CSE 373: Data Structures and Algorithms

Lecture 18: Hashing III

Analysis of linear probing

- the *load factor* λ is the fraction of the table that is full
  empty table$\lambda = 0$ half full table $\lambda = 0.5$ full table $\lambda = 1$

- Expected number of probes per insertion (taking clustering into account) is roughly $(1 + 1/(1-\lambda)^2)/2$
  - empty table $(1 + 1/(1 - 0)^2)/2 = 1$
  - half full $(1 + 1/(1 - .5)^2)/2 = 2.5$
  - 3/4 full $(1 + 1/(1 - .75)^2)/2 = 8.5$
  - 9/10 full $(1 + 1/(1 - .9)^2)/2 = 50.5$

- Expected number of probes per successful search (taking clustering into account) is roughly $(1 + 1/(1-\lambda))/2$

- If hash function is fair *and the table is not too full* (i.e. $\lambda < .50 - .60$), then inserting, deleting, and searching are all O(1) operations

Analysis of double hashing

- Expected number of probes per insertion per insertion is roughly $1 / 1 - \lambda$

empty table	$1 / (1 - 0) = 1$
half full	$1 / (1 - .5) = 2$
3/4 full	$1 / (1 - .75) = 4$
9/10 full	$1 / (1 - .9) = 10$

- Expected number of probes per successful search is roughly $\ln (1 + \lambda)/\lambda$

- If hash function is fair *and the table is not too full* (i.e. $\lambda < .90 - .95$), then inserting, deleting, and searching are all O(1) operations

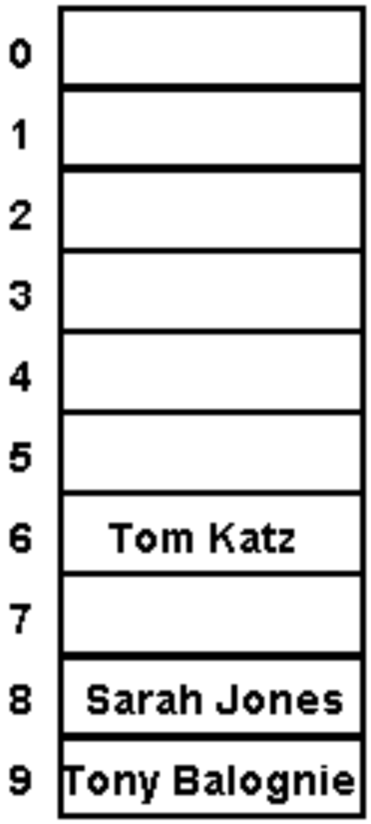
Rehashing, hash table size

- **rehash**: increasing the size of a hash table's array, and re-storing all of the items into the array using the hash function
  - Can we just copy the old contents to the larger array?
  - When should we rehash? Some options:
    - when load reaches a certain level (e.g., $\lambda = 0.5$)
    - when an insertion fails
- What is the cost (Big-Oh) of rehashing?
- What is a good hash table array size?
  - how much bigger should a hash table get when it grows?

How does Java's `HashSet` work?

- it stores `Object`s; every object has a reasonably-unique *hash code*
  - `public int hashCode()` in class `Object`

- `HashSet` stores elements in array by `hashCode()` value
  - searching for this element later, we just have to check that one index to see if it's there (O(1))
    - `"Tom Katz".hashCode() % 10 == 6`
    - `"Sarah Jones".hashCode() % 10 == 8`
    - `"Tony Balognie".hashCode() % 10 == 9`



Membership testing in `HashSets`

- When searching a `HashSet` for a given object (`contains`):
  - the set computes the `hashCode` for the given object
  - it looks in that index of the `HashSet`'s internal array
    - Java compares the given object with the object in the `HashSet`'s array using `equals`; if they are equal, returns `true`

- Hence, an object will be considered to be in the set only if *both*:
  - It has the same `hashCode` as an element in the set, *and*
  - The `equals` comparison returns true

- **General contract: if `equals` is overridden, `hashCode` should be overridden also; equal objects must have equal hash codes**

Common Error: overriding `equals` but not `hashCode`

```
public class Point {
    private int x, y;
    public Point(int x, int y) {
        this.x = x;   this.y = y;
    }
    public boolean equals(Object o) {
        if (o == this) { return true; }
        if (!(o instanceof Point)) { return false;  }
        Point p = (Point)o;
        return p.x == this.x && p.y == this.y;
    }
    // No hashCode!
}
```

- The follow code would surprisingly print `false`!

```
HashSet<Point> p = new HashSet<Point>();
p.add(new Point(7, 11));
System.out.println(p.contains(new Point(7, 11)));
```

Overriding `hashCode`

- Conditions for overriding `hashCode`:
  - return same value for object whose state hasn't changed since last call
  - if `x.equals(y)`, then `x.hashCode() == y.hashCode()`
  - (if `!x.equals(y)`, it is not necessary that `x.hashCode() != y.hashCode()` … why?)

- Advantages of overriding `hashCode`
  - your objects will store themselves correctly in a hash table
  - distributing the hash codes will keep the hash balanced: no one bucket will contain too much data compared to others

```
public int hashCode() {
    int result = 37 * x;
    result = result + y;
    return result;
}
```

Overriding `hashCode`, cont'd.

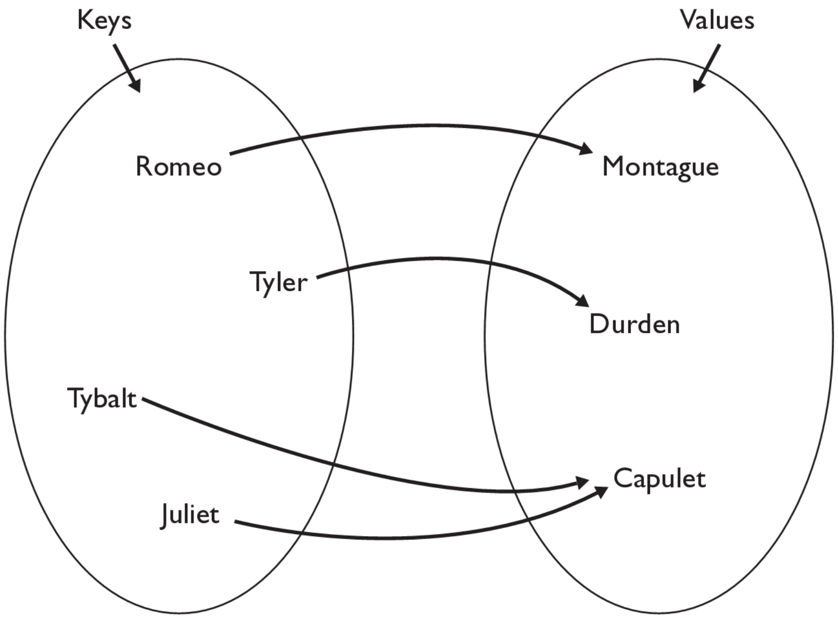
- Things to do in a good `hashCode` implementation
  - make sure the hash code is same for equal objects
  - try to ensure that the hash code will be different for different objects
  - ensure that the hash code depends on every piece of state that is important to the object (every piece of state that is used in `equals`)
  - preferably, weight the pieces so that different objects won't happen to add up to the same hash code

- Override the `Employee`'s `hashCode`.

The Map ADT

- **map:** Holds a set of unique keys and a collection of values, where each key is associated with one value
  - a.k.a. "dictionary", "associative array", "hash"

- basic map operations:
  - **put**(*key*, *value*): Adds a mapping from a key to a value.
  - **get**(*key*): Retrieves the value mapped to the key.
  - **remove**(*key*): Removes the given key and its mapped value.



`myMap.get("Juliet")` returns `"Capulet"`

Maps in computer science

- Compilers
  - symbol table

- Operating Systems
  - page tables
  - file systems (file name → location)

- Real world Examples
  - names to phone numbers
  - URLs to IP addresses
  - student ID to student information

Using `Maps`

- in Java, maps are represented by `Map` interface in `java.util`
- `Map` is implemented by the `HashMap` and `TreeMap` classes
  - `HashMap`: implemented with hash table; uses separate chaining extremely fast: **O(1)** ; keys are stored in unpredictable order
  - `TreeMap`: implemented with balanced binary search tree; very fast: **O(log N)** ; keys are stored in sorted order
  - A map requires 2 type parameters: one for keys, one for values.

```
// maps from String keys to Integer values
Map<String, Integer> votes = new HashMap<String, Integer>();
```

`Map` methods

`put(`**key**, **value**`)`	adds a mapping from the given key to the given value; if the key already exists, replaces its value with the given one
`get(`**key**`)`	returns the value mapped to the given key (`null` if not found)
`containsKey(`**key**`)`	returns `true` if the map contains a mapping for the given key
`remove(`**key**`)`	removes any existing mapping for the given key
`clear()`	removes all key/value pairs from the map
`size()`	returns the number of key/value pairs in the map
`isEmpty()`	returns `true` if the map's size is 0
`toString()`	returns a string such as `"{a=90, d=60, c=70}"`
`keySet()`	returns a set of all keys in the map
`values()`	returns a collection of all values in the map
`putAll(`**map**`)`	adds all key/value pairs from the given map to this map
`equals(`**map**`)`	returns `true` if given map has the same mappings as this one

`keySet` and `values`

- `keySet()` returns a `Set` of all keys in the map
  - can loop over the keys in a foreach loop
  - can get each key's associated value by calling `get` on the map

```
Map<String, Integer> ages = new TreeMap<String, Integer>();
ages.put("Meghan", 29);
ages.put("Kona", 3);  // ages.keySet() returns Set<String>
ages.put("Daisy", 1);
for (String name : ages.keySet()) {            // Daisy -> 1
    int age = ages.get(name);                  // Kona -> 3
    System.out.println(name + " -> " + age);   // Meghan -> 29
}
```

- `values()` returns a collection of values in the map
  - can loop over the values in a foreach loop
  - no easy way to get from a value to its associated key(s)

Implementing Map with Hash Table

- Each map entry adds a new key → value pair to the map
  - entry contains:
    - key element of given type (`null` is a valid key value)
    - value element of given value type
    - additional information needed to maintain hash table

- Organized for super quick access to keys
  - the keys are what we will be hashing on

Implementing Map with Hash Table, cont.

```
public interface Map<K, V> {
    public boolean containsKey(K key);

    public V get(K key);

    public void print();

    public void put(K key, V value);

    public V remove(K key);

    public int size();
}
```

HashMapEntry

```
public class HashMapEntry<K, V> {
    public K key;
    public V value;
    public HashMapEntry<K, V> next;

    public HashMapEntry(K key, V value) {
        this(key, value, null);
    }

    public HashMapEntry(K key, V value, HashMapEntry<K, V> next) {
        this.key = key;
        this.value = value;
        this.next = next;
    }
}
```

Map implementation: put

- Similar to our Set implementation's add method
  - figure out where key would be in the map
  - if it is already there replace the existing value with the new value
  - if the key is not in the map, insert the key, value pair into the map as a new map entry

Map implementation: put

```
public void put(K key, V value) {
    int keyBucket = hash(key);

    HashMapEntry<K, V> temp = table[keyBucket];
    while (temp != null) {
        if ((temp.key == null && key == null)
                || (temp.key != null && temp.key.equals(key))) {
            temp.value = value;
            return;
        }
        temp = temp.next;
    }

    table[keyBucket] =
        new HashMapEntry<K, V>(key, value, table[keyBucket]);
    size++;
}
```