Priority Queues: Binary Min Heaps

CSE 373
Data Structures and Algorithms

Today’s Outline

• Announcements
  – Midterm #1, Friday Oct 22.
  – Assignment #3 coming soon.

• Today’s Topics:
  – Dictionary
    • Balanced Binary Search Trees - (AVL Trees)
  – Priority Queues
    • Binary Min Heap

Priority Queue ADT

1. PQueue data: collection of data with priority

2. PQueue operations
   – insert
   – deleteMin
   (also: create, destroy, is_empty)

3. PQueue property: for two elements in the queue, x and y, if x has a lower priority value than y, x will be deleted before y

Applications of the Priority Q

• Select print jobs in order of decreasing length
• Forward packets on network routers in order of urgency
• Select most frequent symbols for compression
• Sort numbers, picking minimum first

• Anything greedy

Implementations of Priority Queue ADT

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>deleteMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted list (Array)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unsorted list (Linked-List)</td>
<td></td>
<td></td>
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<tr>
<td>Sorted list (Array)</td>
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<tr>
<td>Sorted list (Linked-List)</td>
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<tr>
<td>Binary Search Tree (BST)</td>
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</tbody>
</table>

Representing Complete Binary Trees in an Array

From node i:
- left child: A
- right child: B
- parent: C

implicit (array) implementation:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>
**Heap Order Property**

**Heap order property**: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.

- not a heap

**Heap Operations**

- findMin:
- insert(val): percolate up.
- deleteMin: percolate down.

**Heap – Insert(val)**

Basic Idea:
1. Put val at “next” leaf position
2. Repeatedly exchange node with its parent if needed

**Insert pseudo Code (optimized)**

```java
void insert(Object o) {
    assert(!isFull());
    size++;
    newPos = percolateUp(size, o);
    Heap[newPos] = o;
}
```

```java
int percolateUp(int hole, Object val) {
    while (hole > 1 && val < Heap[hole/2]) {
        Heap[hole] = Heap[hole/2];
        hole /= 2;
    }
    return hole;
}
```

**Heap – Deletemin**

Basic Idea:
1. Remove root (that is always the min!)
2. Put “last” leaf node at root
3. Find smallest child of node
4. Swap node with its smallest child if needed.
5. Repeat steps 3 & 4 until no swaps needed.
DeleteMin pseudo Code (Optimized)

```java
Object deleteMin() {
    assert(!isEmpty());
    returnVal = Heap[1];
    size--;
    newPos = percolateDown(1, Heap[size+1]);
    Heap[newPos] = Heap[size + 1];
    return returnVal;
}
```

`int percolateDown(int hole, Object val) {
    while (2*hole <= size) {
        left = 2*hole;
        right = left + 1;
        if (right <= size && Heap[right] < Heap[left])
            target = right;
        else
            target = left;
        if (Heap[target] < val) {
            Heap[hole] = Heap[target];
            hole = target;
        } else
            break;
    } return hole;
}
```

DeleteMin: percolate down

![DeleteMin percolate down diagram]

Other Priority Queue Operations

- **decreaseKey**
  - given a pointer to an object in the queue, reduce its priority value
  - Solution: change priority and ______________________

- **increaseKey**
  - given a pointer to an object in the queue, increase its priority value
  - Solution: change priority and ______________________

Why do we need a pointer? Why not simply data value?

Other Heap Operations

- **decreaseKey**(objPtr, amount): raise the priority of a object, percolate up
- **increaseKey**(objPtr, amount): lower the priority of a object, percolate down
- **remove**(objPtr): remove a object, move to top, them delete.
  1) decreaseKey(objPtr, ∞)
  2) deleteMin()

Worst case Running time for all of these:
- FindMax?
- ExpandHeap – when heap fills, copy into new space.

Binary Min Heaps (summary)

- **insert**: percolate up. Θ(log N) time.
- **deleteMin**: percolate down. Θ(log N) time.

- **Build Heap?**
BuildHeap: Floyd’s Method

Add elements arbitrarily to form a complete tree. Pretend it’s a heap and fix the heap-order property!

Buildheap pseudocode

```java
private void buildHeap() {
    for (int i = currentSize/2; i > 0; i--) {
        percolateDown(i);
    }
}
```

Finally…

Facts about Binary Min Heaps

Observations:
- finding a child/parent index is a multiply/divide by two
- operations jump widely through the heap
- each percolate step looks at only two new nodes
- inserts are at least as common as deleteMins

Realities:
- division/multiplication by powers of two are equally fast
- looking at only two new pieces of data: bad for cache!
- with huge data sets, disk accesses dominate