Graphs: Definitions and Representations
(Chapter 9)

CSE 373
Data Structures and Algorithms

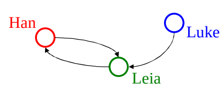
Today's Outline

- **Admin**:
  - Midterm #2, Wed Nov 17th.
  - HW #5 due Monday, Nov 22 at 11:45pm
- **Graphs**
  - **Representations**
  - **Topological Sort**

Graph… ADT?

- Not quite an ADT… operations not clear
- A formalism for representing relationships between objects

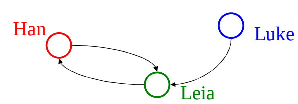
  Graph $G = (V,E)$
  - *Set of vertices*: $V = \{v_1, v_2, \ldots, v_n\}$
  - *Set of edges*: $E = \{e_1, e_2, \ldots, e_m\}$ where each e_i connects two vertices (v_{i1}, v_{i2})



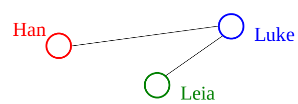

```
V = {Han, Leia, Luke}
E = {(Luke, Leia),
     (Han, Leia),
     (Leia, Han)}
```


Graph Definitions

In *directed* graphs, edges have a specific direction:



In *undirected* graphs, they don't (edges are two-way):



v is *adjacent* to u if $(u,v) \in E$

More Definitions: Simple Paths and Cycles

A *simple path* repeats no vertices (except that the first can be the last):
p = {Seattle, Salt Lake City, San Francisco, Dallas}
p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}

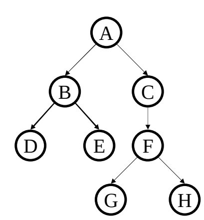
A *cycle* is a path that starts and ends at the same node:
p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}
p = {Seattle, Salt Lake City, Seattle, San Francisco, Seattle}

A *simple cycle* is a cycle that repeats no vertices except that the first vertex is also the last (in undirected graphs, no edge can be repeated)

Trees as Graphs

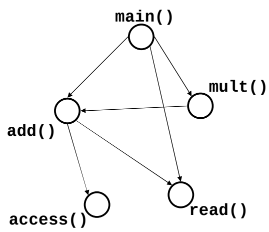
- Every tree is a graph!
- Not all graphs are trees!

A graph is a tree if
- There are *no cycles* (directed or undirected)
- There is a *path* from the root *to every node*



Directed Acyclic Graphs (DAGs)

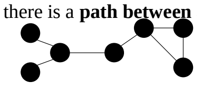
DAGs are directed graphs with no (directed) cycles.



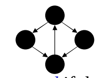
Aside: *If program call-graph is a DAG, then all procedure calls can be in-lined*

Graph Connectivity

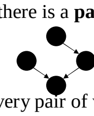
Undirected graphs are *connected* if there is a **path between any two vertices**



Directed graphs are *strongly connected* if there is a **path from any one vertex to any other**

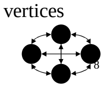


Directed graphs are *weakly connected* if there is a **path between any two vertices, *ignoring direction***

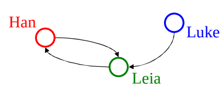


A *complete* graph has an **edge** between every pair of vertices

(i.e. max possible # of edges for directed or undirected)



Graph Representations



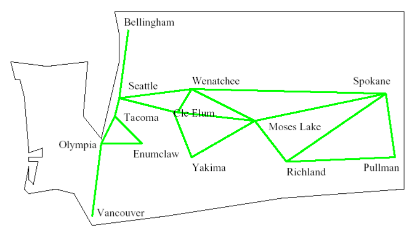
0. List of vertices + list of edges
1. 2-D matrix of vertices (marking edges in the cells)
   "adjacency matrix"
2. List of vertices each with a list of adjacent vertices
   "adjacency list"

Things we might want to do:

- iterate over vertices
- iterate over edges
- iterate over vertices adj. to a vertex
- check whether an edge exists

Vertices and edges may be labeled

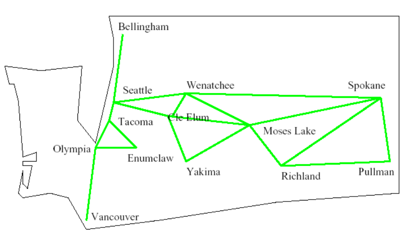
Some Applications: Moving Around Washington



What's the *shortest way* to get from Seattle to Pullman?

Edge labels:

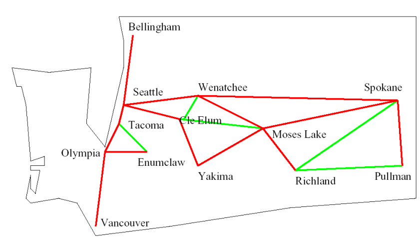
Some Applications: Moving Around Washington



What's the *fastest way* to get from Seattle to Pullman?

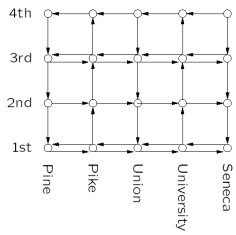
Edge labels:

Some Applications: Reliability of Communication



If Wenatchee's phone exchange *goes down*, can Seattle still talk to Pullman?

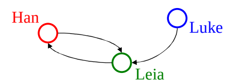
Some Applications: Bus Routes in Downtown Seattle



If we're at 3rd and Pine, how can we get to 1st and University using Metro?

Representation 1: Adjacency Matrix

A `|V| x |V|` array in which an element `(u,v)` is true if and only if there is an edge from **u** to **v**



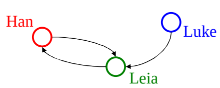
	Han	Luke	Leia
Han			
Luke			
Leia			

space requirements:

runtime:

Representation 2: Adjacency List

A `|V|`-ary list (array) in which each entry stores a list (linked list) of all adjacent vertices



Han	
Luke	
Leia	

space requirements:

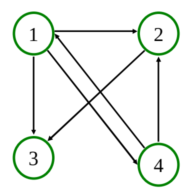
runtime:

Representation

- adjacency **matrix**:

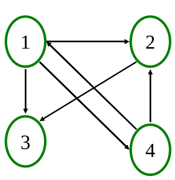
$$A[u][v] = \begin{cases} \text{weight} & \text{, if } (u, v) \in E \\ 0 & \text{, if } (u, v) \notin E \end{cases}$$

	1	**2**	**3**	**4**
1				
2				
3				
4				



Representation

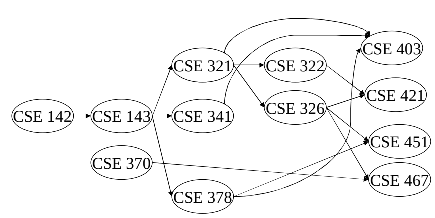
- adjacency **list:**



1	
2	
3	
4	

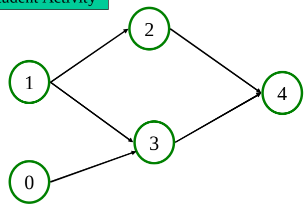
Application: Topological Sort

Given a directed graph, `G = (V,E)`, output all the vertices in **V** such that no vertex is output before any other vertex with an edge to it.



Is the output unique?

Student Activity



Valid Topological Sorts:

Student Activity

```
void Graph::topsort(){
  Vertex v, w;

  labelEachVertexWithItsIn-degree();

  for(int count=0; count<NUM_VERTICES; count++){
    v = findNewVertexOfDegreeZero();

    v.topoNum = count;
    for each w adjacent to v
      w.indegree--;
  }
}
```

Runtime:

Student Activity

```
void Graph::topsort(){
  Queue q(NUM_VERTICES);  int counter = 0; Vertex v, w;
  labelEachVertexWithItsIn-degree();

  q.makeEmpty();
  for each vertex v
    if (v.indegree == 0)
      q.enqueue(v);

  while (!q.isEmpty()){
    v = q.dequeue();
    v.topologicalNum = ++counter;
    for each w adjacent to v
      if (--w.indegree == 0)
        q.enqueue(w);
  }
}
```

intialize the queue

get a vertex with indegree 0

insert new eligible vertices

Runtime: