CSE 373: Data Structures and Algorithms

Lecture 11: Priority Queues (Heaps)

Motivating examples

- **Bandwidth management**: A router is connected to a line with limited bandwidth. If there is insufficient bandwidth, the router maintains a queue for incoming data such that the most important data will get forwarded first as bandwidth becomes available.

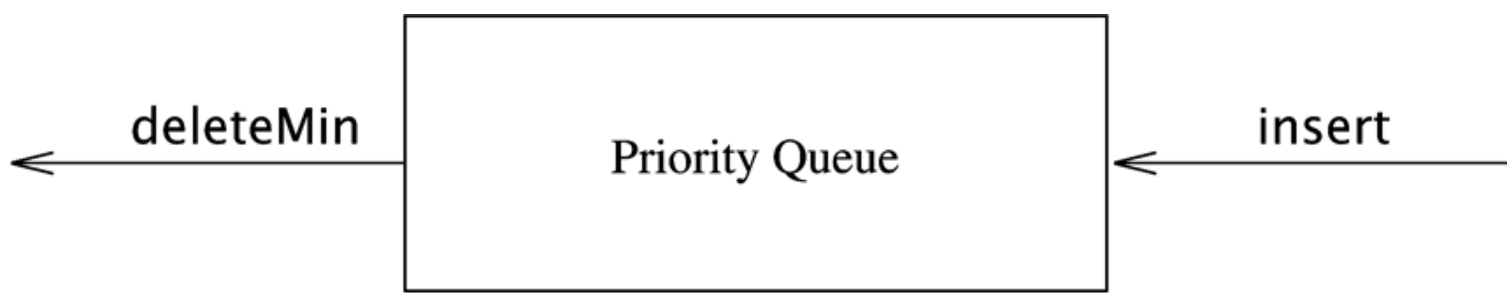
- **Printing**: A shared server has a list of print jobs to print. It wants to print them in chronological order, but each print job also has a *priority*, and higher-priority jobs always print before lower-priority jobs

- **Algorithms**: We are writing a ghost AI algorithm for Pac-Man. It needs to search for the best path to find Pac-Man; it will enqueue all possible paths with priorities (based on guesses about which one will succeed), and try them in order.

Priority Queue ADT

- **priority queue**: A collection of elements that provides fast access to the minimum (or maximum) element
  - a mix between a queue and a BST

- basic priority queue operations:
  - **insert**: Add an element to the priority queue (priority matters)
  - **remove (i.e. deleteMin)**: Removes/returns minimum element



Using `PriorityQueue`s

`PriorityQueue<E>()`	constructs a `PriorityQueue` that orders the elements according to their `compareTo` (element type must implement `Comparable`)
`add(`**element**`)`	inserts the element into the `PriorityQueue`
`remove()`	removes and returns the element at the head of the queue
`peek()`	returns, but does not remove, the element at the head of the queue

```
Queue<String> pq = new PriorityQueue<String>();
pq.add("Kona");
pq.add("Daisy");
```

– implements `Queue` interface

Potential Implementations

	insert	deleteMin
Unsorted list (Array)		
Unsorted list (Linked-List)		
Sorted list (Array)		
Sorted list (Linked-List)		
Binary Search Tree		
AVL Trees		

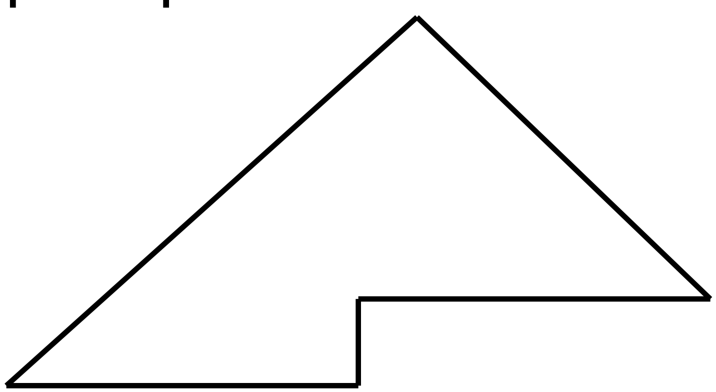
Potential Implementations

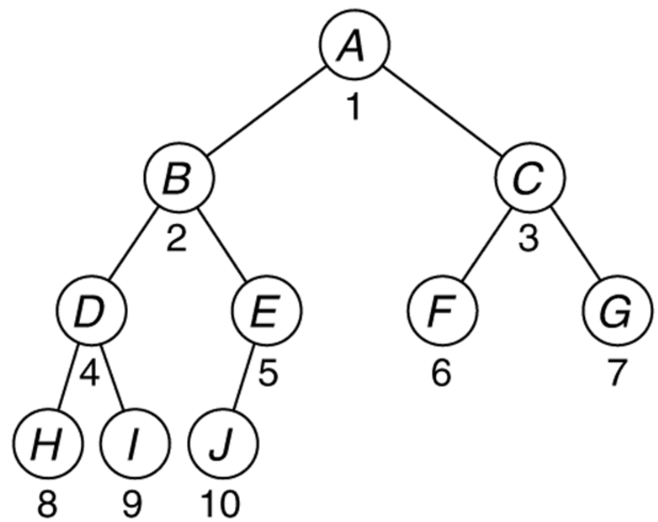
	insert	deleteMin
Unsorted list (Array)	$\Theta(1)$	$\Theta(n)$
Unsorted list (Linked-List)	$\Theta(1)$	$\Theta(n)$
Sorted list (Array)	$\Theta(n)$	$\Theta(1)$*
Sorted list (Linked-List)	$\Theta(n)$	$\Theta(1)$
Binary Search Tree	$\Theta(n)$ worst	$\Theta(n)$ worst
AVL Trees	$\Theta(\log n)$	$\Theta(\log n)$

Heap properties

- **heap**: a tree with the following two properties:
  - 1. *completeness*
    complete tree: every level is full except possibly the lowest level, which must be filled from left to right with no leaves to the right of a missing node (i.e., a node may not have any children until all of its possible siblings exist)

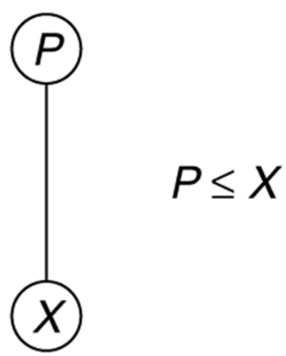
Heap shape:

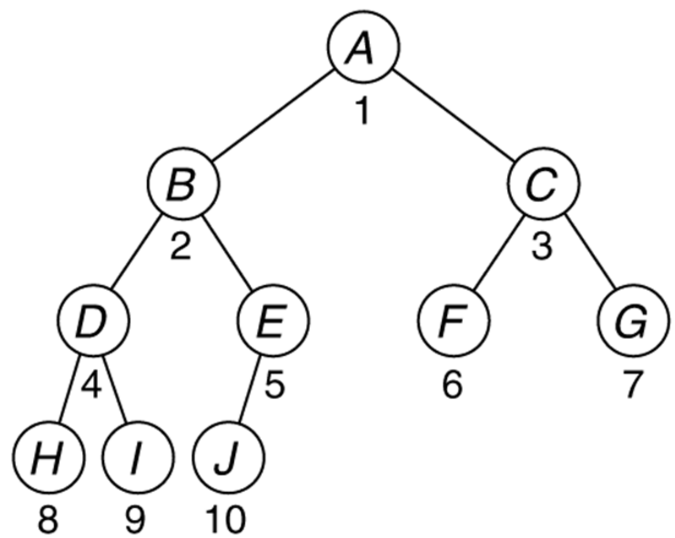




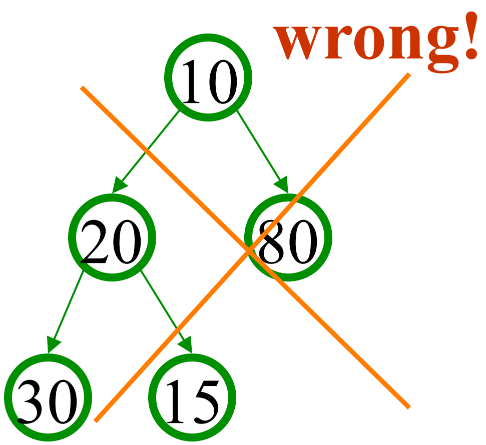
Heap properties 2

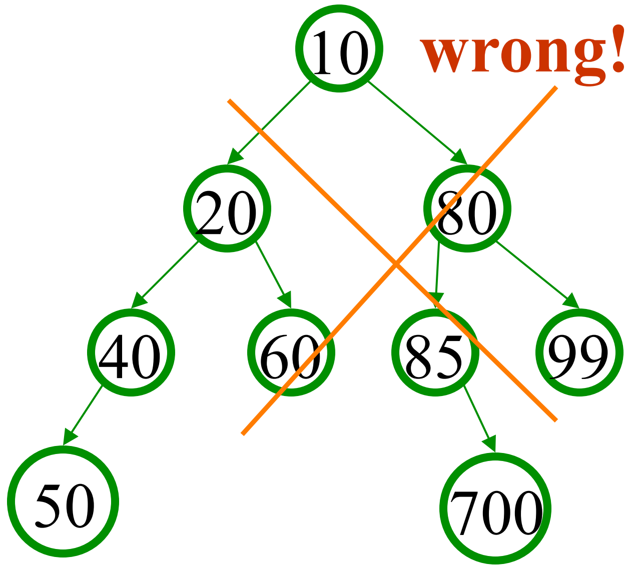
– 2. *heap ordering*
  a tree has *heap ordering* if *P* <= *X* for every element *X* with parent *P*
  - in other words, in heaps, parents' element values are always smaller than those of their children
  - implies that minimum element is always the root
  - is every a heap a BST? Are any heaps BSTs?

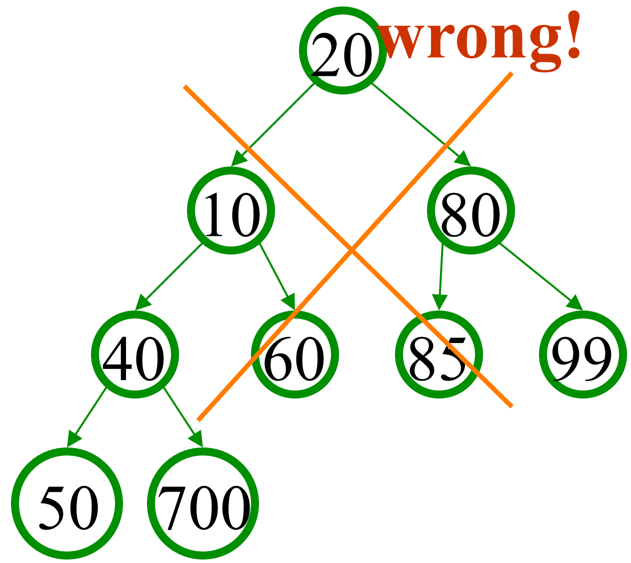


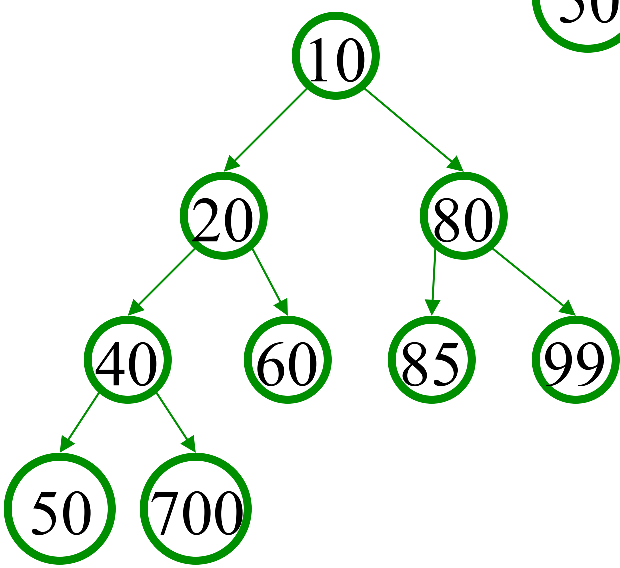


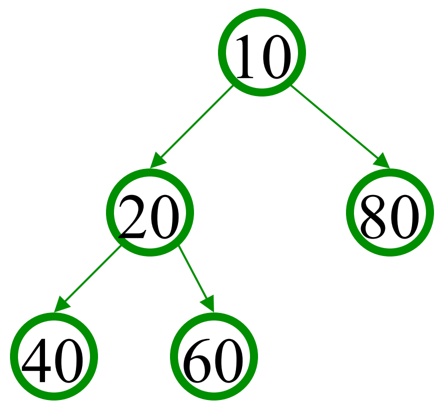
Which are min-heaps?

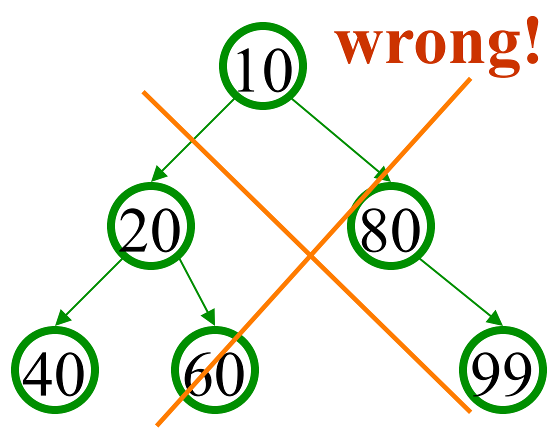




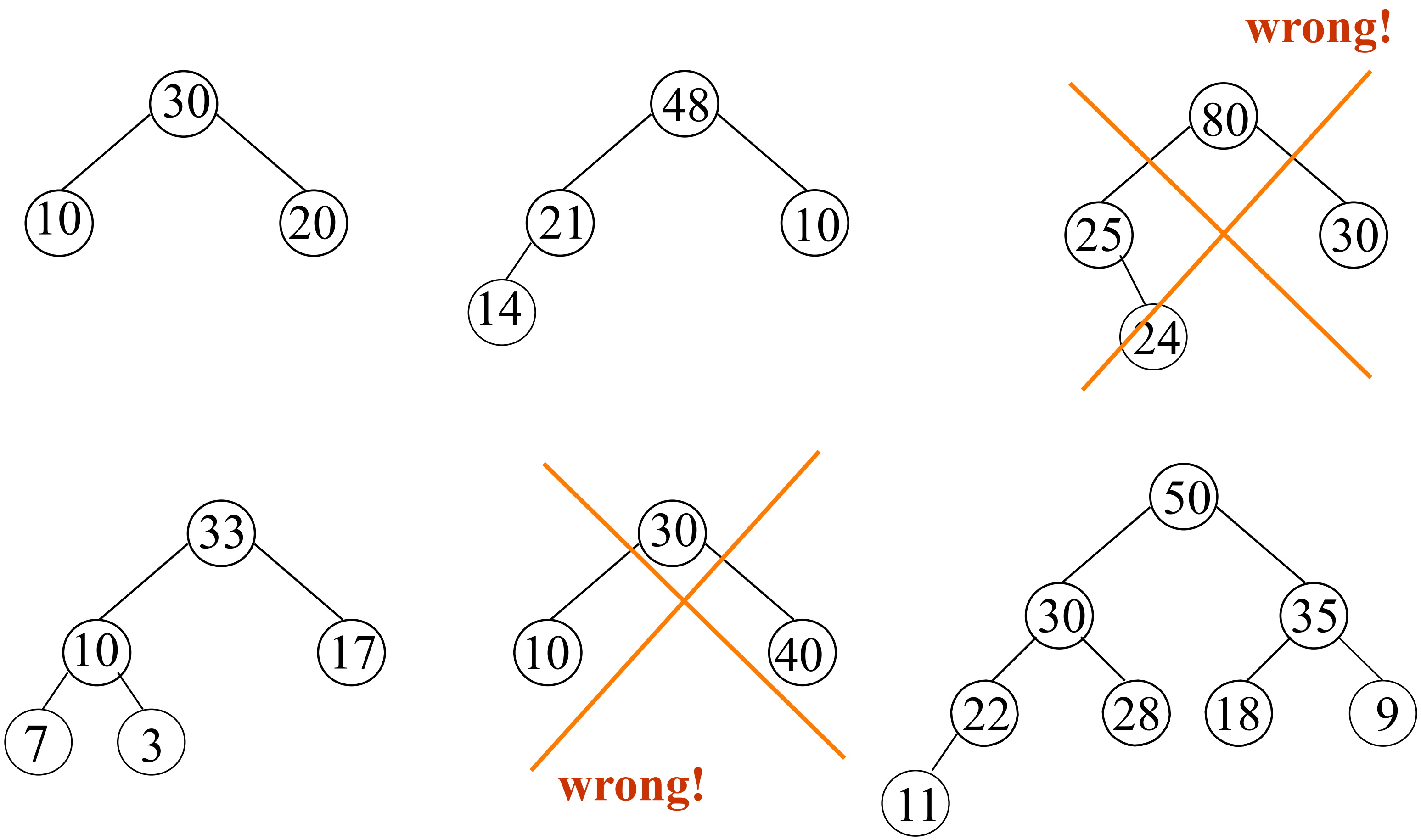






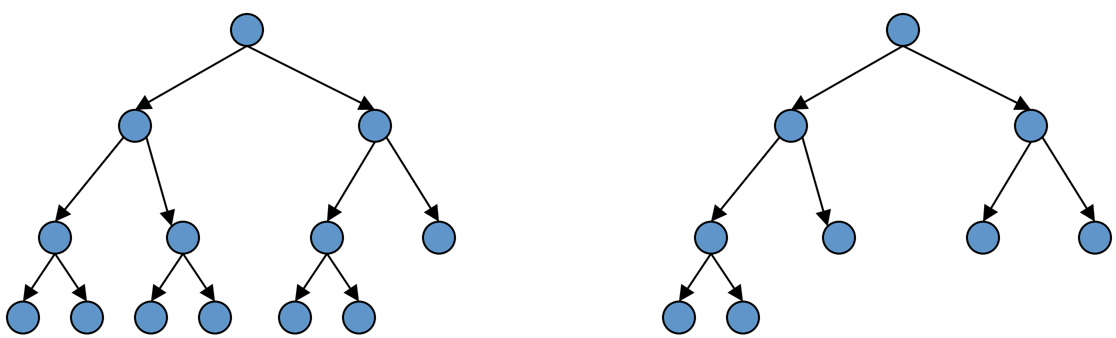


Which are max-heaps?



Heap height and runtime

- height of a complete tree is always log *n*, because it is always balanced
  - because of this, if we implement a priority queue using a heap, we can provide the O(log *n*) runtime required for the `add` and `remove` operations



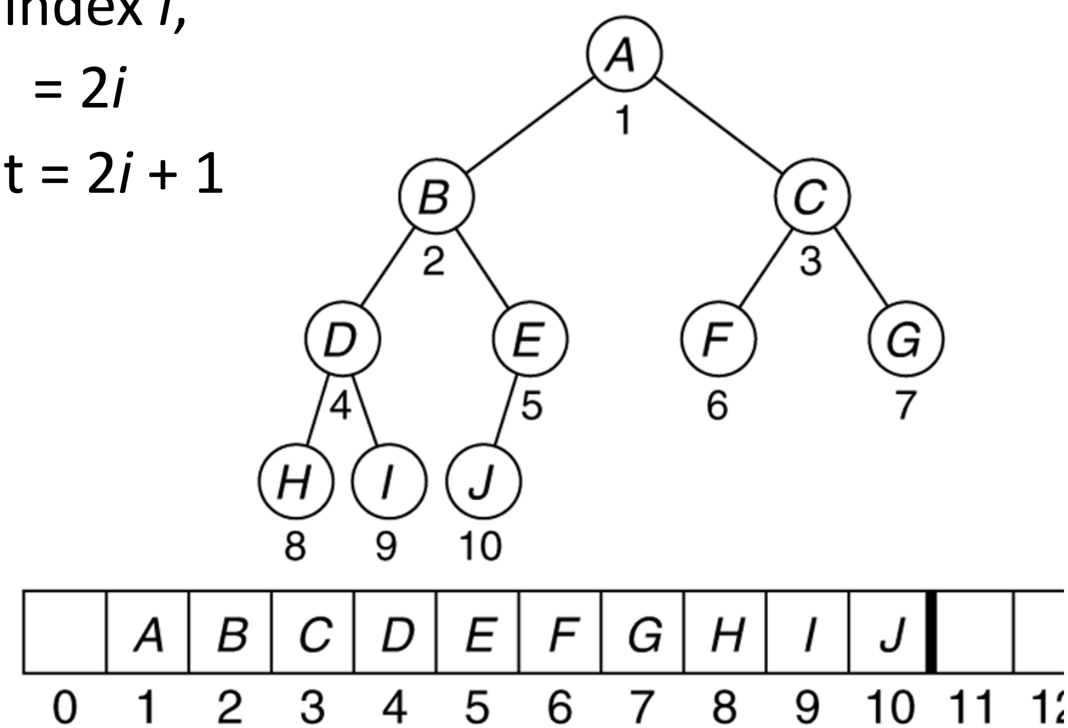
n-node complete tree of height h:

$2^h \leq n \leq 2^{h+1} - 1$

$h = \lfloor \log n \rfloor$

Implementation of a heap

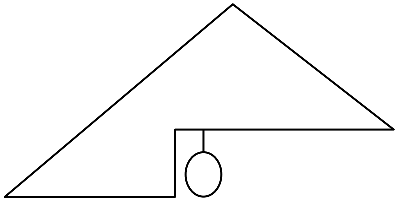
- when implementing a complete binary tree, we actually can "cheat" and just use an array
  - index of root = 1   (leave 0 empty for simplicity)
  - for any node *n* at index *i*,
    - index of *n*.left = 2*i*
    - index of *n*.right = 2*i* + 1
  - parent index?



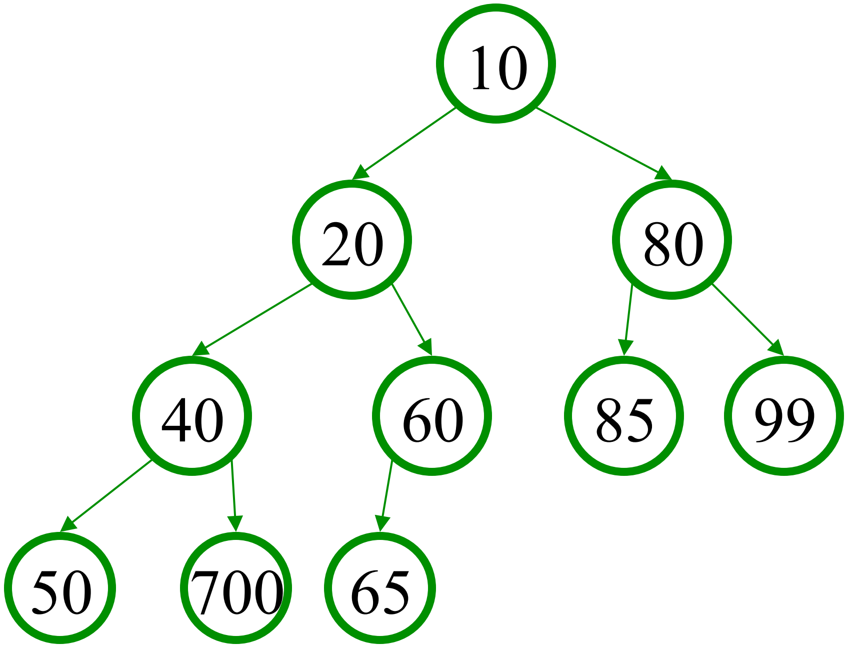
Implementing Priority Queue with Binary Heap

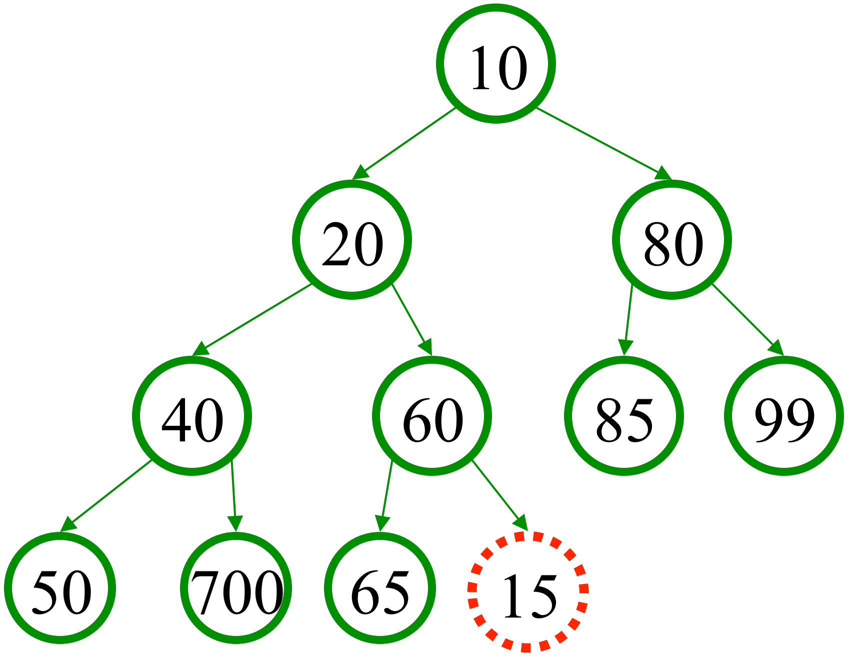
```
public interface IntPriorityQueue {
    public void add(int value);
    public boolean isEmtpy();
    public int peek();
    public int remove();
}

public class IntBinaryHeap implements IntPriorityQueue {
    private static final int DEFAULT_CAPACITY = 10;
    private int[] array;
    private int size;

    public IntBinaryHeap () {
        array = new int[DEFAULT_CAPACITY];
        size = 0;
    }
    ...
}
```

Adding to a heap



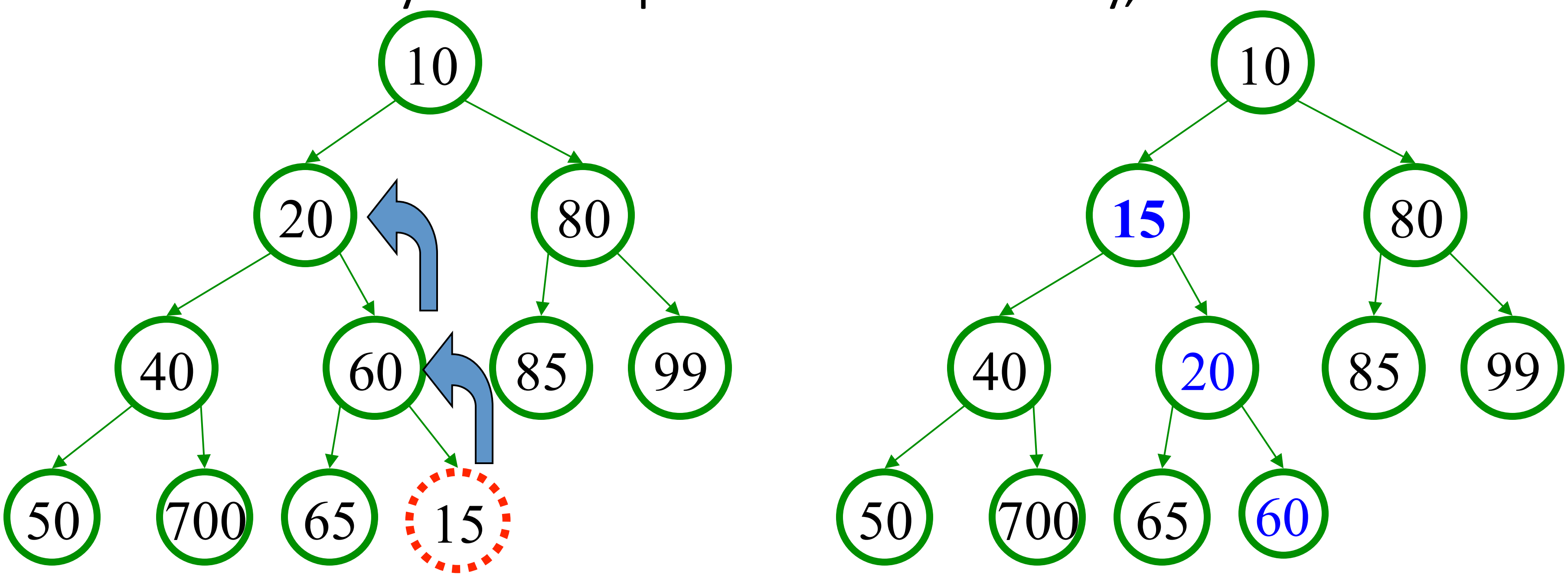
- when an element is added to a heap, it should be initially placed as the rightmost leaf (to maintain the completeness property)
  - heap ordering property becomes broken!





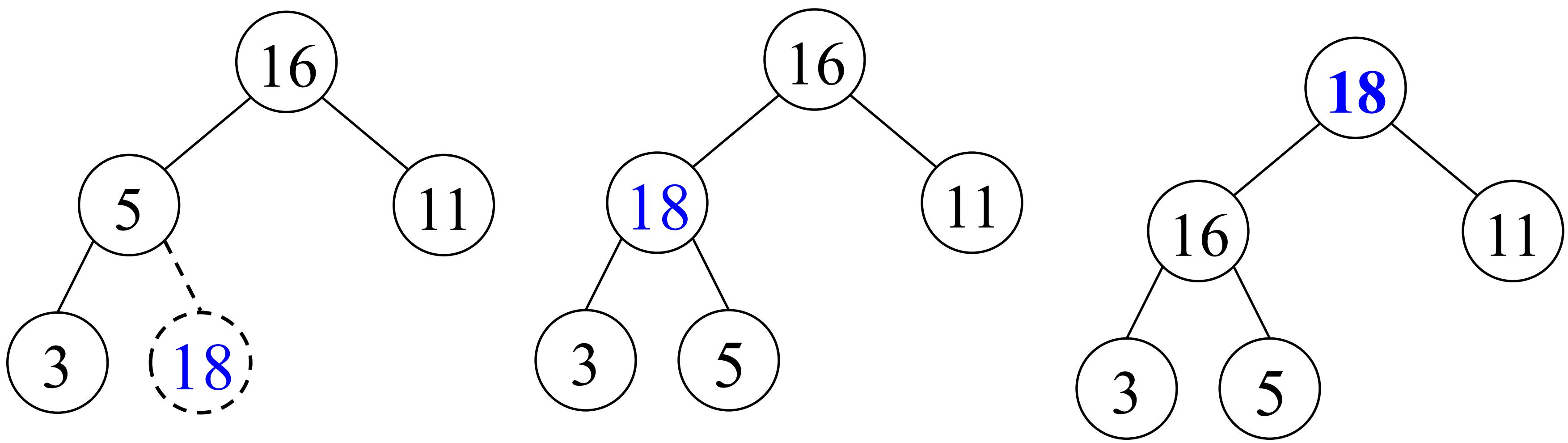
Adding to a heap, cont'd.

- to restore heap ordering property, the newly added element must be shifted upward ("bubbled up") until it reaches its proper place
  - bubble up (book: "percolate up") by swapping with parent
  - how many bubble-ups could be necessary, at most?



Adding to a max-heap

- same operations, but must bubble up *larger* values to top



Heap practice problem

- Draw the state of the min-heap tree after adding the following elements to it:

  6, 50, 11, 25, 42, 20, 104, 76, 19, 55, 88, 2

Code for add method

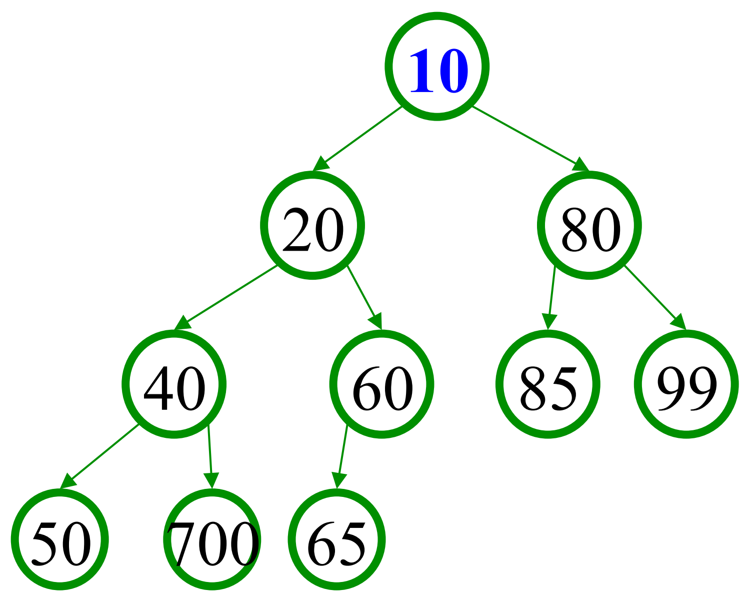
```
public void add(int value) {
    // grow array if needed
    if (size >= array.length - 1) {
        array = this.resize();
    }

    // place element into heap at bottom
    size++;
    int index = size;
    array[index] = value;

    bubbleUp();
}
```

The `bubbleUp` helper

```
private void bubbleUp() {
    int index = this.size;

    while (hasParent(index)
            && (parent(index) > array[index])) {
        // parent/child are out of order; swap them
        swap(index, parentIndex(index));
        index = parentIndex(index);
    }
}

// helpers
private boolean hasParent(int i) { return i > 1; }
private int parentIndex(int i)   { return i/2; }
private int parent(int i)  { return array[parentIndex(i)]; }
```

The `peek` operation

- peek on a min-heap is trivial; because of the heap properties, the minimum element is always the root
  - peek is O(1)
  - peek on a max-heap would be O(1) as well, but would return you the maximum element and not the minimum one



Code for peek method

```
public int peek() {
    if (this.isEmpty()) {
        throw new IllegalStateException();
    }

    return array[1];
}
```