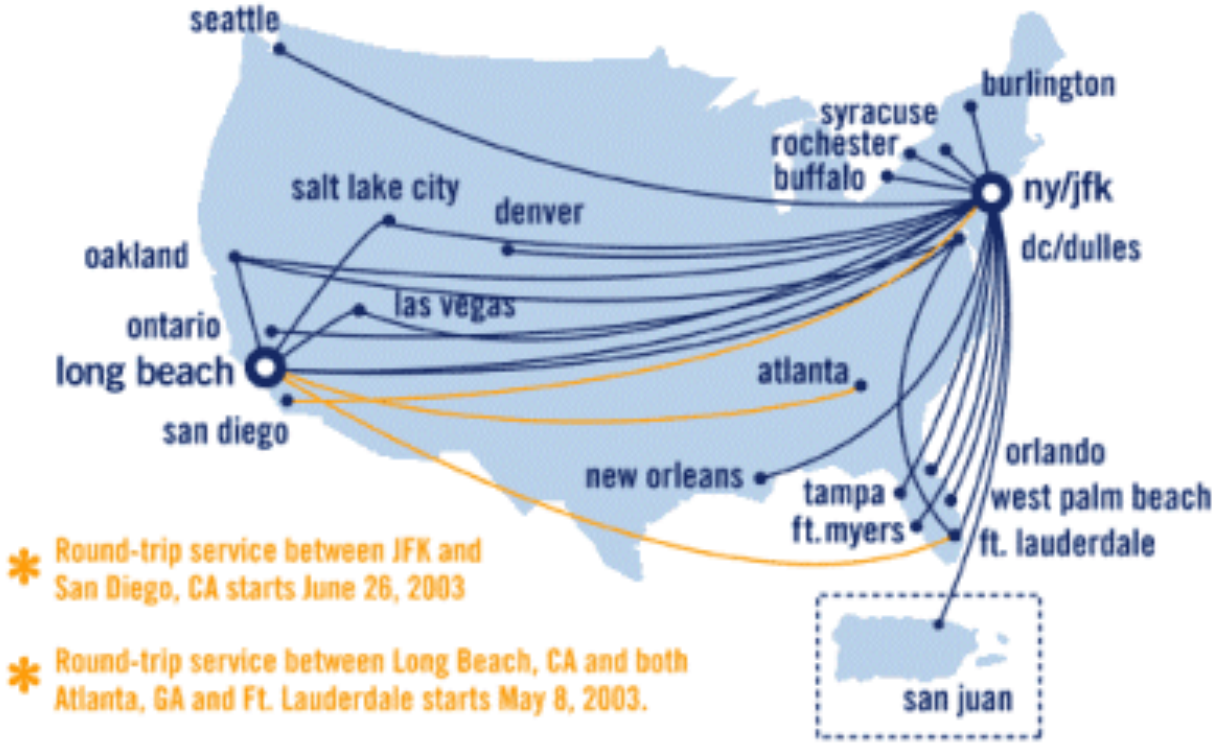
What are graphs?

- Yes, this is a graph….



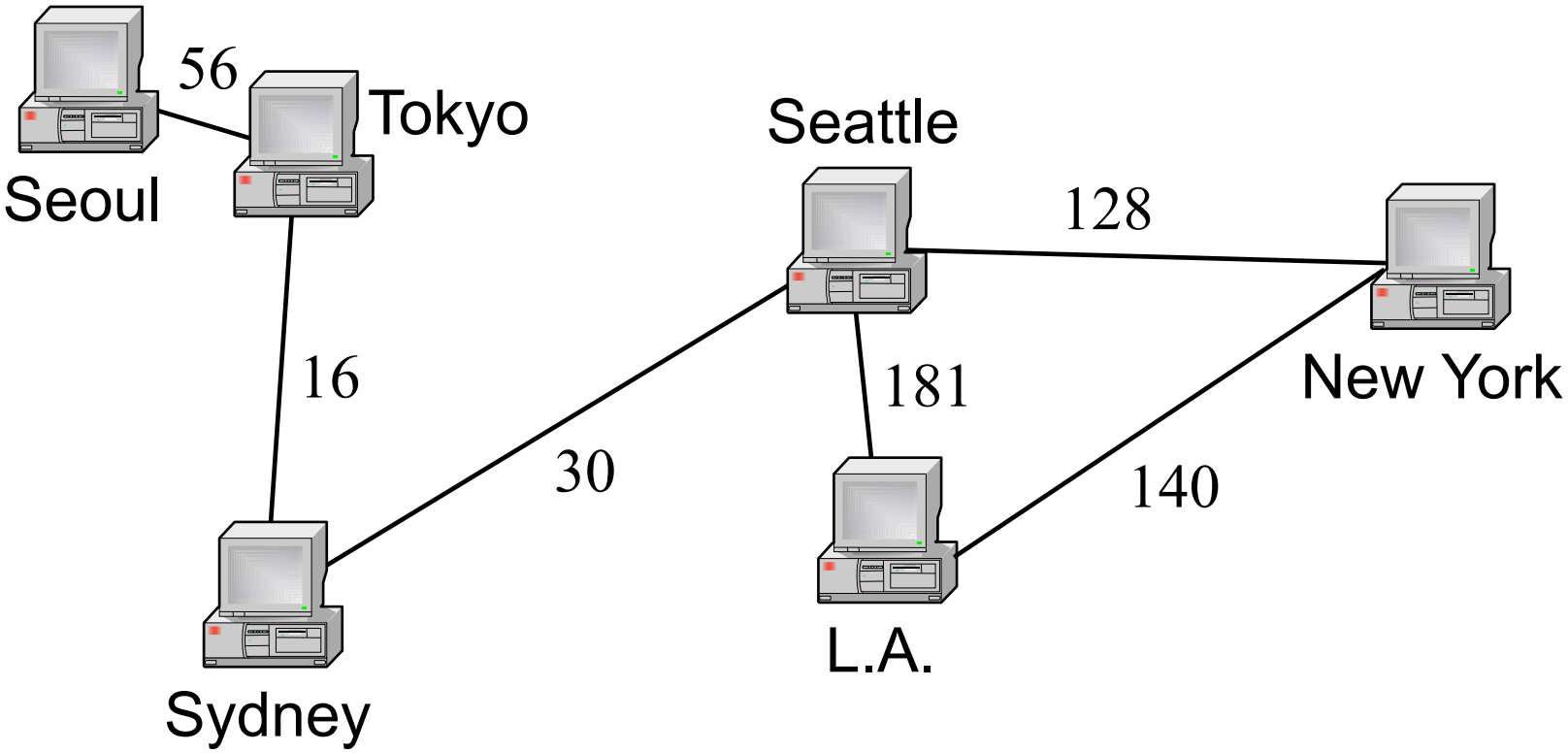
- But we are interested in a different kind of “graph”

Airline Routes



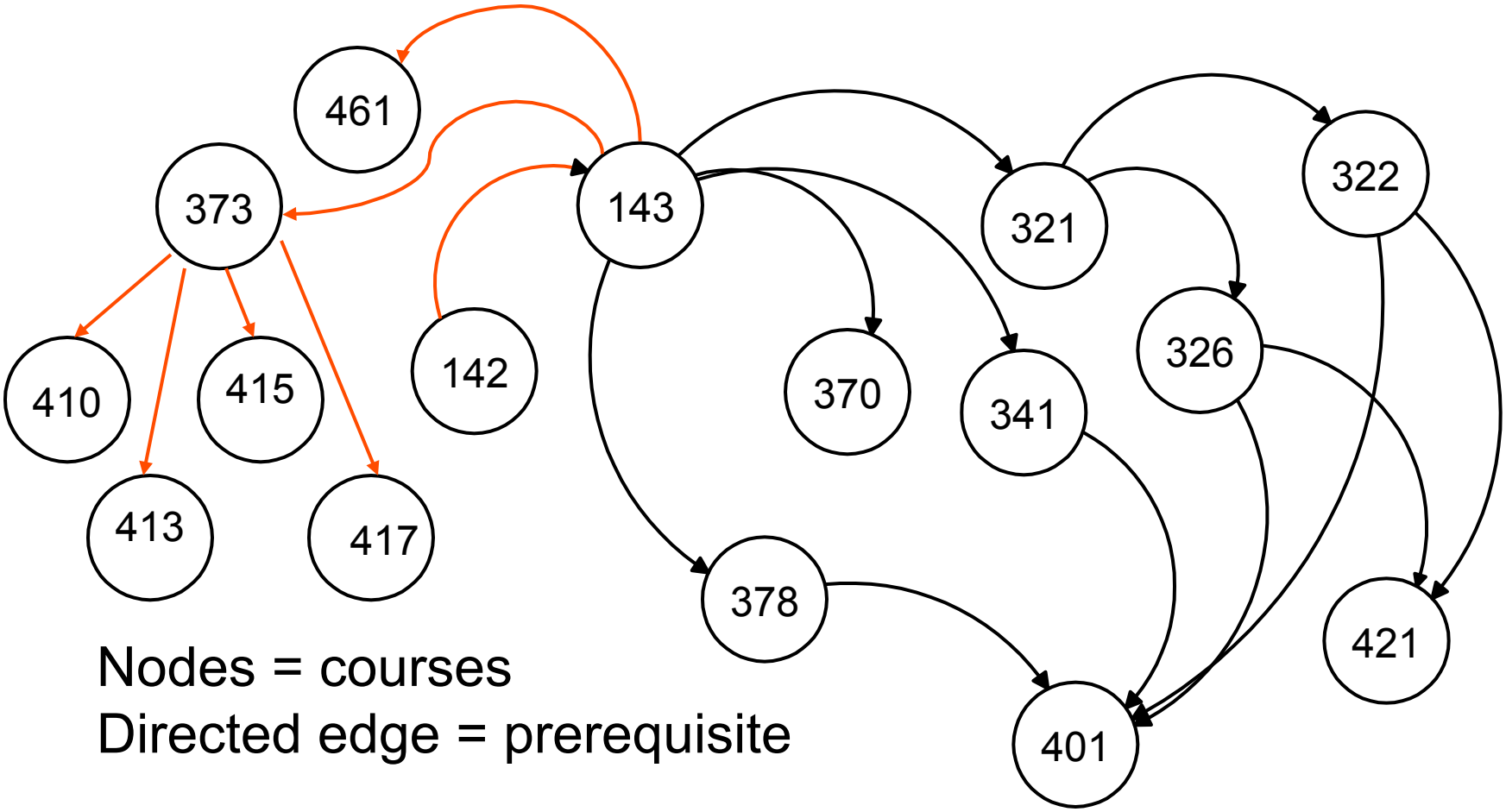
Nodes = cities
Edges = direct flights

Computer Networks



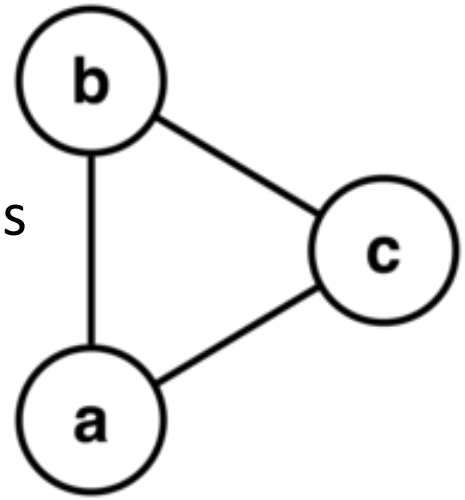
Nodes = computers
Edges = transmission rates

CSE Course Prerequisites at UW



Graphs

- **graph**: a data structure containing
  - a set of vertices *V*
  - a set of edges *E*, where an edge represents a connection between 2 vertices
  - *G* = (*V*, *E*)
  - edge is a pair (*v*, *w*) where *v*, *w* in *V*



- the graph at right: *V* = {a, b, c} and *E* = {(a, b), (b, c), (c, a)}

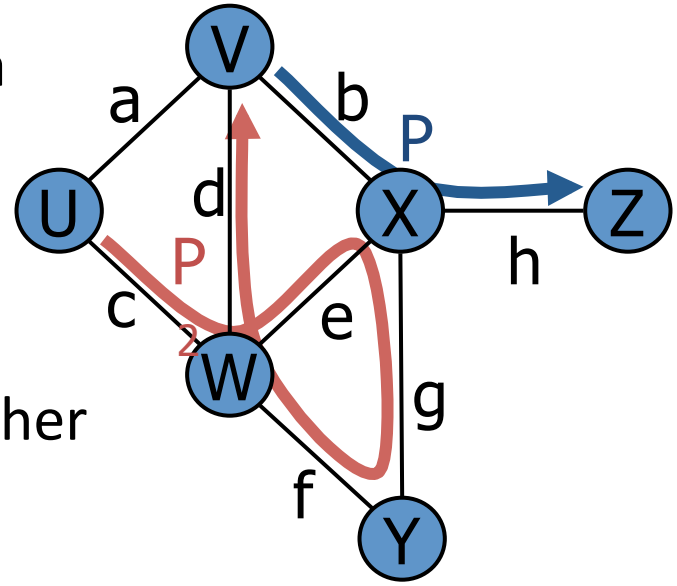
  - Assuming that a graph can only have one edge between a pair of vertices and cannot have an edge to itself, what is the maximum number of edges a graph can contain, relative to the size of the vertex set V?

Paths

- **path**: a path from vertex *A* to *B* is a sequence of edges that can be followed starting from *A* to reach *B*
  - can be represented as vertices visited or edges taken
  - example: path from V to Z: {b, h} or {V, X, Z}

- **reachability**: v_1 is *reachable* from v_2 if a path exists from V1 to V2

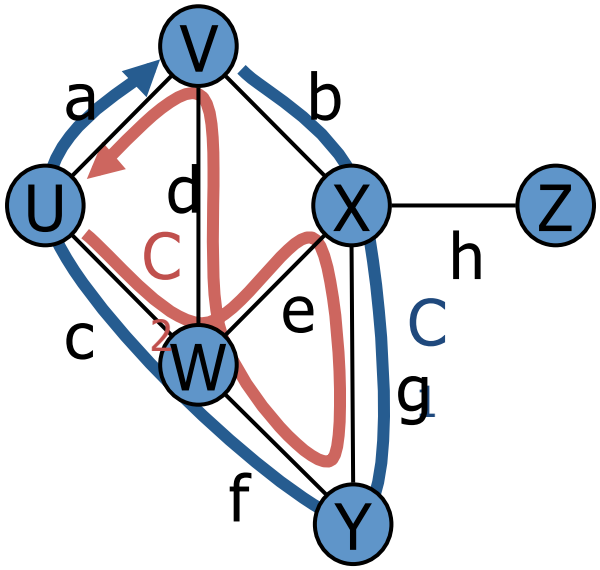
- **connected** graph: one in which it's possible to reach any node from any other
  - is this graph connected?



Cycles

- **cycle**: path from one node back to itself
  - example: {b, g, f, c, a} or {V, X, Y, W, U, V}

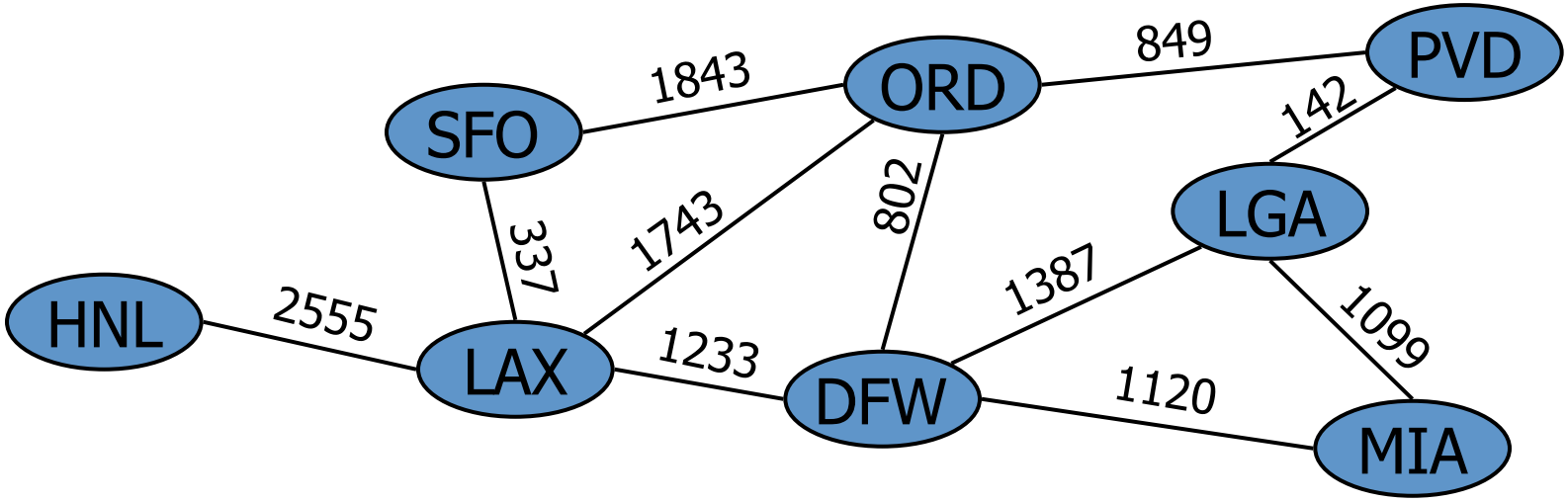
- **loop**: edge directly from node to itself
  - many graphs don't allow loops



Weighted graphs

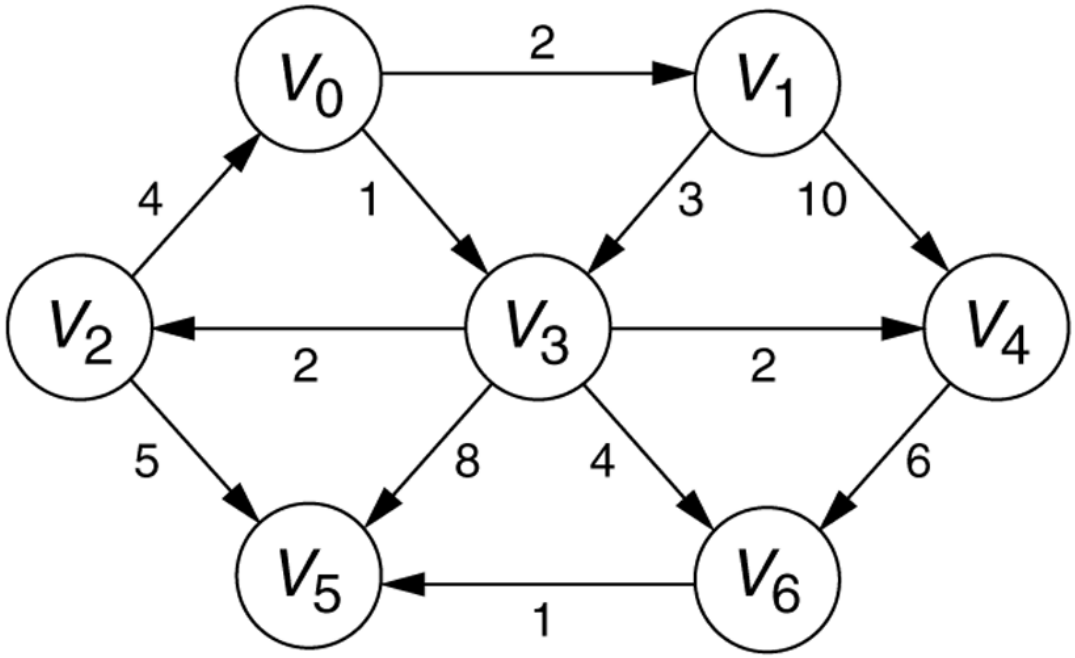
- **weight**: (optional) cost associated with a given edge

- example: graph of airline flights
  - if we were programming this graph, what information would we have to store for each vertex / edge?



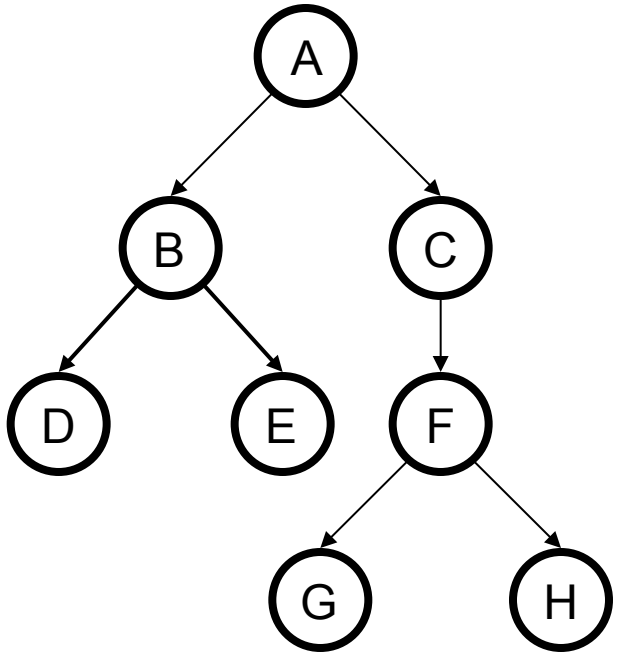
Directed graphs

- **directed graph (digraph)**: edges are one-way connections between vertices
  - if graph is directed, a vertex has a separate *in/out degree*



Trees as Graphs

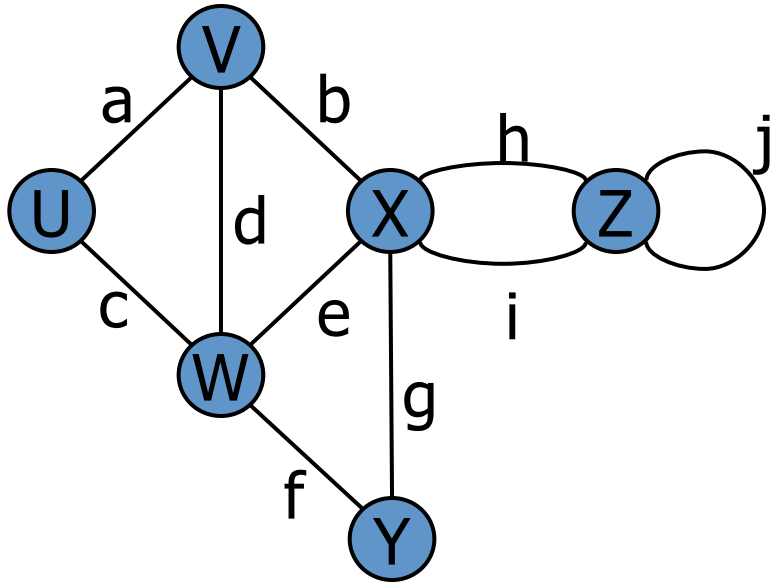
- Every tree is a graph with some restrictions:
  - the tree is directed
  - there is exactly one directed path from the root to every node



More terminology

- **degree**: number of edges touching a vertex
  - example: W has degree 4
  - what is the degree of X? of Z?

- **adjacent** vertices: connected directly by an edge



Graph questions

- Are the following graphs directed or not directed?
  - Buddy graphs of instant messaging programs? (vertices = users, edges = user being on another's buddy list)
  - bus line graph depicting all of Seattle's bus stations and routes
  - graph of movies in which actors have appeared together

- Are these graphs potentially cyclic? Why or why not?

Graph exercise

- Consider a graph of instant messenger buddies.
  - What do the vertices represent? What does an edge represent?
  - Is this graph directed or undirected? Weighted or unweighted?
  - What does a vertex's degree mean? In degree? Out degree?
  - Can the graph contain loops? cycles?

- Consider this graph data:
  - Jessica's buddy list: Meghan, Alan, Martin.
  - Meghan's buddy list: Alan, Lori.
  - Toni's buddy list: Lori, Meghan.
  - Martin's buddy list: Lori, Meghan.
  - Alan's buddy list: Martin, Jessica.
  - Lori's buddy list: Meghan.
  - Compute the in/out degree of each vertex. Is the graph connected?
  - Who is the most popular? Least? Who is the most antisocial?
  - If we're having a party and want to distribute the message the most quickly, who should we tell first?