Priority Queues:
Binary Min Heaps

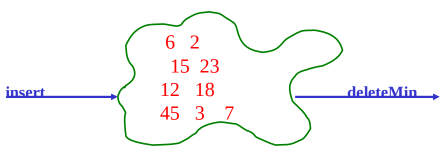
CSE 373
Data Structures and Algorithms

Today's Outline

- **Announcements**
  - Assignment #3 coming soon, due Thurs, May 7th.
- **Today's Topics:**
  - **Dictionary**
    - **Balanced Binary Search Trees - (AVL Trees)**
  - **Priority Queues**
    - **Binary Min Heap**

Priority Queue ADT

- Checkout line at the supermarket ???
- Printer queues ???
- operations: insert, deleteMin



Priority Queue ADT

1. **PQueue <u>data</u>** : collection of data with priority
2. **PQueue <u>operations</u>**
   - insert
   - deleteMin

   (also: create, destroy, is_empty)
3. **PQueue <u>property</u>**: for two elements in the queue, *x* and *y*, if *x* has a **<u>lower</u>** priority value than *y*, *x* will be deleted before *y*

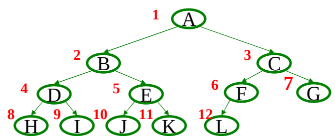
Applications of the Priority Q

- Select print jobs in order of decreasing length
- Forward packets on network routers in order of urgency
- Select most frequent symbols for compression
- Sort numbers, picking minimum first
- **Anything *greedy***

Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted list (Array)		
Unsorted list (Linked-List)		
Sorted list (Array)		
Sorted list (Linked-List)		
Binary Search Tree (BST)		

Representing Complete Binary Trees in an Array



From node **i**:

left child:
right child:
parent:

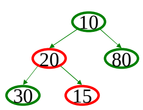
implicit (array) implementation:

	A	B	C	D	E	F	G	H	I	J	K	L	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

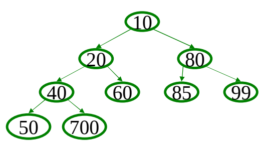
Why better than tree with pointers?

Heap **Order** Property

Heap order property: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.

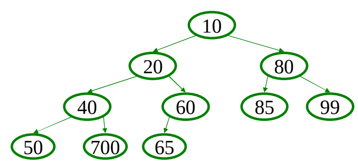


not a heap



Heap Operations

- findMin:
- insert(val): percolate up.
- deleteMin: percolate down.



Heap – Insert(val)

Basic Idea:

1. Put val at "next" leaf position
2. Repeatedly exchange node with its parent if needed

Insert pseudo Code (optimized)

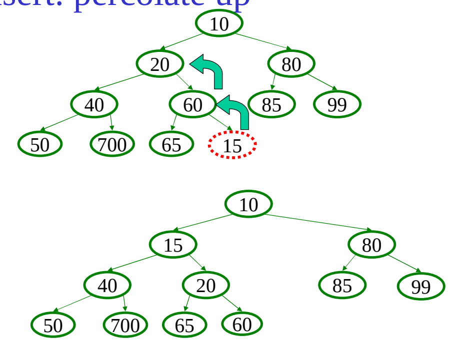
```
void insert(Object o) {
  assert(!isFull());
  size++;
  newPos =
    percolateUp(size,o);
  Heap[newPos] = o;
}
```

```
int percolateUp(int hole,
                Object val) {
  while (hole > 1 &&
         val < Heap[hole/2])
    Heap[hole] = Heap[hole/2];
    hole /= 2;
  }
  return hole;
}
```

runtime:

(Java code in book)

Insert: percolate up



Heap – Deletemin

Basic Idea:

1. Remove root (that is always the min!)
2. Put “last” leaf node at root
3. Find smallest child of node
4. Swap node with its smallest child if needed.
5. Repeat steps 3 & 4 until no swaps needed.

DeleteMin pseudo Code (Optimized)

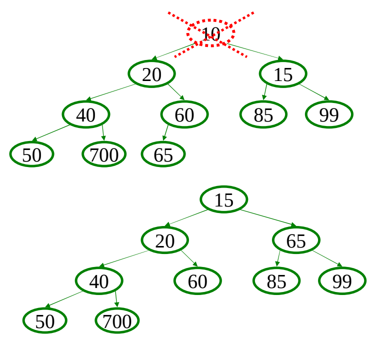
```
Object deleteMin() {
  assert(!isEmpty());
  returnVal = Heap[1];
  size--;
  newPos =
    percolateDown(1,
        Heap[size+1]);
  Heap[newPos] =
    Heap[size + 1];
  return returnVal;
}
```

runtime:

(Java code in book)

```
int percolateDown(int hole,
                  Object val) {
while (2*hole <= size) {
    left = 2*hole;
    right = left + 1;
    if (right ≤ size &&
        Heap[right] < Heap[left])
      target = right;
    else
      target = left;
    if (Heap[target] < val) {
      Heap[hole] = Heap[target];
      hole = target;
    }
    else
      break;
  }
  return hole;
}
```


DeleteMin: percolate down



Insert: 16, 32, 4, 69, 105, 43, 2

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

Other Priority Queue Operations

- **decreaseKey**
  – given a pointer to an object in the queue, reduce its priority value

  Solution: change priority and ___________________________
- **increaseKey**
  – given a pointer to an object in the queue, increase its priority value

  Solution: change priority and ___________________________

Why do we need a *pointer*? Why not simply data value?

Other Heap Operations

decreaseKey(objPtr, amount): raise the priority of a object, percolate up

increaseKey(objPtr, amount): lower the priority of a object, percolate down

remove(objPtr): remove a object, move to top, them delete.

1) decreaseKey(**objPtr**, ∞)

2) deleteMin()

Worst case Running time for all of these:

FindMax?

ExpandHeap – when heap fills, copy into new space.

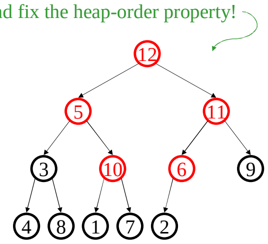
Binary Min Heaps (summary)

- **insert**: percolate up. $\Theta(\log N)$ time.
- **deleteMin**: percolate down. $\Theta(\log N)$ time.
- **Build Heap?**

BuildHeap: Floyd's Method

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!

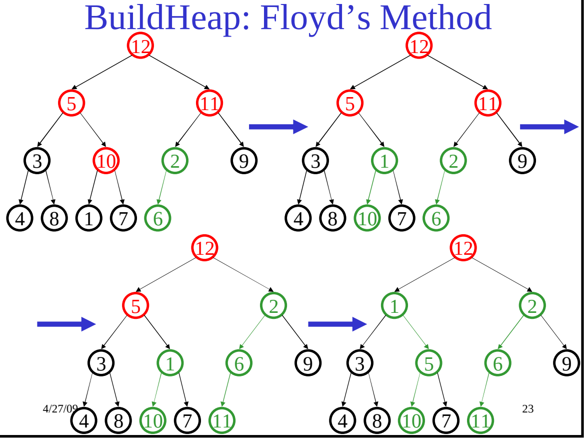


Buildheap pseudocode

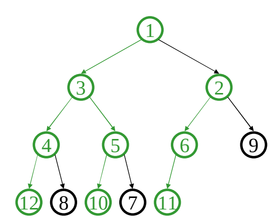
```
private void buildHeap() {
  for ( int i = currentSize/2; i > 0; i-- )
      percolateDown( i );
}
```

runtime:

BuildHeap: Floyd's Method



Finally…



runtime:

Facts about Binary Min Heaps

Observations:

- finding a child/parent index is a multiply/divide by two
- operations jump widely through the heap
- each percolate step looks at only two new nodes
- inserts are *at least* as common as deleteMins

Realities:

- division/multiplication by *powers* of two are equally fast
- looking at only **two** new pieces of data: bad for cache!
- with huge data sets, disk accesses dominate

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