Math Review

CSE 373
Data Structures & Algorithms
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Winter 2009

Today's Outline

- **Announcements**
  - Assignment #1 due Thurs, Jan 15 at 11:45pm
  - Email sent to cse373 mailing list – did you get it?
  - Have you installed Eclipse and Java yet?
- **Queues and Stacks**
- **Math Review**
  - **Powers of 2**
  - **Binary numbers**
  - **Floor and Ceiling**
  - **Exponents and Logs**

Background Survey Info: When did you take cse143?

0 - autumn 08	9	12.33%
1 - summer 08	5	6.85%
2 - spring 08	9	12.33%
3 - winter 08	13	17.81%
4 - autumn 07	6	8.22%
5 - summer 07	2	2.74%
6 - Before summer 07	17	23.29%
7 - I did not take cse 143 at UW (AP or transfer credit)	8	10.96%
Other:	4	5.48%

Homework 1 – Sound Blaster!

Play your favorite song in reverse!

Aim:

1. Implement stack ADT two different ways
2. Use to reverse a sound file

Due: Thurs, Jan 15, 2009
Electronic: at 11:45pm

Powers of 2

- Many of the numbers we use in Computer Science are powers of 2
- Binary numbers (base 2) are easily represented in digital computers
  - each "bit" is a 0 or a 1
  - an n-bit wide field can represent how many different things?

0000000000101011

N bits can represent how many things?

# Bits	Patterns	# of patterns
1		
2		

Unsigned binary numbers

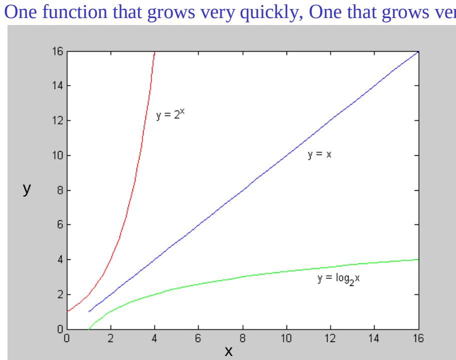
- For **unsigned** numbers in a fixed width field
  - the minimum value is 0
  - the maximum value is 2^n-1, where n is the number of bits in the field
  - The value is $\sum_{i=0}^{i=n-1} a_i 2^i$
- Each bit position represents a power of 2 with $a_i = 0$ or $a_i = 1$

Signed Numbers?

Logarithms and Exponents

- Definition: $\log_2 x = y$ if and only if $x = 2^y$
  $8 = 2^3$, so $\log_2 8 = 3$
  $65536 = 2^{16}$, so $\log_2 65536 = 16$
- Notice that $\log_2 n$ tells you how many bits are needed to distinguish among n different values.
  8 bits can hold any of 256 numbers, for example: 0 to 2^8-1, which is 0 to 255
  $\log_2 256 = 8$

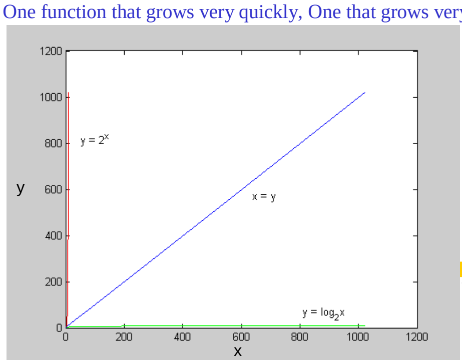
One function that grows very quickly, One that grows very slowly




```
x = 0:.1:4
y = 2.^x
plot(x,y,'r')
hold on
plot(y,x,'g')
plot(y,y,'b')
```

x, 2^x and $\log_2 x$

One function that grows very quickly, One that grows very slowly




```
x = 0:10
y = 2.^x
plot(x,y,'r')
hold on
plot(y,x,'g')
plot(y,y,'b')
```

2^x and $\log_2 x$

Floor and Ceiling

$\lfloor X \rfloor$ Floor function: the largest integer $\leq$ X

$\lfloor 2.7 \rfloor = 2$ $\lfloor -2.7 \rfloor = -3$ $\lfloor 2 \rfloor = 2$

$\lceil X \rceil$ Ceiling function: the smallest integer $\geq$ X

$\lceil 2.3 \rceil = 3$ $\lceil -2.3 \rceil = -2$ $\lceil 2 \rceil = 2$

Facts about Floor and Ceiling

1. $X - 1 < \lfloor X \rfloor \le X$
2. $X \le \lceil X \rceil < X + 1$
3. $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$ if n is an integer

Properties of logs

- We will assume logs to base 2 unless specified otherwise.
- $8 = 2^3$, so $\log_2 8 = 3$, so $2^{(\log_2 8)} =$ ________

Show:

log (A • B) = log A + log B

$A = 2^{\log_2 A}$ and $B = 2^{\log_2 B}$

$A \bullet B = 2^{\log_2 A} \bullet 2^{\log_2 B} = 2^{\log_2 A + \log_2 B}$

So: $\log_2 AB = \log_2 A + \log_2 B$

- Note: $\log AB \neq \log A \bullet \log B$!!

Other log properties

- log A/B = log A – log B
- log (A^B) = B log A
- log log X < log X < X for all X > 0
  - log log X = Y means: $2^{2^Y} = X$
  - log X grows more slowly than X
    - called a “sub-linear” function

Note: $\log \log X \neq \log^2 X$

$\log^2 X = (\log X)(\log X)$ aka “log-squared”

A log is a log is a log

- “Any base B log is equivalent to base 2 log within a constant factor.”

$B = 2^{\log_2 B}$

$x = 2^{\log_2 x}$

$\log_B X = \log_B X$

$B^{\log_B X} = X$ (substitution; $B^{\log_B X} = X$ by def. of logs)

$(2^{\log_2 B})^{\log_B X} = 2^{\log_2 X}$

$2^{\log_2 B \log_B X} = 2^{\log_2 X}$

$\log_2 B \log_B X = \log_2 X$

$$\log_B X = \frac{\log_2 X}{\log_2 B}$$