Math Review

CSE 373
Data Structures & Algorithms
Ruth Anderson
Winter 2009

Today’s Outline

• Announcements
  – Assignment #1 due Thurs, Jan 15 at 11:45pm
  – Email sent to cse373 mailing list – did you get it?
  – Have you installed Eclipse and Java yet?

• Queues and Stacks
• Math Review
  – Powers of 2
  – Binary numbers
  – Floor and Ceiling
  – Exponents and Logs

Background Survey Info:
When did you take cse143?

- 0 - autumn 08 9 12.33%
- 1 - summer 08 5 6.85%
- 2 - spring 08 9 12.33%
- 3 - winter 08 13 17.81%
- 4 - autumn 07 6 8.22%
- 5 - summer 07 2 2.74%
- 6 - Before summer 07 17 23.29%
- 7 - I did not take cse 143 at UW (AP or transfer credit) 8 10.96%
- Other: 4 5.48%

Homework 1 – Sound Blaster!

Play your favorite song in reverse!

Aim:
1. Implement stack ADT two different ways
2. Use to reverse a sound file

Due: Thurs, Jan 15, 2009
  Electronic: at 11:45pm

Powers of 2

- Many of the numbers we use in Computer Science are powers of 2
- Binary numbers (base 2) are easily represented in digital computers
  - each "bit" is a 0 or a 1
  - an n-bit wide field can represent how many different things?

N bits can represent how many things?

<table>
<thead>
<tr>
<th># Bits</th>
<th>Patterns</th>
<th># of patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>000000000101011</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unsigned binary numbers

- For **unsigned** numbers in a fixed width field
  - the minimum value is 0
  - the maximum value is $2^n - 1$, where $n$ is the number of bits in the field
  - The value is $\sum_{i=0}^{n-1} a_i 2^i$
- Each bit position represents a power of 2 with $a_i = 0$ or $a_i = 1$

Signed Numbers?

Logarithms and Exponents

- Definition: $\log_2 x = y$ if and only if $x = 2^y$
  - $8 = 2^3$, so $\log_2 8 = 3$
  - $65536 = 2^{16}$, so $\log_2 65536 = 16$
- Notice that $\log_2 n$ tells you how many bits are needed to distinguish among $n$ different values.
  - 8 bits can hold any of 256 numbers, for example: 0 to $2^8 - 1$, which is 0 to 255
  - $\log_2 256 = 8$

Floor and Ceiling

- $\lfloor X \rfloor$ Floor function: the largest integer $\leq X$
  - $\lfloor 2.7 \rfloor = 2$  $\lfloor -2.7 \rfloor = -3$  $\lfloor 2 \rfloor = 2$
- $\lceil X \rceil$ Ceiling function: the smallest integer $\geq X$
  - $\lceil 2.3 \rceil = 3$  $\lceil -2.3 \rceil = -2$  $\lceil 2 \rceil = 2$
Facts about Floor and Ceiling

1. \(X - 1 < \lfloor X \rfloor \leq X\)
2. \(X \leq \lceil X \rceil < X + 1\)
3. \(\lfloor n/2 \rfloor + \lceil n/2 \rceil = n\) if \(n\) is an integer

Properties of logs

- We will assume logs to base 2 unless specified otherwise.
- \(8 = 2^3\), so \(\log_2 8 = 3\), so \(2^{\log_2 8} = \) __________

Show:
\[
\log(A \cdot B) = \log A + \log B
\]

\(A = 2^{\log_2 A}\) and \(B = 2^{\log_2 B}\)
\(A \cdot B = 2^{\log_2 A} \cdot 2^{\log_2 B} = 2^{\log_2 A + \log_2 B}\)

So:
\(\log_2 AB = \log_2 A + \log_2 B\)

- Note: \(\log AB \neq \log A \cdot \log B\) !!

Other log properties

- \(\log A/B = \log A - \log B\)
- \(\log (A^B) = B \log A\)
- \(\log X < \log X < X\) for all \(X > 0\)
  - \(\log \log X = Y\) means: \(2^Y = X\)
  - \(\log X\) grows more slowly than \(X\)
    - called a “sub-linear” function

Note: \(\log \log X \neq \log^2 X\)
\(\log^2 X = (\log X)(\log X)\) aka “log-squared”

A log is a log is a log

- “Any base \(B\) log is equivalent to base 2 log within a constant factor.”

\[
B^{\log_B X} = X \quad \text{and} \quad 2^{\log_2 X} = X
\]

\[
\log_B X = \frac{\log_2 X}{\log_2 B}
\]