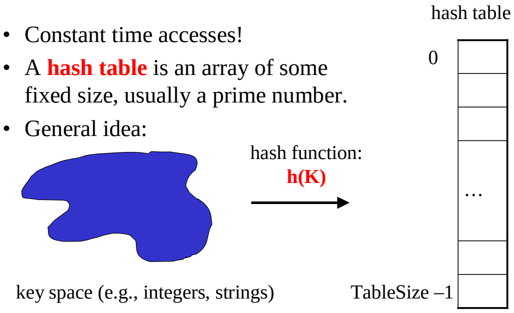
# Hashing

CSE 373
Data Structures and Algorithms

## Hash Tables

- Constant time accesses!
- A **hash table** is an array of some fixed size, usually a prime number.
- General idea:






## Example

- key space = integers
- TableSize = 10
- **h**(K) = K mod 10
- **Insert**: 7, 18, 41, 94

| | |
|---|---|
| **0** | |
| **1** | |
| **2** | |
| **3** | |
| **4** | |
| **5** | |
| **6** | |
| **7** | |
| **8** | |
| **9** | |



## Another Example

- key space = integers
- TableSize = 6
- **h**(K) = K mod 6
- **Insert**: 7, 18, 41, 34

| | |
|---|---|
| **0** | |
| **1** | |
| **2** | |
| **3** | |
| **4** | |
| **5** | |

Student Activity



## Hash Functions

1. **simple/fast** to compute,
2. Avoid **collisions**
3. have keys distributed **evenly** among cells.

Perfect Hash function:



## Sample Hash Functions:

- key space = strings
- $s = s_0\ s_1\ s_2 \ldots s_{k-1}$

1. $h(s) = s_0$ mod TableSize
2. $h(s) = \left(\sum_{i=0}^{k-1} s_i\right)$ mod TableSize
3. $h(s) = \left(\sum_{i=0}^{k-1} s_i \cdot 37^i\right)$ mod TableSize

## Designing a Hash Function for web URLs

s = $s_0\ s_1\ s_2 \ldots s_{k-1}$

Issues to take into account:

h(s) =

Student Activity



## Compare Hash Functions for web URLs

h(s) = $\left(\sum_{i=0}^{10} s.charAt(i)\right)$ mod TableSize

Pros:

Cons:

h(s) = $\left(\sum_{i=0}^{n} s.charAt(i)\cdot 39^{i}\right)$ mod TableSize

Pros:

Cons:



## Collision Resolution

**Collision**: when two keys map to the same location in the hash table.

Two ways to resolve collisions:

1. Separate Chaining
2. Open Addressing (linear probing, quadratic probing, double hashing)



## Separate Chaining

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

**Insert**:
10
22
107
12
42

- **Separate chaining**: All keys that map to the same hash value are kept in a list ("bucket").



## Analysis of find

- Defn: The load factor, $\lambda$, of a hash table is the ratio: $\frac{N}{M}$ ← no. of elements ← table size

For separate chaining, $\lambda$ = average # of elements in a bucket

- unsuccessful:
- successful:



## How big should the hash table be?

- For Separate Chaining:

## tableSize: Why Prime?

- Suppose
  - data stored in hash table: 7160, 493, 60, 55, 321, 900, 810
  - tableSize = 10
    data hashes to 0, 3, <u>0</u>, 5, 1, <u>0</u>, <u>0</u>
  - tableSize = 11
    data hashes to 10, 9, 5, 0, 2, <u>9</u>, 7

Real-life data tends to have a pattern

Being a multiple of 11 is usually *not* the pattern ☺



## Open Addressing

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

**Insert**:
38
19
8
109
10

- <u>**Linear Probing**</u>: after checking spot h(k), try spot h(k)+1, if that is full, try h(k)+2, then h(k)+3, etc.



## Terminology Alert!

"**Open** Hashing" equals "Separate Chaining"

"Closed Hashing" equals "**Open** Addressing"

Weiss



## Linear Probing

- f(i) = i
- Probe sequence:

  $0^{th}$ probe = h(k) mod TableSize
  $1^{th}$ probe = (h(k) + 1) mod TableSize
  $2^{th}$ probe = (h(k) + 2) mod TableSize
  . . .
  $i^{th}$ probe = (h(k) + i) mod TableSize



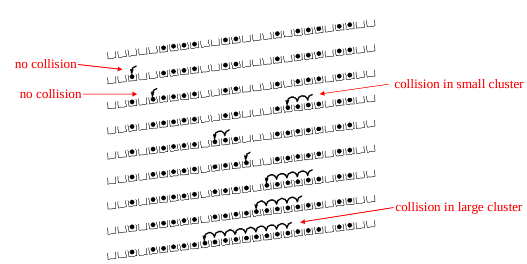
## Write pseudocode for find(k) for Open Addressing with linear probing

- Find(k) returns i where T(i) = k

Student Activity



## Linear Probing – Clustering




[R. Sedgewick]

## Load Factor in Linear Probing

- For *any* $\lambda < 1$, linear probing *will* find an empty slot
- Expected # of probes (for large table sizes)
  - successful search:

$$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$$

  - unsuccessful search:

$$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$$

- Linear probing suffers from ***primary clustering***
- Performance quickly degrades for $\lambda > 1/2$



## Quadratic Probing

Less likely to encounter Primary Clustering

$f(i) = i^2$

- Probe sequence:

  $0^{th}$ probe = h(k) mod TableSize
  $1^{th}$ probe = (h(k) + 1) mod TableSize
  $2^{th}$ probe = (h(k) + 4) mod TableSize
  $3^{th}$ probe = (h(k) + 9) mod TableSize
  . . .
  $i^{th}$ probe = (h(k) + $i^2$) mod TableSize



## Quadratic Probing

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

Insert:
89
18
49
58
79



## Quadratic Probing Example

insert(76) 76%7 = 6
insert(40) 40%7 = 5
insert(48) 48%7 = 6
insert(5) 5%7 = 5
insert(55) 55%7 = 6

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | 76 |

But… insert(47) 47%7 = 5



## Quadratic Probing: Success guarantee for $\lambda < ½$

- If size is prime and $\lambda < ½$, then quadratic probing will find an empty slot in size/2 probes or fewer.
  - show for all **0 ≤ i,j ≤ size/2** and **i ≠ j**

    **(h(x) + i²) mod size ≠ (h(x) + j²) mod size**

  - by contradiction: suppose that for some i ≠ j:

    **(h(x) + i²) mod size = (h(x) + j²) mod size**
    **⇒ i² mod size = j² mod size**
    **⇒ (i² - j²) mod size = 0**
    **⇒ [(i + j)(i - j)] mod size = 0**

  BUT size does not divide **(i-j)** or **(i+j)**



## Quadratic Probing: Properties

- For *any* $\lambda < ½$, quadratic probing will find an empty slot; for bigger $\lambda$, quadratic probing *may* find a slot
- Quadratic probing does not suffer from *primary* clustering: keys hashing to the same *area* are not bad
- But what about keys that hash to the same *spot*?
  - ***Secondary Clustering!***

## Quadratic Probing Works for $\lambda < 1/2$

- If HSize is prime then

$(h(x) + i^2) \bmod HSize \neq (h(x) + j^2) \bmod HSize$ for $i \neq j$ and $0 \leq i,j < HSize/2$.

- Proof

$(h(x) + i^2) \bmod HSize = (h(x) + j^2) \bmod HSize$

$(h(x) + i^2) - (h(x) + j^2) \bmod HSize = 0$

$(i^2 - j^2) \bmod HSize = 0$

$(i-j)(i+j) \bmod HSize = 0$

$\Rightarrow\Leftarrow$ HSize does not divide (i-j) or (i+j)



## Double Hashing

$f(i) = i * g(k)$

where g is a second hash function

- Probe sequence:

$0^{th}$ probe = h(k) mod TableSize

$1^{th}$ probe = (h(k) + g(k)) mod TableSize

$2^{th}$ probe = (h(k) + 2*g(k)) mod TableSize

$3^{th}$ probe = (h(k) + 3*g(k)) mod TableSize

. . .

$i^{th}$ probe = (h(k) + i*g(k)) mod TableSize



## Double Hashing Example

h(k) = k mod 7 and g(k) = 5 – (k mod 5)

| | 76 | 93 | 40 | 47 | 10 | 55 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | | | | 47 | 47 | 47 |
| 2 | | 93 | 93 | 93 | 93 | 93 |
| 3 | | | | | 10 | 10 |
| 4 | | | | | | 55 |
| 5 | | | 40 | 40 | 40 | 40 |
| 6 | 76 | 76 | 76 | 76 | 76 | 76 |
| Probes | 1 | 1 | 1 | 2 | 1 | 2 |



## Resolving Collisions with Double Hashing

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

Hash Functions:

$H(k) = k \bmod M$

$H_2(k) = 1 + ((k/M) \bmod (M-1))$

M =

**Insert these values into the hash table in this order. Resolve any collisions with double hashing:**

13

28

33

147

43



## Rehashing

**Idea:** When the table gets too full, create a bigger table (usually 2x as large) and hash all the items from the original table into the new table.

- When to rehash?
  - half full ($\lambda = 0.5$)
  - when an insertion fails
  - some other threshold
- Cost of rehashing?



## Hashing Summary

- Hashing is one of the most important data structures.
- Hashing has many applications where operations are limited to find, insert, and delete.
- Dynamic hash tables have good amortized complexity.