CSE 373: Data Structures and Algorithms

Lecture 22: Graphs IV

Dijkstra's algorithm

- **Dijkstra's algorithm**: finds shortest (minimum weight) path between a particular pair of vertices in a *weighted* directed graph with nonnegative edge weights
  - solves the "one vertex, shortest path" problem
  - basic algorithm concept: create a table of information about the currently known best way to reach each vertex (distance, previous vertex) and improve it until it reaches the best solution

- in a graph where:
  - vertices represent cities,
  - edge weights represent driving distances between pairs of cities connected by a direct road,
    Dijkstra's algorithm can be used to find the shortest route between one city and any other

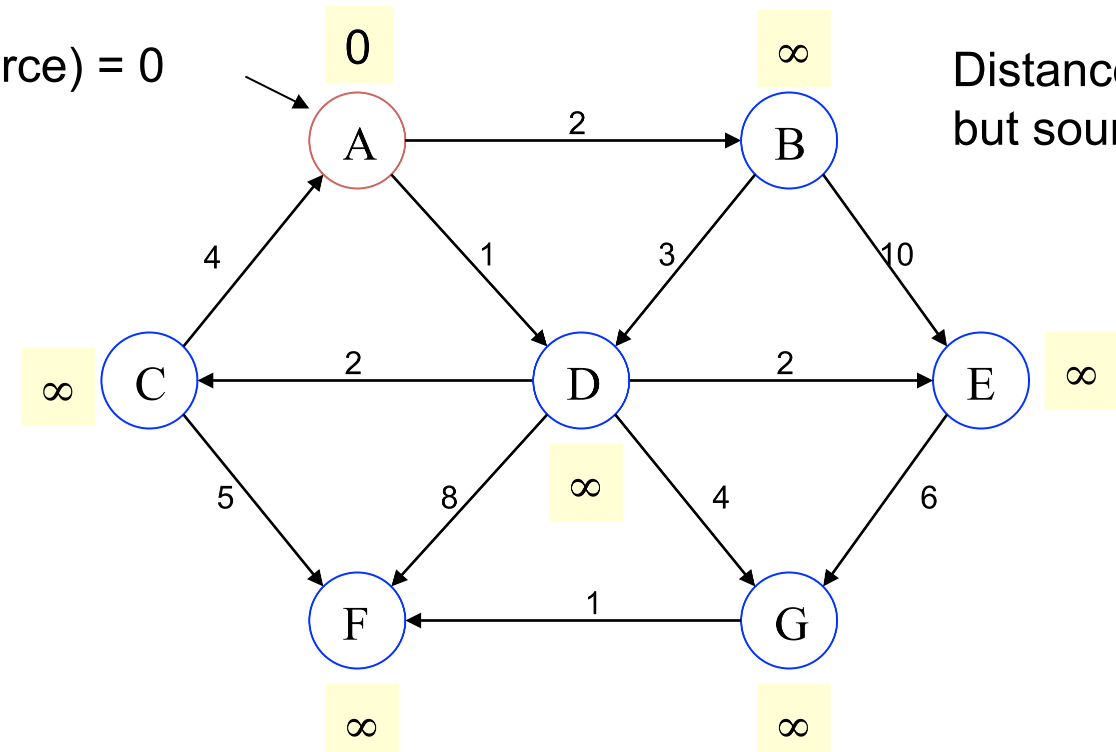
Dijkstra pseudocode

```
Dijkstra(v1, v2):
  for each vertex v:                      // Initialization
      v's distance := infinity.
      v's previous := none.
  v1's distance := 0.
  List := {all vertices}.

  while List is not empty:
      v := remove List vertex with minimum distance.
      mark v as known.
      for each unknown neighbor n of v:
          dist := v's distance + edge (v, n)'s weight.

          if dist is smaller than n's distance:
              n's distance := dist.
              n's previous := v.

  reconstruct path from v2 back to v1,
  following previous pointers.
```

Example: Initialization

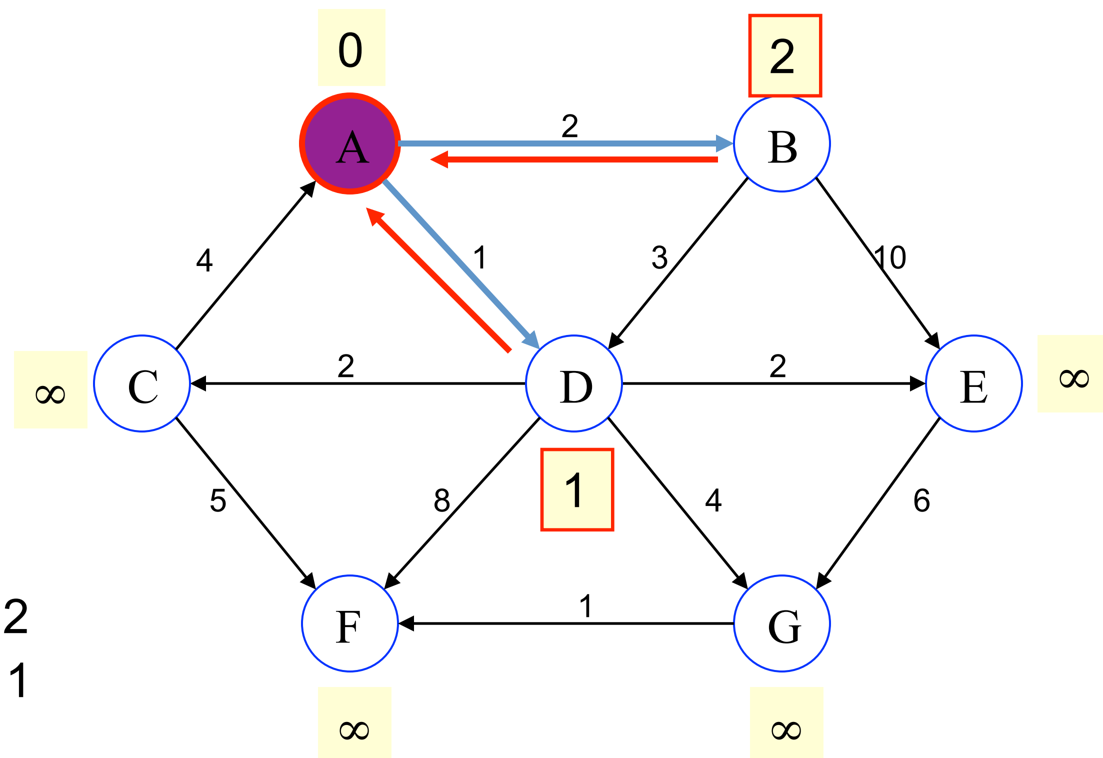
Distance(source) = 0

Distance (all vertices but source) = ∞



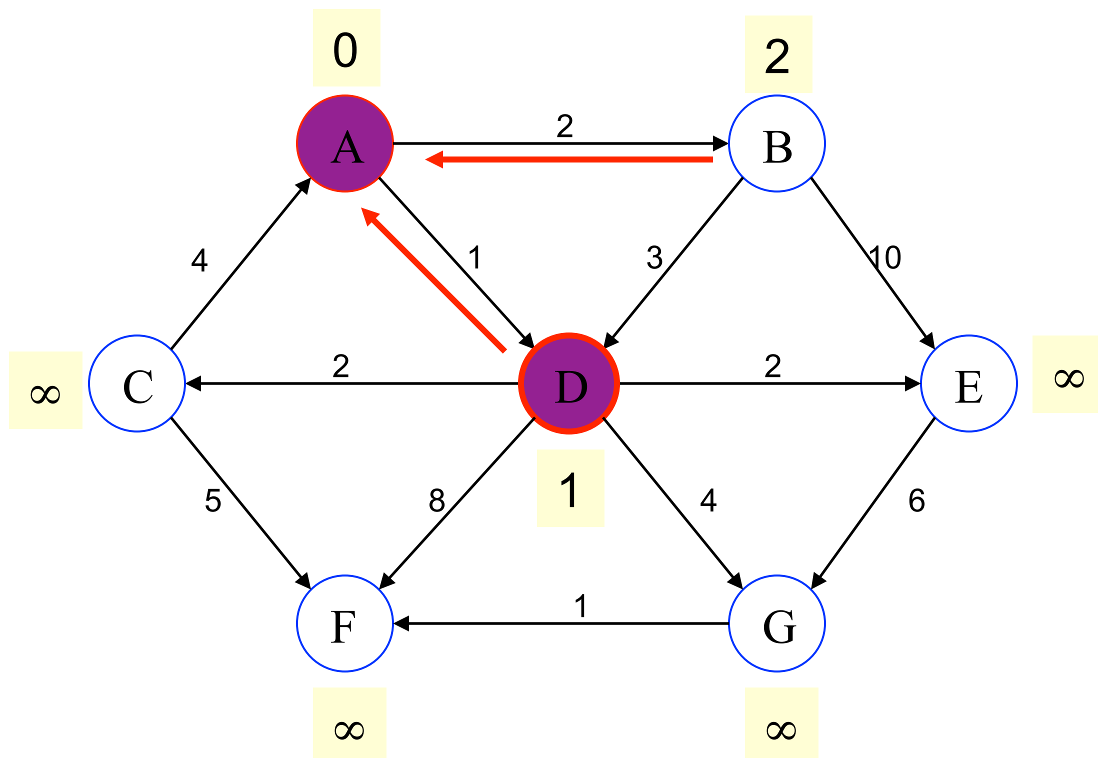
Pick vertex in List with minimum distance.

Example: Update neighbors' distance



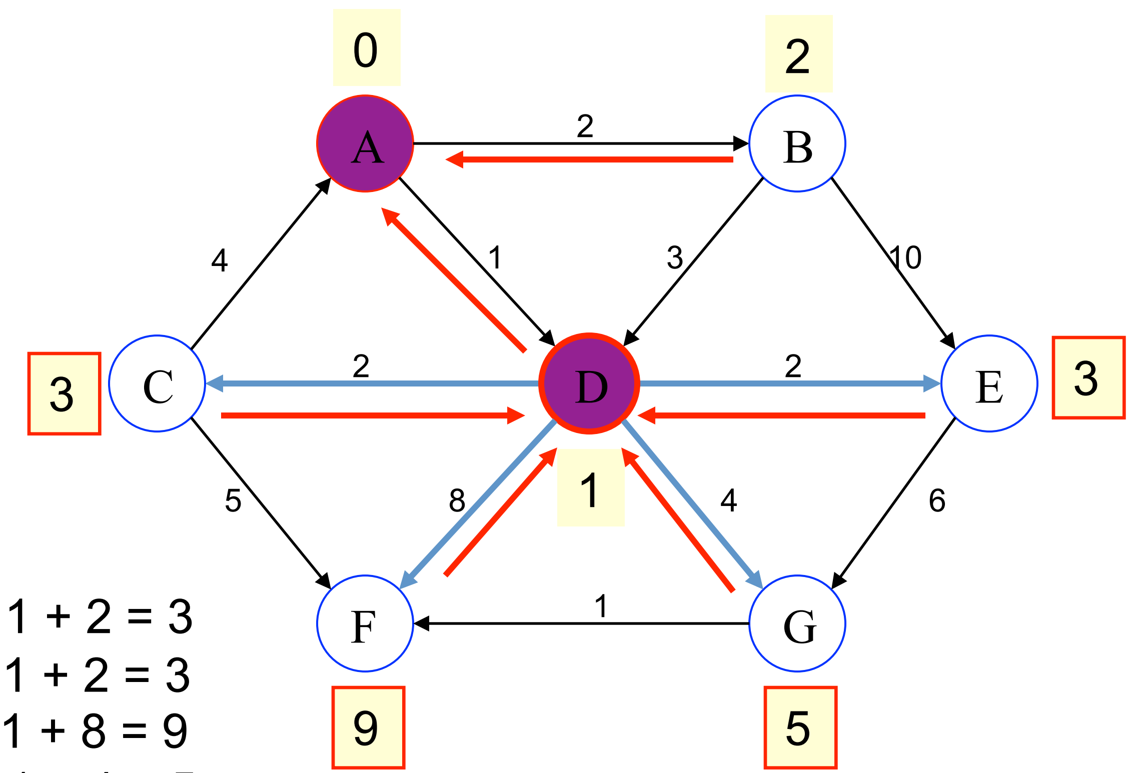
Distance(B) = 2
Distance(D) = 1

Example: Remove vertex with minimum distance



Pick vertex in List with minimum distance, i.e., D

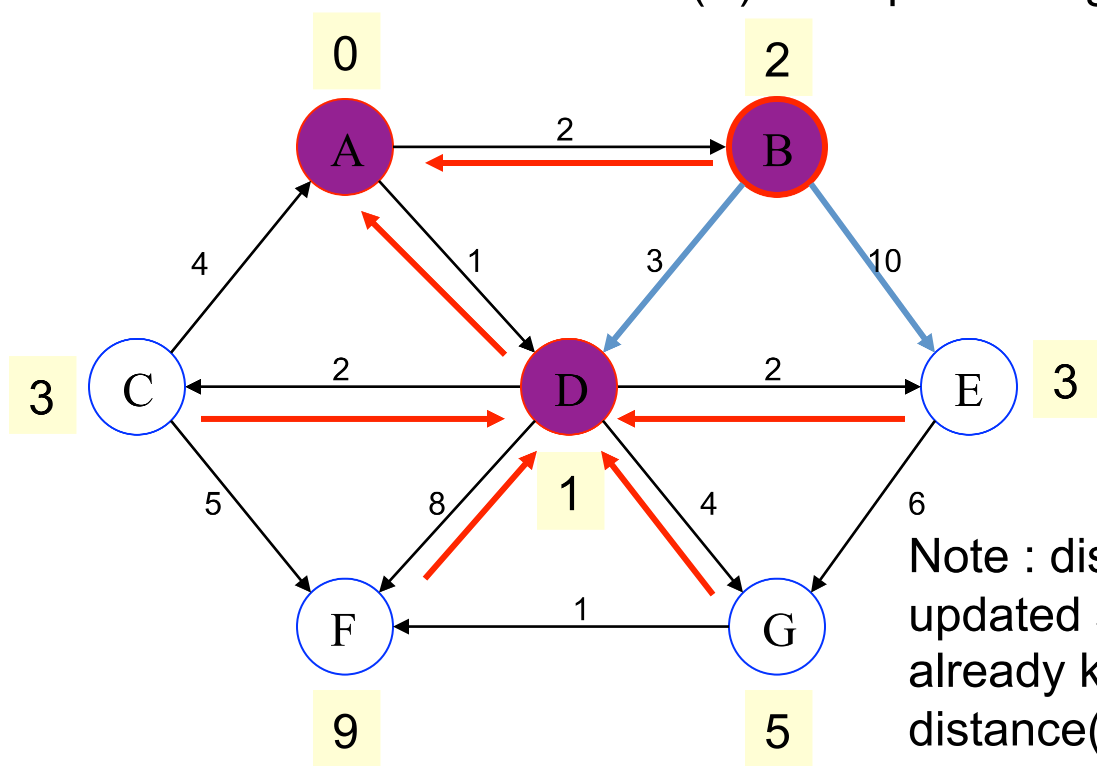
Example: Update neighbors



Distance(C) = 1 + 2 = 3
Distance(E) = 1 + 2 = 3
Distance(F) = 1 + 8 = 9
Distance(G) = 1 + 4 = 5

Example: Continued...

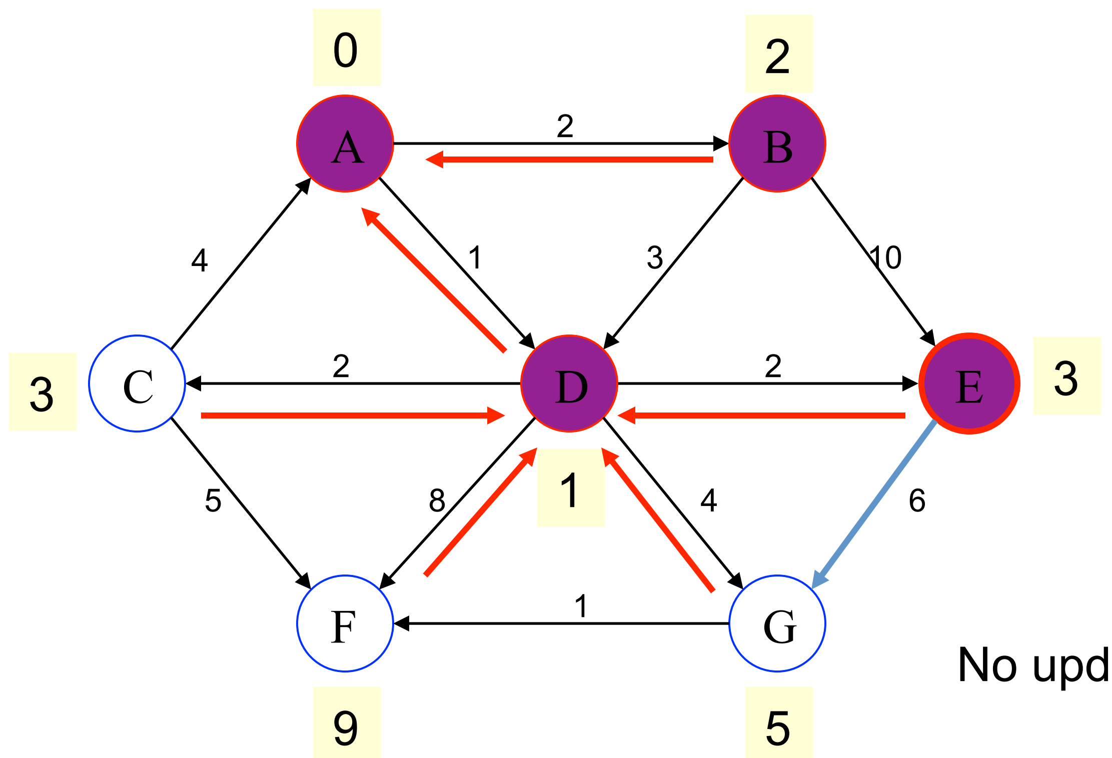
Pick vertex in List with minimum distance (B) and update neighbors



Note : distance(D) not updated since D is already known and distance(E) not updated since it is larger than previously computed

Example: Continued...

Pick vertex List with minimum distance (E) and update neighbors

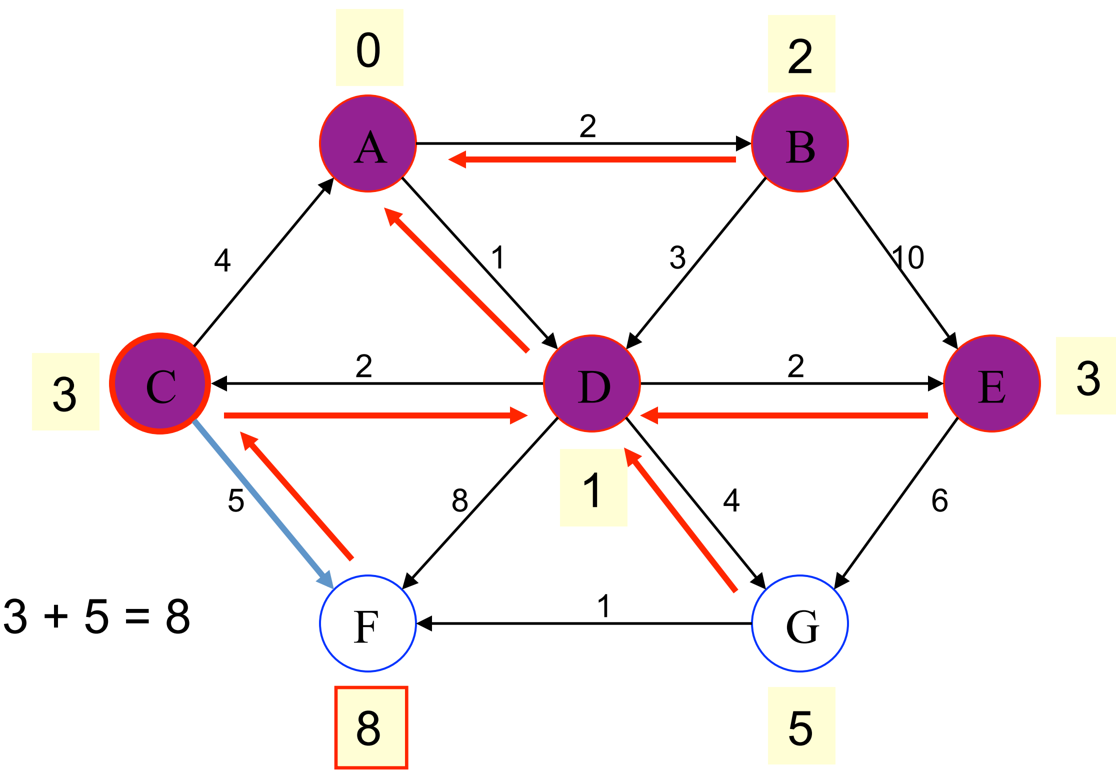


No updating

Example: Continued...

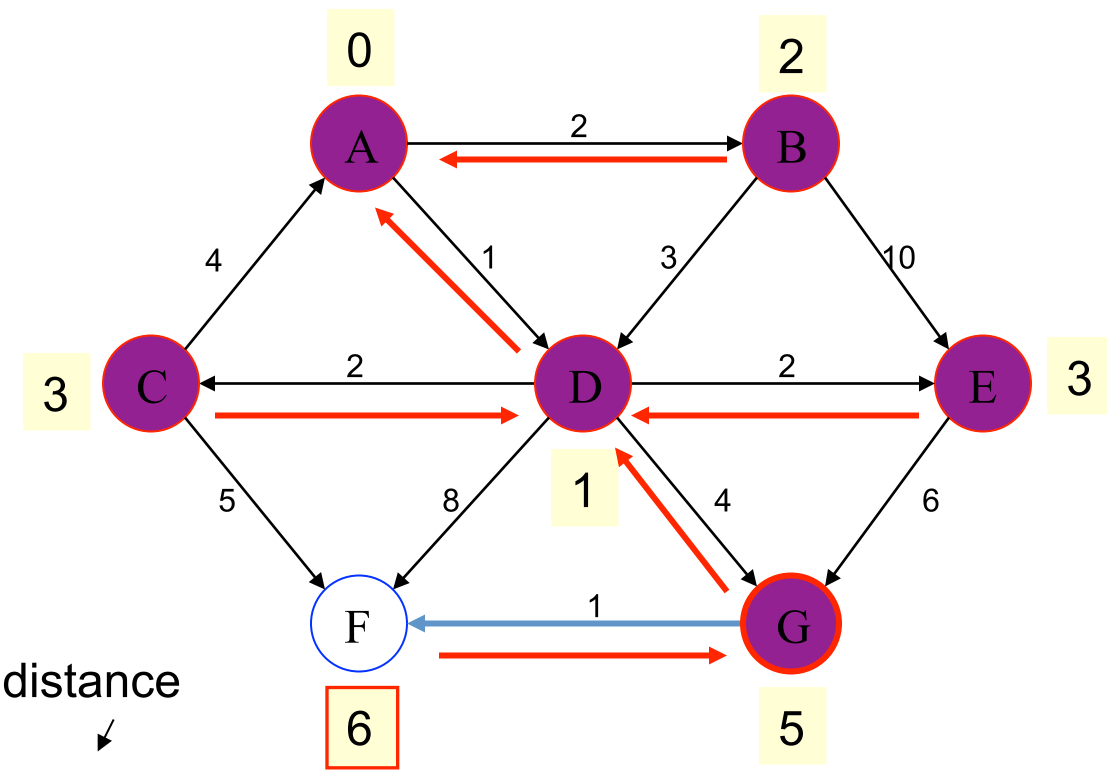
Pick vertex List with minimum distance (C) and update neighbors

Distance(F) = 3 + 5 = 8



Example: Continued...

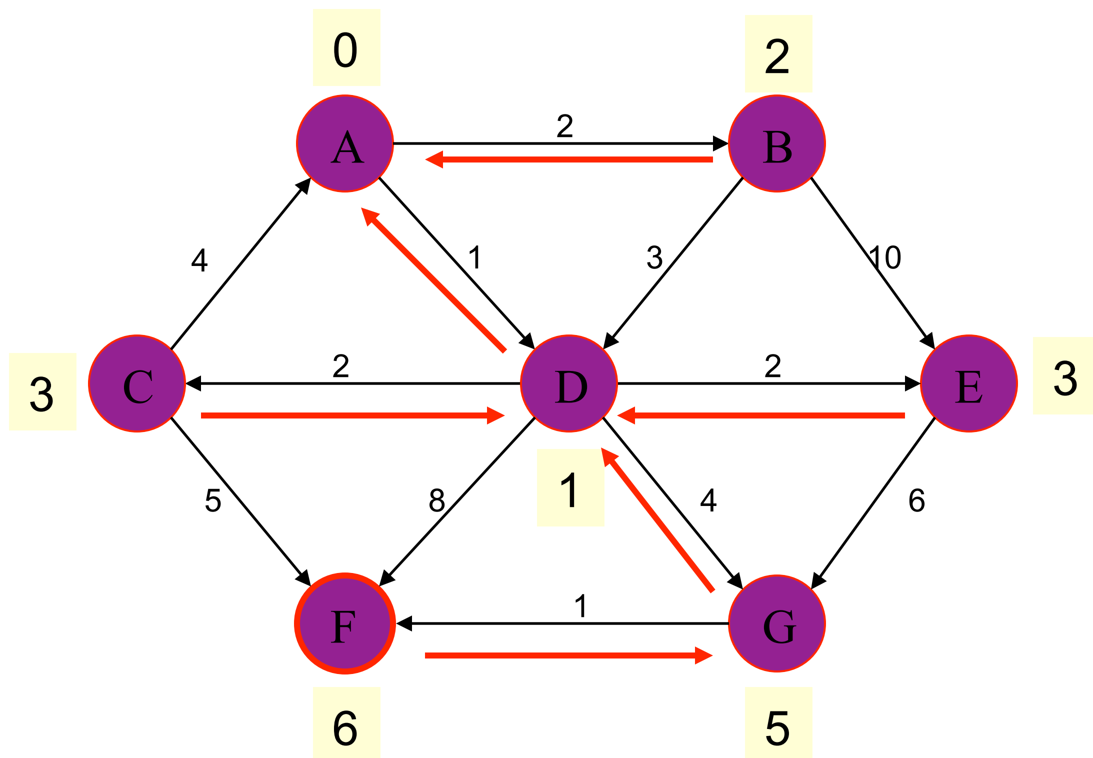
Pick vertex List with minimum distance (G) and update neighbors



Previous distance

Distance(F) = min (8, 5+1) = 6

Example (end)

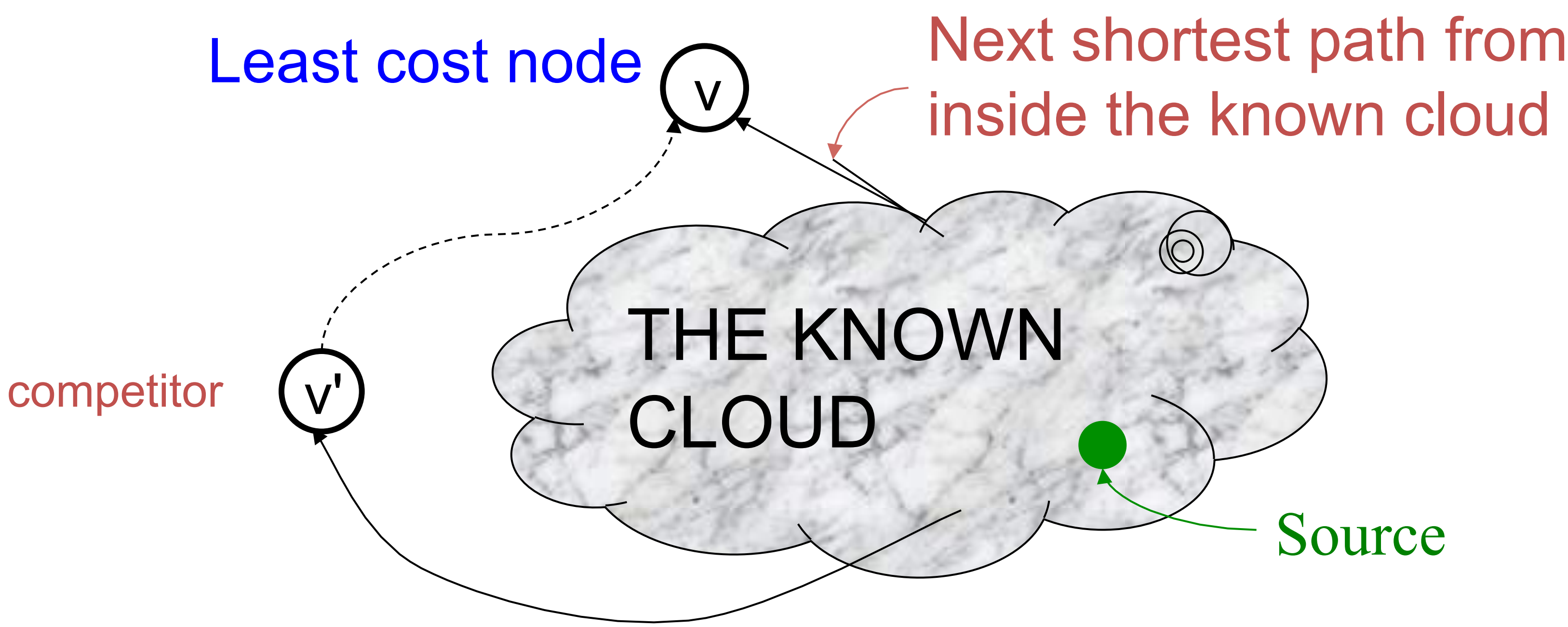


Pick vertex not in S with lowest cost (F) and update neighbors

Correctness

- Dijkstra's algorithm is a greedy algorithm
  - make choices that currently seem the best
  - locally optimal does not always mean globally optimal
- Correct because maintains following two properties:
  - for every known vertex, recorded distance is shortest distance to that vertex from source vertex
  - for every unknown vertex *v*, its recorded distance is shortest path distance to *v* from source vertex, considering only currently known vertices and *v*

“Cloudy” Proof: The Idea



- If the path to v is the next shortest path, the path to v' must be at least as long. Therefore, any path through v' to v cannot be shorter!