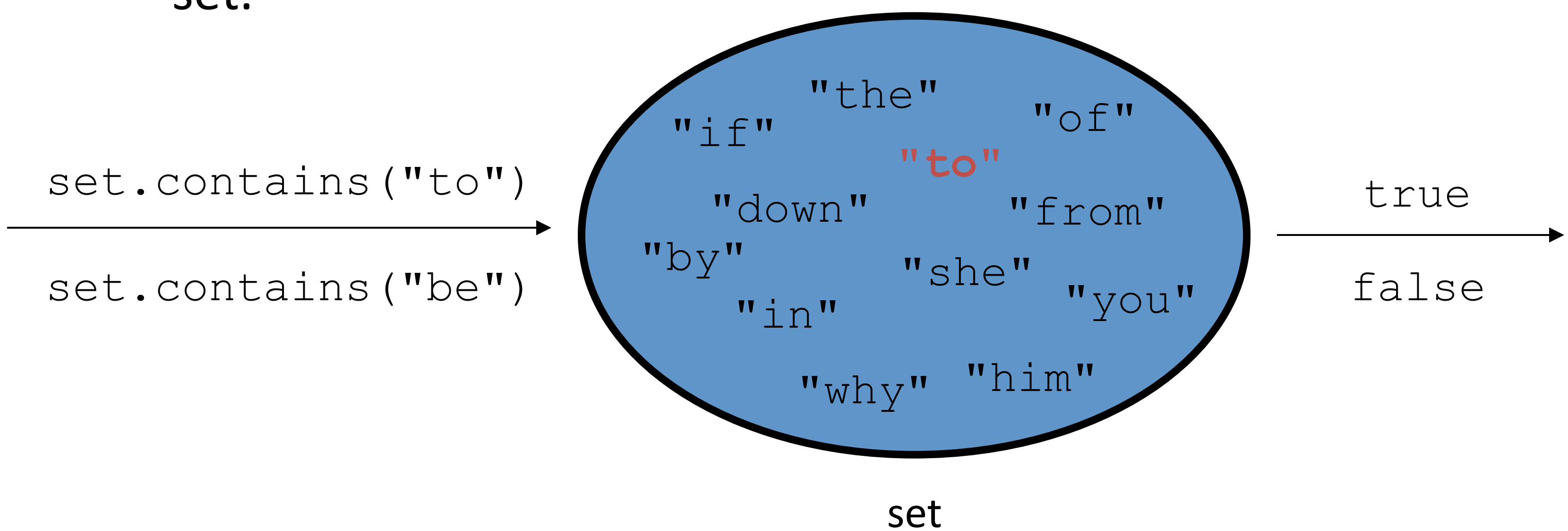
CSE 373: Data Structures and Algorithms

Lecture 8: Trees

Set ADT

- **set**: A collection that does not allow duplicates
  - We don't think of a set as having indexes; we just add things to the set in general and don't worry about order

- basic set operations:
  - **insert**: Add an element to the set (order doesn't matter).
  - **remove**: Remove an element from the set.
  - **search**: Efficiently determine if an element is a member of the set.



Sets in computer science

- Databases:
  - set of records in a table
- Search engines:
  - set of URLs/webpages on the Internet
- Real world examples:
  - set of all products for sale in a store inventory
  - set of friends on Facebook
  - set of email addresses

Using `Sets`

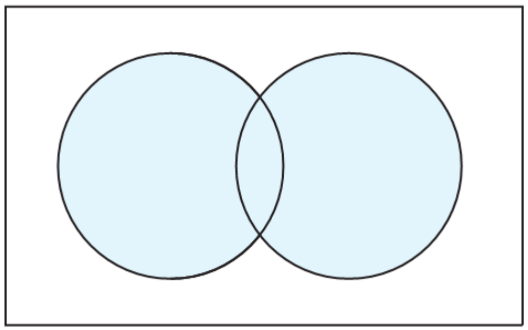
`add(`**value**`)`	adds the given value to the set
`contains(`**value**`)`	returns `true` if the given value is found in this set
`remove(`**value**`)`	removes the given value from the set
`clear()`	removes all elements of the set
`size()`	returns the number of elements in list
`isEmpty()`	returns `true` if the set's size is 0
`toString()`	returns a string such as `"[3, 42, -7, 15]"`

```
List<String> list = new ArrayList<String>();
…
Set<Integer> set = new TreeSet<Integer>();     // empty
Set<String> set2 = new HashSet<String>(list);
```

– can construct an empty set, or one based on a given collection

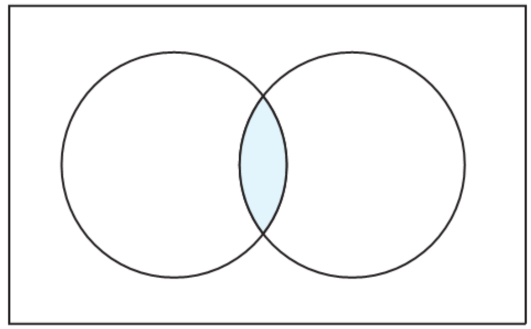
More `Set` operations

A ∪ B Union



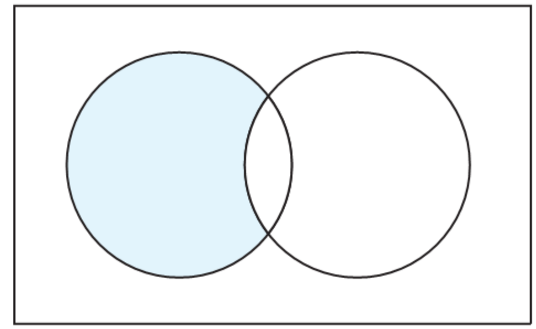
`addAll`

A ∩ B Intersection



`retainAll`

A - B Difference



`removeAll`

`addAll(`**collection**`)`	adds all elements from the given collection to this set
`containsAll(`**coll**`)`	returns `true` if this set contains every element from given set
`equals(`**set**`)`	returns `true` if given other set contains the same elements
`iterator()`	returns an object used to examine set's contents
`removeAll(`**coll**`)`	removes all elements in the given collection from this set
`retainAll(`**coll**`)`	removes elements *not* found in given collection from this set
`toArray()`	returns an array of the elements in this set

Accessing elements in a Set

```
for (type name : collection) {
    statements;
}
```

- Provides a clean syntax for looping over the elements of a `Set`, `List`, array, or other collection

```
Set<Double> grades = new TreeSet<Double>();
…

for (double grade : grades) {
    System.out.println("Student grade: " + grade);
}
```

  - needed because sets have no indexes; can't `get` element `i`

Sets and ordering

- `HashSet` : elements are stored in an unpredictable order

```
Set<String> names = new HashSet<String>();
names.add("Jake");
names.add("Robert");
names.add("Marisa");
names.add("Kasey");
System.out.println(names);
// [Kasey, Robert, Jake, Marisa]
```

- `TreeSet` : elements are stored in their "natural" sorted order

```
Set<String> names = new TreeSet<String>();
...
// [Jake, Kasey, Marisa, Robert]
```

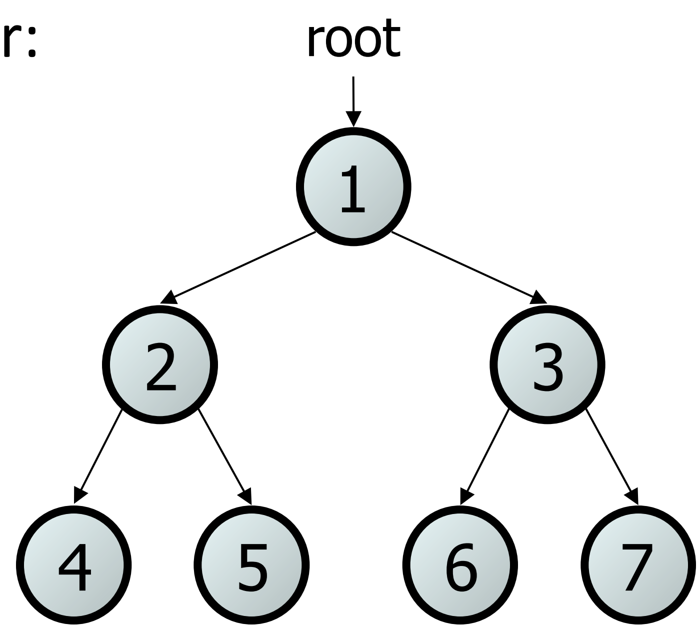
Implementing Set ADT

	Insert	Remove	Search
Unsorted array	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Sorted array	$\Theta(\log(n)+n)$	$\Theta(\log(n) + n)$	$\Theta(\log(n))$
Linked list	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$

Trees

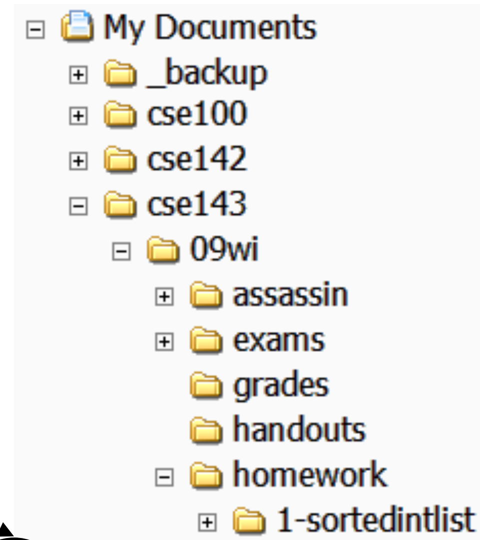
- **tree**: A directed, acyclic structure of linked nodes.
  - *directed*: Has one-way links between nodes.
  - *acyclic*: No path wraps back around to the same node twice.
  - **binary tree**: One where each node has at most two children.

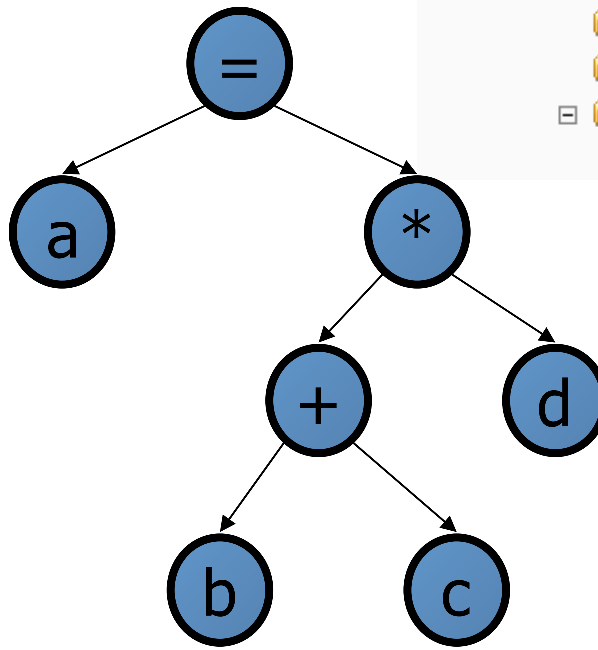
- A binary tree can be defined as either:
  - empty (`null`), or
  - a **root** node that contains:
    - **data**,
    - a **left** subtree, and
    - a **right** subtree.
  - (The left and/or right subtree could be empty.)

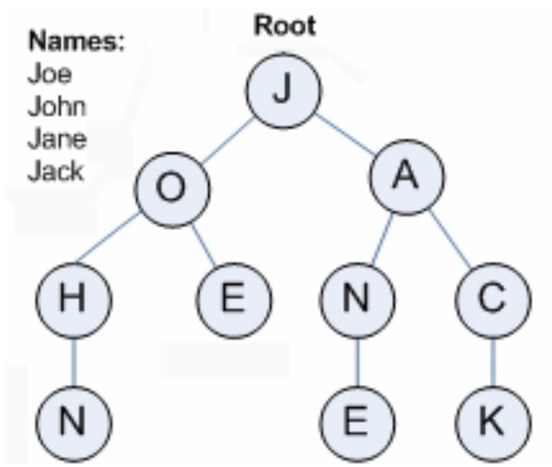


Trees in computer science

- folders/files on a computer
- family genealogy; organizational charts
- AI: decision trees
- compilers: parse tree
  - a = (b + c) * d;
- cell phone T9

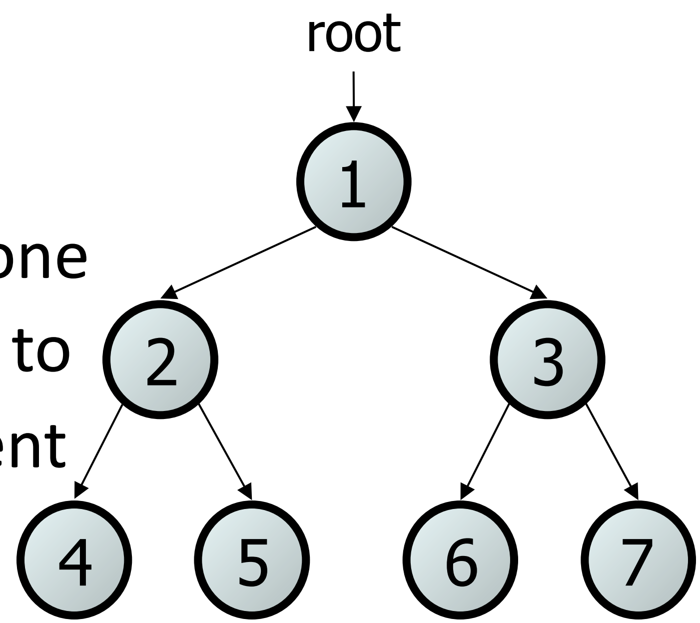






Terminology

- **node**: an object containing a data value and left/right children
- **root**: topmost node of a tree
- **leaf**: a node that has no children
- **branch**: any internal node; neither the root nor a leaf



- **parent**: a node that refers to this one
- **child**: a node that this node refers to
- **sibling**: a node with common parent

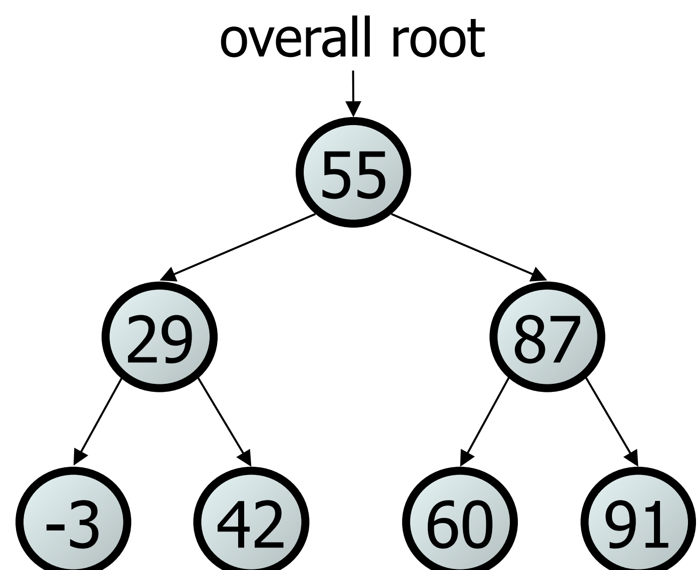
StringTreeNode class

```
// A StringTreeNode object is one node in a binary tree of Strings.
public class StringTreeNode {
    public String data;                 // data stored at this node
    public StringTreeNode left;         // reference to left subtree
    public StringTreeNode right;        // reference to right subtree

    // Constructs a leaf node with the given data.
    public StringTreeNode(String data) {
        this(data, null, null);
    }

    // Constructs a leaf or branch node with the given data and links.
    public StringTreeNode(String data, StringTreeNode left,
        StringTreeNode right) {
        this.data = data;
        this.left = left;
        this.right = right;
    }
}
```

Binary search trees

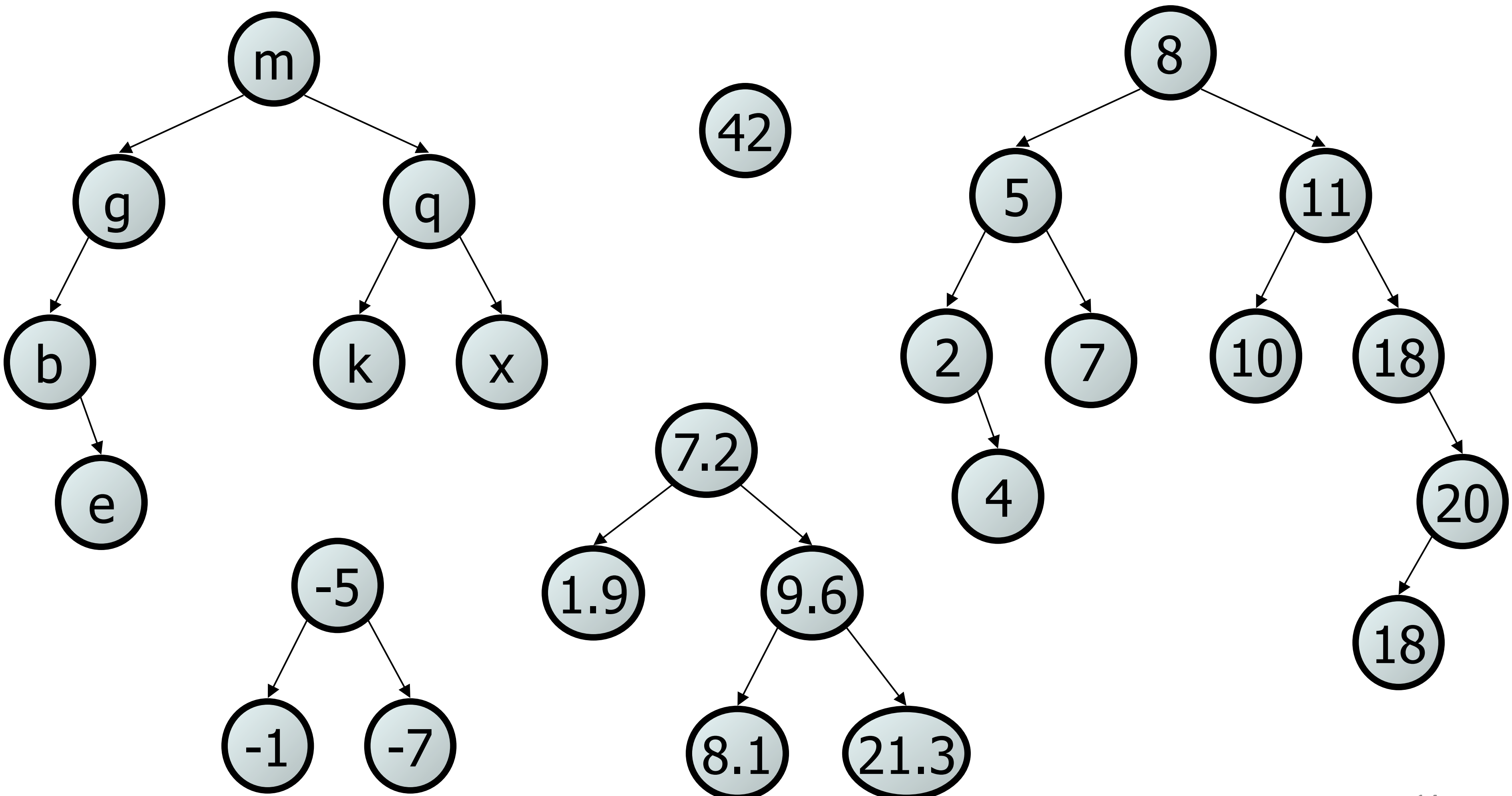
- **binary search tree** ("BST"): a binary tree that is either:
  - empty (`null`), or
  - a root node R such that:
    - every element of R's left subtree contains data "less than" R's data,
    - every element of R's right subtree contains data "greater than" R's,
    - R's left and right subtrees are also binary search trees.

- BSTs store their elements in sorted order, which is helpful for searching/sorting tasks.



Exercise

- Which of the trees shown are legal binary search trees?



Programming with Binary Trees

- Many tree algorithms are recursive
  - Process current node, recur on subtrees
  - Base case is empty tree (`null`)

- **traversal**: An examination of the elements of a tree.
  - A pattern used in many tree algorithms and methods

- Common orderings for traversals:
  - **pre-order**: process root node, then its left/right subtrees
  - **in-order**: process left subtree, then root node, then right
  - **post-order**: process left/right subtrees, then root node

Tree Traversal (in order)

```
// Returns a String representation of StringTreeSet with elements in
// their "natural order" (e.g., [Jake, Kasey, Marisa, Robert]).
public String toString() {
    String str = "[" + toString(root);
    if (str.length() > 1) { str = str.substring(0, str.length()-2); }
    return str + "]";
}

// recursive helper; in-order traversal
private String toString(StringTreeNode root) {
    String str = "";
    if (root != null) {
        str += toString(root.left);
        str += root.data + ", ";
        str += toString(root.right);
    }
    return str;
}
```

Implementing Set with BST

- Each Set entry adds a node to tree
  - Node contains String element, references to left/right subtree
- Tree organized for binary search
  - Quickly search or place to insert/remove element

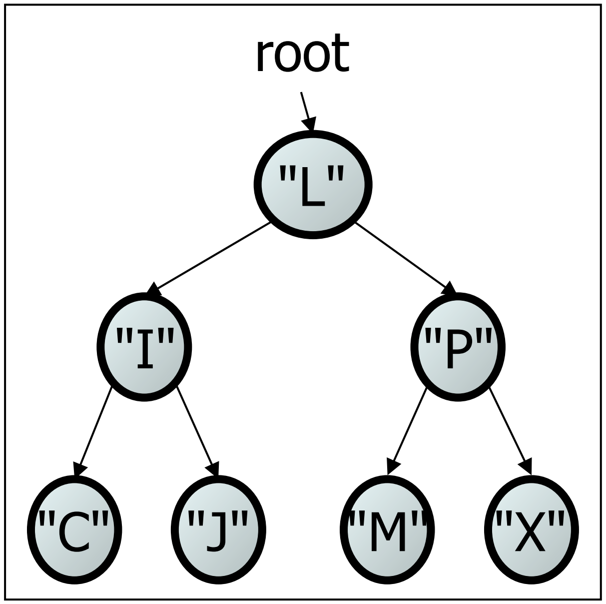
Implementing Set with BST (cont.)

```
public interface StringSet {
    public boolean add(String value);

    public boolean contains(String value);

    public void print();

    public boolean remove(String value);

    public int size();
}
```

StringTreeSet class

```
// A StringTreeSet represents a Set of Strings.
public class StringTreeSet {
    private StringTreeNode root;    // null for an empty set

    methods
}
```

- Client code talks to the `StringTreeSet`, not to the node objects inside it
- Methods of the `StringTreeSet` create and manipulate the nodes, their data and links between them



Set implementation: search

```
public boolean contains(String value) {
    return contains(root, value);
}

private boolean contains(StringTreeNode root, String value) {
    if (root == null) {
        return false;                                  // not in set
    } else if (root.data.compareTo(value) == 0) {
        return true;                                   // found!
    } else if (root.data.compareTo(value) > 0) {
        return contains(root.left, value);             // search left
    } else {
        return contains(root.right, value);            // search right
    }
}
```

Set implementation: insert

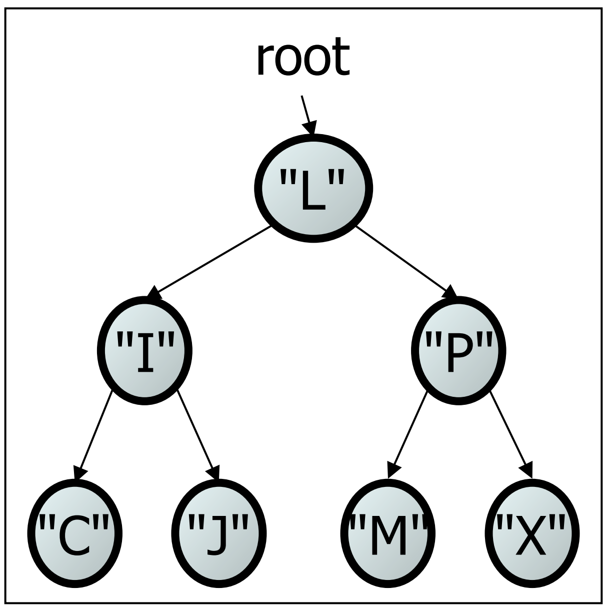
- Starts like `contains`
  - Trace out path where node should be
- Add node as new leaf
  - Don't change any other nodes or references
  - Correct place to maintain binary search tree property

Set implementation: insert

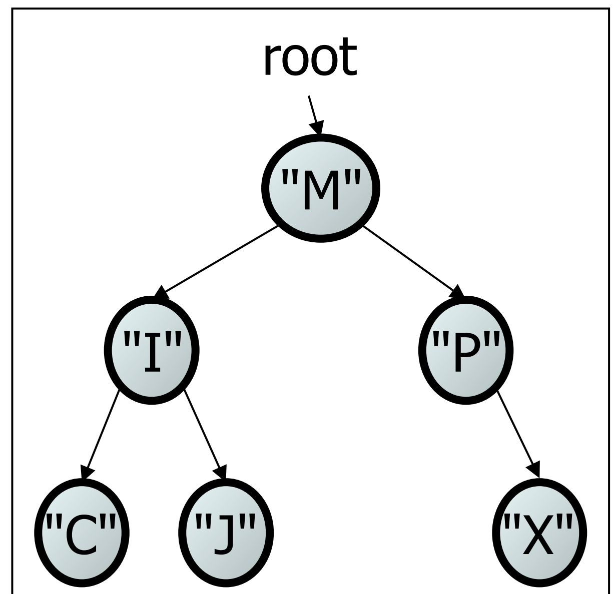
```
public boolean add(String value) {
    int oldSize = size();
    this.root = add(root, value);
    return oldSize != size();
}

private StringTreeNode add(StringTreeNode root, String value) {
    if (root == null) {
        root = new StringTreeNode(value);
        numElements++;
    } else if (root.data.compareTo(value) == 0) {
      return root;
    } else if (root.data.compareTo(value) > 0) {
        root.left = add(root.left, value);
    } else { root.right = add(root.right, value); }
    return root;
}
```

Set implementation: remove

- Possible states for the node to be removed:
  - a leaf: replace with null
  - a node with a left child only: replace with left child
  - a node with a right child only: replace with right child
  - a node with both children: replace with min value from right




```
set.remove("L");
```



Set implementation: remove

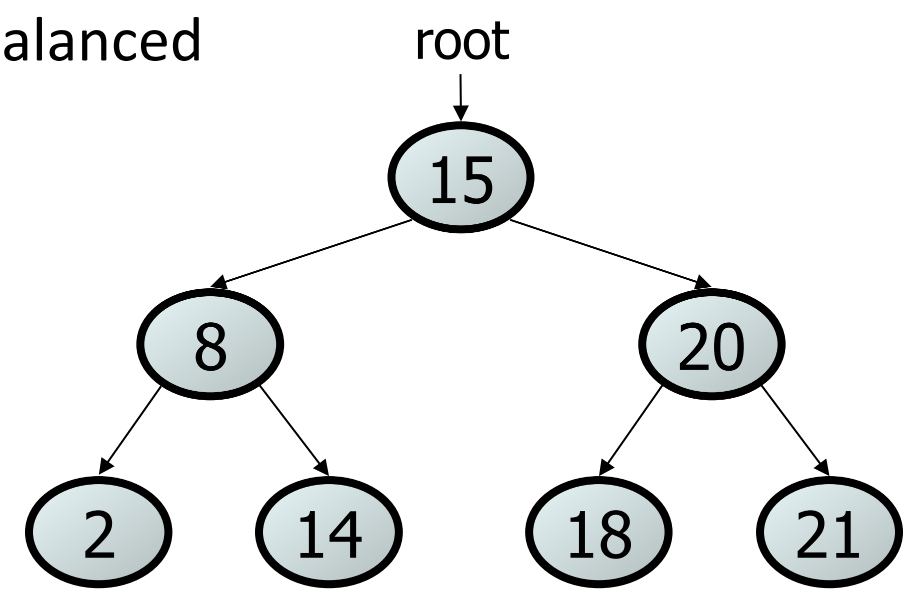
```
public boolean remove(String value) {
    int oldSize = size();
    root = remove(root, value);
    return oldSize != size();
}

private StringTreeNode remove(StringTreeNode root, String value) {
    if (root == null) { return root; }
    else if (root.data.compareTo(value) > 0) {
        root.left = remove(root.left, value);
    } else if (root.data.compareTo(value) < 0) {
        root.right = remove(root.right, value);
    } else { numElements--;
        if (root.right != null && root.left != null) {
            root.data = findMin(root.right).data;
            root.right = remove(root.right, root.data);
        } else if (root.right != null) { root = root.right;
        } else { root = root.left; }
    }
    return root;
}
```

Evaluate Set as BST

- Space used
  - Overhead of two references per entry
  - BST adds nodes as needed; no excess capacity
- Runtime
  - `add`, `contains` take time proportional to tree height
  - height expected to be O(log N)

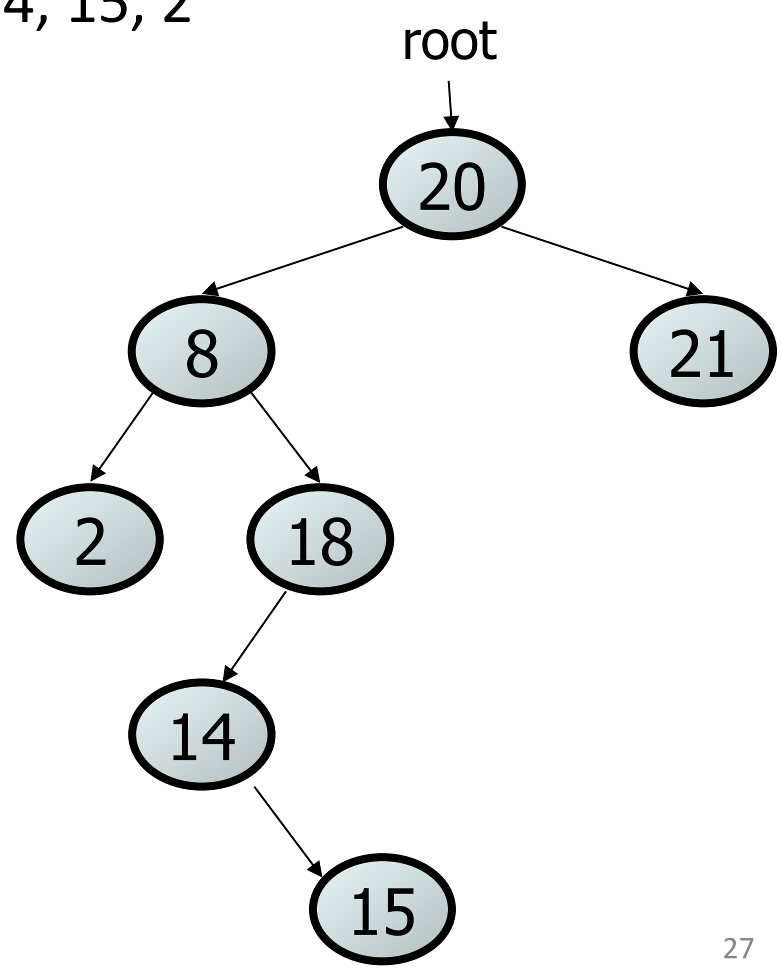
A Balanced Tree

- Values: 2 8 14 15 18 20 21
  - Order added: 15, 8, 2, 20, 21, 14, 18
- Different tree structures possible
  - Depends on order inserted
- 7 nodes, expected height log 7 ≈ 3
- Perfectly balanced



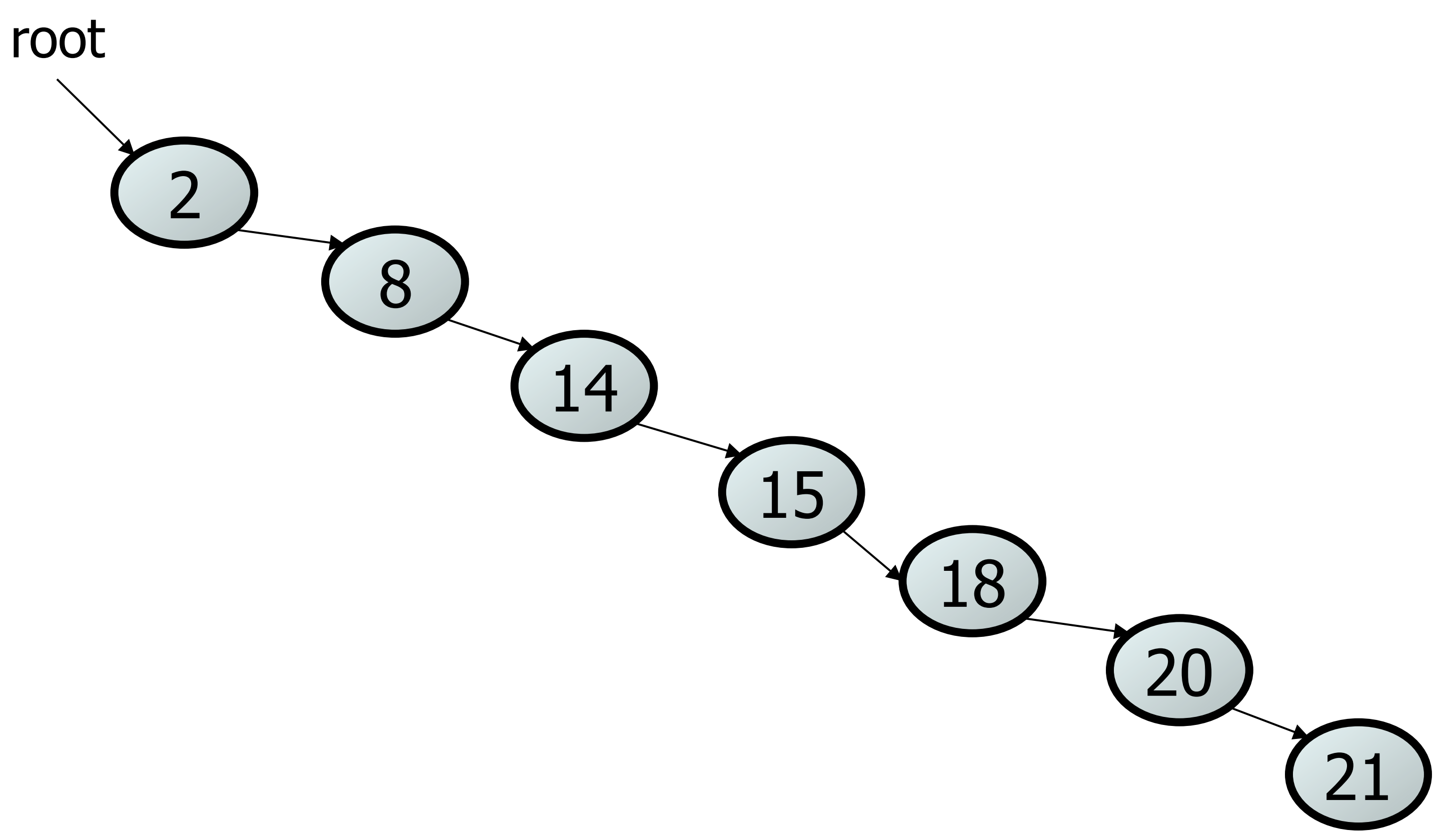
Mostly Balanced Tree

- Same Values: 2 8 14 15 18 20 21
  - Order added: 20, 8, 21, 18, 14, 15, 2
- Mostly balanced, height 4/5



Degenerate Tree

- Same Values: 2 8 14 15 18 20 21
  - Order added: 2, 8, 14, 15, 18, 20, 21
- Totally unbalanced, height 7



Binary Trees: Some Numbers

Recall: height of a tree = length of longest path from the root to a leaf.

For binary tree of height *h*:

– max # of leaves: 2^h

– max # of nodes: $2^{(h+1)} - 1$

– min # of leaves: 1

– min # of nodes: $h + 1$

We're not going to do better than log(n) height, and we need something to keep us away from n.

Implementing Set ADT (Revisited)

	Insert	Remove	Search
Unsorted array	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Sorted array	$\Theta(\log(n)+n)$	$\Theta(\log(n) + n)$	$\Theta(\log(n))$
Linked list	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
BST (if balanced)	O(log n)	O(log n)	O(log n)