Hash Tables

CSE 373
Data Structures & Algorithms
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Today's Outline

- **Announcements**
  - Assignment #4 due this Friday Feb 13th at the beginning of lecture.
- **Today's Topics:**
  - **Disjoint Sets & Dynamic Equivalence**
  - **Hashing**

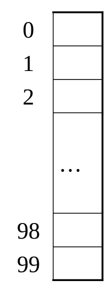
Dictionary Implementations

	Unsorted linked list	Sorted Array	Binary Search Tree	AVL Tree
Insert				O(log N)
Find	O(N)			
Delete			O(N)	O(log N)

Constant Time Access

Data Set:
- 100 students
- Keys = Student numbers between 0 and 99.

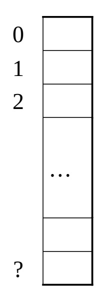
Solution:
- Array of size 0-99.
- One-to-one mapping: e.g. student number 2 goes in location 2



Constant Time Access?

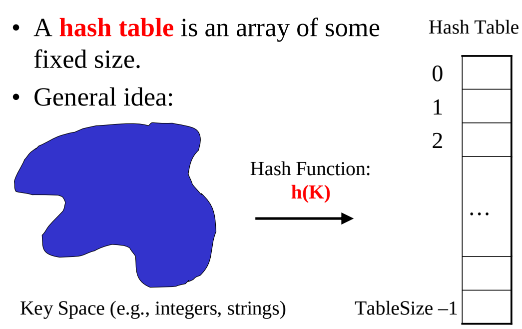
Data Set:
- 100 students
- Keys = Student numbers between 0 and 999999999.

Solution:
- Array of size ?
- Mapping ?



Hash Tables

- A **hash table** is an array of some fixed size.
- General idea:



Example

- Key space = integers
- TableSize = 10
- **h**(K) = K mod 10
- **Insert**: 207, 18, 41, 194, 19, 43

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

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Another Example

- key space = integers
- TableSize = 6
- **h**(K) = K mod 6
- **Insert**: 7, 18, 41, 34

0	
1	
2	
3	
4	
5	

Student Activity 8

Hash Functions

1. **simple/fast** to compute,
2. Avoid **collisions**
3. have keys distributed **evenly** among cells.

Perfect Hash function:

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Sample Hash Functions:

key space = strings A=0, B=1,...Z=25

s = $s_0\ s_1\ s_2 \ldots s_{k-1}$

1. h(s) = s_0 mod TableSize
2. h(s) = $\left(\sum_{i=0}^{k-1} s_i \right)$ mod TableSize
3. h(s) = $\left(\sum_{i=0}^{k-1} s_i \cdot 26^i \right)$ mod TableSize

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Designing a Hash Function for web URLs

s = $s_0\ s_1\ s_2 \ldots s_{k-1}$

Issues to take into account:

h(s) =

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Collision Resolution

Collision: when two keys map to the same location in the hash table.

Two ways to resolve collisions:

1. Separate Chaining
2. Open Addressing (linear probing, quadratic probing, double hashing)

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Separate Chaining

h(K) = K mod 10

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
10
22
107
12
42

Separate chaining: All keys that map to the same hash value are kept in a list (or "bucket").

Analysis of Find

The load factor, λ, of a hash table is the ratio:

$$\frac{N}{\text{TableSize}} \quad \begin{array}{l} \leftarrow \text{\# of elements} \\ \leftarrow \text{table size} \end{array}$$

For separate chaining,
λ = average # of elements in a bucket

Average # of values needed to examine for a:

- unsuccessful find:
- successful find:

How Big Should the Hash Table Be?

For Separate Chaining, if we want $\lambda = 1$
(e.g. the average # of values per bucket = 1)

- How large should I make the hash table, in terms of N?

TableSize =

tableSize: Why Prime?

- Suppose
  - data stored in hash table: 7160, 493, 60, 55, 321, 900, 810
  - tableSize = 10
    data hashes to 0, 3, <u>0</u>, 5, 1, <u>0</u>, <u>0</u>
  - tableSize = 11
    data hashes to 10, 9, 5, 0, 2, <u>9</u>, 7

Real-life data tends to have a pattern

Being a multiple of 11 is usually *not* the pattern ☺

Open Addressing

h(K) = K mod 10

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
38
19
8
109
10

Linear Probing: after checking h(k), try h(k)+1, if that is full, try h(k)+2, then try h(k)+3, etc.

Terminology Alert!

	"**Open** Hashing"	"Closed Hashing"
	equals	equals
Weiss	"Separate Chaining"	"**Open** Addressing"

Linear Probing

$f(i) = i$

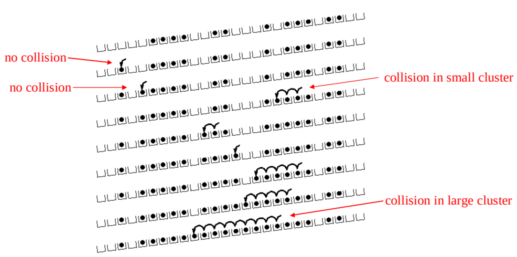
- Probe sequence:
  - 0^{th} probe = h(k) mod TableSize
  - 1^{th} probe = (h(k) + 1) mod TableSize
  - 2^{th} probe = (h(k) + 2) mod TableSize
  - . . .
  - i^{th} probe = (h(k) + i) mod TableSize

Write pseudocode for find(k) for Open Addressing with linear probing

- Find(k) returns i where T(i) = k

Student Activity

Linear Probing – Clustering



[R. Sedgewick]

Load Factor in Linear Probing

- For *any* $\lambda < 1$, linear probing *will* find an empty slot
- Expected # of probes (for large table sizes)
  - successful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$
  - unsuccessful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$
- Linear probing suffers from ***primary clustering***
- Performance quickly degrades for $\lambda > 1/2$

Quadratic Probing

Less likely to encounter Primary Clustering

$f(i) = i^2$

- Probe sequence:
  - 0^{th} probe = h(k) mod TableSize
  - 1^{th} probe = (h(k) + 1) mod TableSize
  - 2^{th} probe = (h(k) + 4) mod TableSize
  - 3^{th} probe = (h(k) + 9) mod TableSize
  - . . .
  - i^{th} probe = $(h(k) + i^2)$ mod TableSize

Quadratic Probing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
89
18
49
58
79

Quadratic Probing Example

insert(76)	insert(40)	insert(48)	insert(5)	insert(55)
76%7 = 6	40%7 = 5	48%7 = 6	5%7 = 5	55%7 = 6

0	
1	
2	
3	
4	
5	
6	76

But… insert(47)
47%7 = 5

Quadratic Probing: Success guarantee for $\lambda < ½$

- If size is prime and $\lambda < ½$, then quadratic probing will find an empty slot in size/2 probes or fewer.
  - show for all `0 ≤ i,j ≤ size/2` and `i ≠ j`
    `(h(x) + i²) mod size ≠ (h(x) + j²) mod size`
  - by contradiction: suppose that for some i ≠ j:
    `(h(x) + i²) mod size = (h(x) + j²) mod size`
    `⇒ i² mod size = j² mod size`
    `⇒ (i² - j²) mod size = 0`
    `⇒ [(i + j)(i - j)] mod size = 0`

  BUT size does not divide `(i-j)` or `(i+j)`

Quadratic Probing: Properties

- For *any* $\lambda < ½$, quadratic probing will find an empty slot; for bigger λ, quadratic probing *may* find a slot
- Quadratic probing does not suffer from *primary* clustering: keys hashing to the same *area* are not bad
- But what about keys that hash to the same *spot*?
  - ***Secondary Clustering!***

Double Hashing

f(i) = i * g(k)
where g is a second hash function

- Probe sequence:

  0th probe = h(k) mod TableSize
  1th probe = (h(k) + g(k)) mod TableSize
  2th probe = (h(k) + 2*g(k)) mod TableSize
  3th probe = (h(k) + 3*g(k)) mod TableSize
  . . .
  ith probe = (h(k) + i*g(k)) mod TableSize

Double Hashing Example

h(k) = k mod 7 and g(k) = 5 – (k mod 5)

	76	93	40	47	10	55
0						
1				47	47	47
2		93	93	93	93	93
3					10	10
4						55
5			40	40	40	40
6	76	76	76	76	76	76
Probes	1	1	1	2	1	2

Resolving Collisions with Double Hashing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

<u>Hash Functions</u>:
H(K) = K mod M
$H_2(K) = 1 + ((K/M) \bmod (M-1))$
M =

Insert these values into the hash table in this order. Resolve any collisions with double hashing:
13
28
33
147
43

Rehashing

Idea: When the table gets too full, create a bigger table (usually 2x as large) and hash all the items from the original table into the new table.

- When to rehash?
  - half full ($\lambda = 0.5$)
  - when an insertion fails
  - some other threshold
- Cost of rehashing?

Hashing Summary

- Hashing is one of the most important data structures.
- Hashing has many applications where operations are limited to find, insert, and delete.
- Dynamic hash tables have good amortized complexity.